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IVT - Seminar

Speed or spacing? Cumulative variables, and convolution of model errors and time in traffic flow models validation and calibration

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Outlines

- Background and motivations
  1. Inadequacy of error statistics to measure the discrepancy between a model and the reality
  2. What measure of performance to adopt in calibration/validation
- Definitions
- Error propagation from a variable to its cumulative sum through *SAE* and *SSE*
- Physical interpretation
- Implications on model calibration and validation
- Case study on car-following model calibration
- Conclusions

Motivation (1)

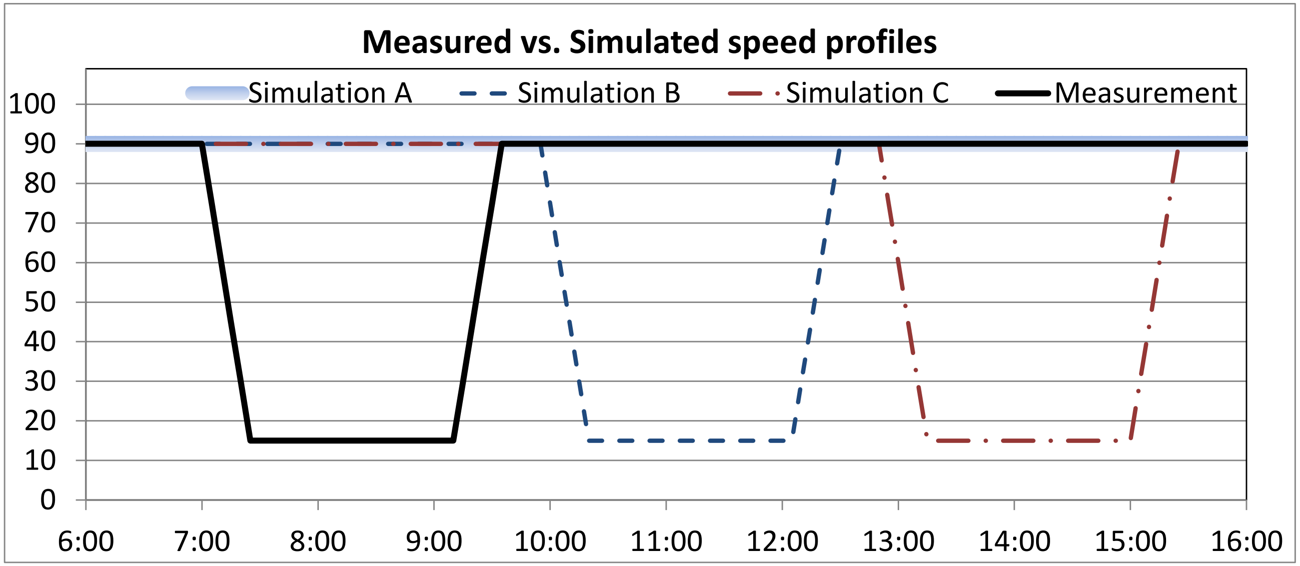
- The measures of **discrepancy** between a simulation and the real world are at the basis of scientific modelling:
  - ✓ model calibration and validation

- As for **dynamic** models, discrepancy is mainly measured on the interest variables **time-series**.

- Usual **global error statistics**: *MAE, RMSE, RMSPE, Theil's U, etc.*
  - ✓ *Hollander and Liu, 2008; Brackstone and Punzo, 2014; Buisson et al., 2014*

- Common assumption: the temporal evolution of model residuals and their features (e.g. autocorrelation) do not affect the error statistics

Motivation (1)



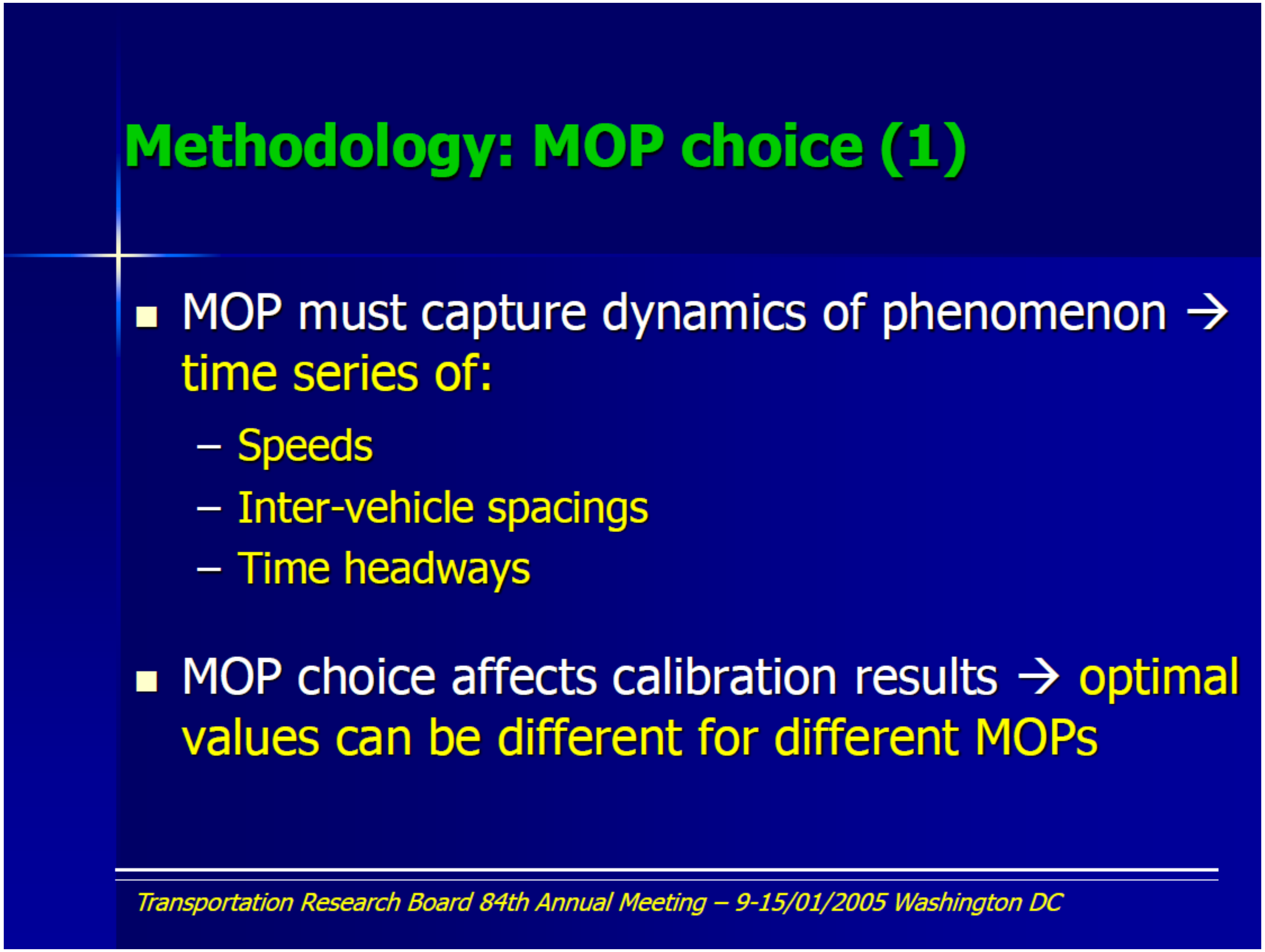
- Expectations about the ranking?
- Ranking according to any global error statistic (e.g. RMSE):

1. Simulation A — RMSE(A)=33.7
2. Simulation B ≡ Simulation C — RMSE(B)=RMSE(C)=47.6

Motivation (1) in the TFT literature

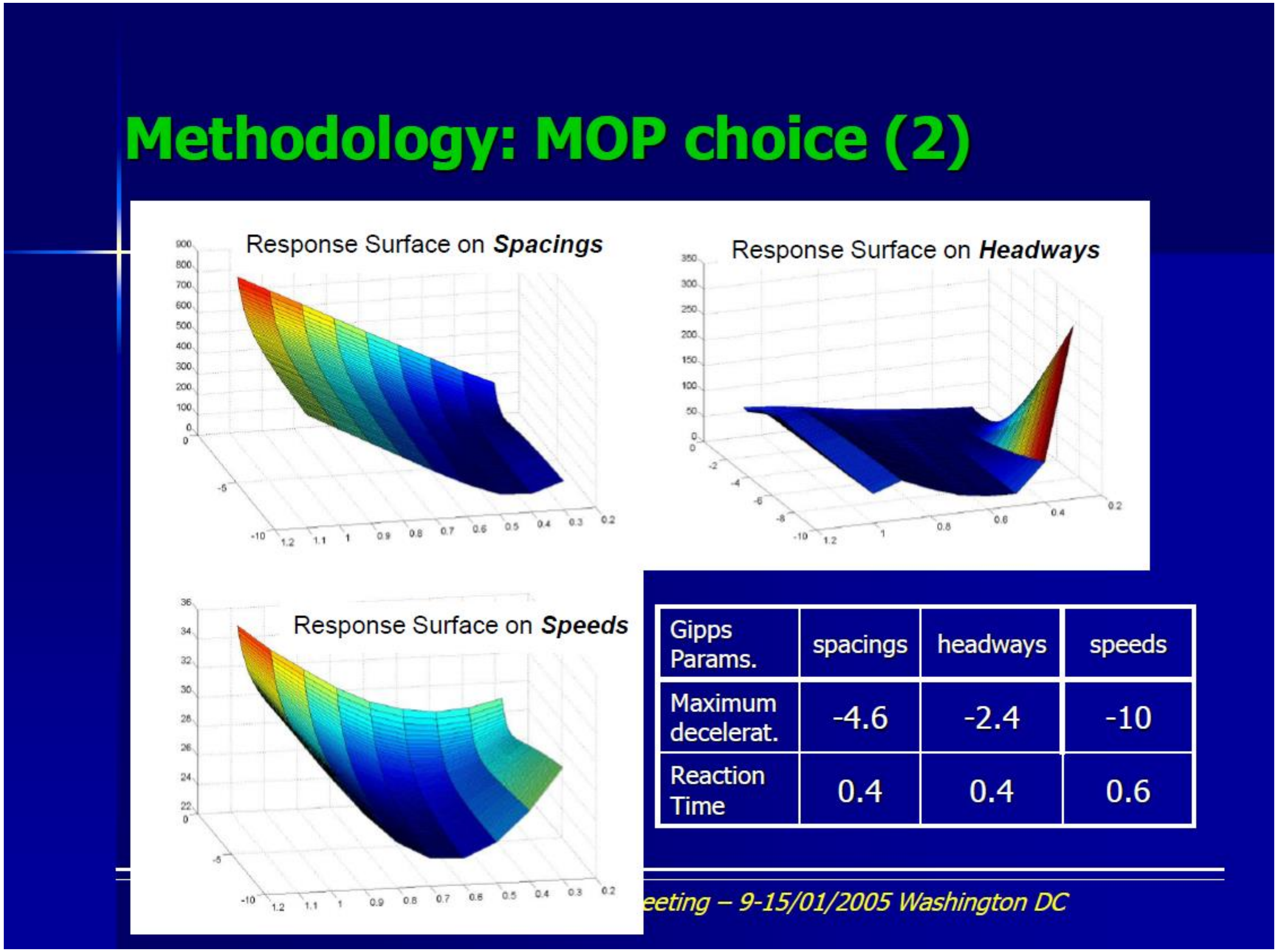
- ✓ Model residuals autocorrelation affects microscopic traffic flow model calibration *(Hoogendoorn and Hoogendoorn, 2010)*
- ✓ Frequency-domain statistics make the most of the information about residuals autocorrelation in time-series data *(Montanino et al., 2012)*

Motivation (2)



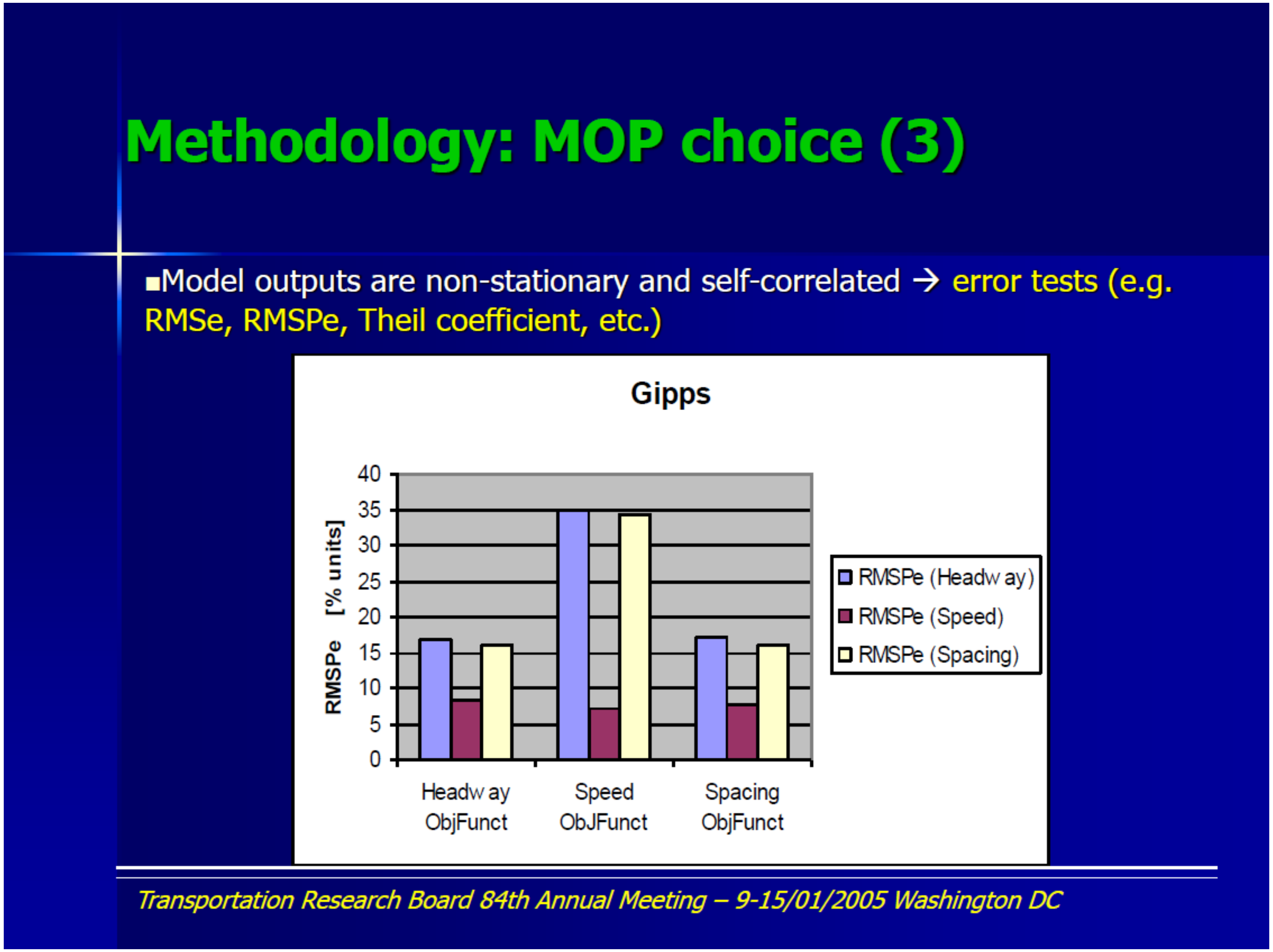
- Punzo and Simonelli, 2005, TRR

Motivation (2)



- Punzo and Simonelli, 2005, TRR

Motivation (2)



- Punzo and Simonelli, 2005, TRR

Motivation (2) in the TFT literature

- **Speed or spacing**? (or acceleration…)
- Many studies making use of one variable or the other, and some even of **their combination** (see e.g. Ossen and Hoogendoorn, 2008; Kim and Mahmassani, 2011).
- **No sound mathematical explanation** of results
- Ranjiktar et al. observed that calibration on speed provides values of error statistics after calibration lower than those from spacing
- Punzo and Simonelli (2005) claimed that spacing is preferable to speed and provided an intuitive explanation
- Ossen and Hoogendoorn (2007), showed that calibrating against speed some of the Gipps' model parameters cannot be estimated
- Kesting and Treiber (2008) also suggest the use of spacing and compare error measures (see also Hamdar et al., 2015).
- in the exploratory study of Punzo et al. (2012), the preference for spacing was confirmed through substantial empirical evidence.

Rationale (1)

- ✓ Model residuals autocorrelation affects microscopic traffic flow model calibration *(Hoogendoorn and Hoogendoorn, 2010)*

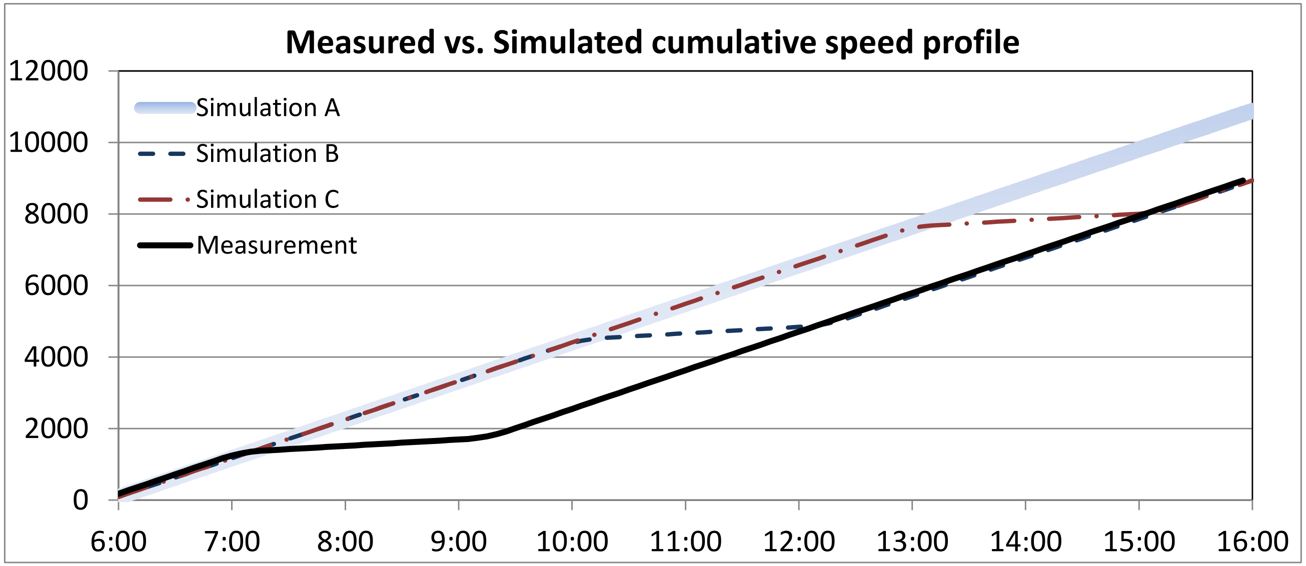
- ✓ Frequency-domain statistics make the most of the information about residuals autocorrelation in time-series data *(Montanino et al., 2012)*

- ✓ Feasible approach in the time-domain: assigning weights to the residuals depending on their occurrence time → **convolution of residuals and time**

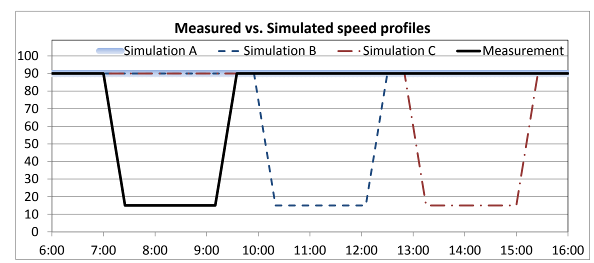
$$(f*g)(t)=\int_{-\infty}^{+\infty} f(\tau)g(t-\tau)\,d\tau=\int_{-\infty}^{+\infty} f(t-\tau)g(\tau)\,d\tau$$

$$(f*g)[h]=\sum_{k=1}^{N}[h-k]g[k] \qquad (f*g)[N]=\sum_{k=1}^{N}[N-k]g[k]$$

Rationale (2)



- if we calculate discrepancy measures of the cumulative speed profiles A, B and C we obtain the ‘right’ ranking!
- Ranking according to e.g. RMSE:
  1. Simulation B RMSE(B)=907
  2. Simulation C RMSE(C)=1387
  3. Simulation A RMSE(A)=1673



Definitions (1)

- Given a time-discrete representation of a variable $z(t)$:

$$z[k] \overset{def.}{=} z(k \cdot \Delta t) \quad \forall k \in \{0,\dots,N\}$$

- we define a time-discrete variable $\mathbf{y}[\boldsymbol{k}]$ as the **cumulative sum** of **z** until $\boldsymbol{k}$:

$$y[k] = \sum_{i=0}^{k} z[i]$$

- z_k^{obs} and z_k^{sim} are, respectively, the real world observation and the simulated value of the variable z in k;

- We define the model error or residual on the variables z_k and y_k as:

$$\varepsilon_k^Z = z_k^{sim} - z_k^{obs} \qquad \varepsilon_k^Y = y_k^{sim} - y_k^{obs}$$

Definitions (2)

- Assuming that a simulation starts at time k=0, with $\varepsilon_0^z = \varepsilon_0^y = 0$ the model error evolution for the variables z_k and y_k can be derived recursively as follows:

$$\begin{cases} z_1^{sim} = z_1^{obs} + \varepsilon_1^Z \\ y_1^{sim} = y_0^{sim} + z_1^{sim} = y_0^{obs} + z_1^{obs} + \varepsilon_1^Z = y_1^{obs} + \varepsilon_1^Z \end{cases}$$

$$\begin{cases} z_2^{sim} = z_2^{obs} + \varepsilon_2^Z \\ y_2^{sim} = y_1^{sim} + z_2^{sim} = y_0^{obs} + z_1^{obs} + z_2^{obs} + \varepsilon_1^Z + \varepsilon_2^Z = y_2^{obs} + \sum_{i=1}^{2} \varepsilon_i^Z \end{cases}$$

$$\begin{cases} \ldots \\ \ldots \end{cases}$$

$$\begin{cases} z_k^{sim} = z_k^{obs} + \varepsilon_k^Z \\ y_k^{sim} = y_k^{obs} + \sum_{i=1}^{k} \varepsilon_i^Z \end{cases}$$

$$\varepsilon_k^Y = y_k^{sim} - y_k^{obs} \Rightarrow \boxed{\varepsilon_k^Y = \sum_{i=1}^{k} \varepsilon_i^Z}$$

Objective

- ✓ As a discrepancy measure, let's first assume the *Sum of the Absolute Errors*, *SAE,* that for our variables is expressed as:

$$SAE^Z = \sum_{k=1}^{N} \left| \epsilon_k^Z \right| \qquad SAE^Y = \sum_{k=1}^{N} \left| \epsilon_k^Y \right|$$

- ✓ We aim to derive a general **model for the propagation of model errors** from a variable z_k to its cumulative y_k, through the *Sum of the Absolute Errors.*

Error propagation through *SAE* (1)

- To this aim, the SAE^Y can be expressed as a function of z_k through a recursive application of:

$$\varepsilon_k^Y = \sum_{i=1}^{k} \varepsilon_i^Z$$

- That is:

$$\left|\varepsilon_1^Y\right| = \left|\varepsilon_1^Z\right|$$

$$\left|\varepsilon_2^Y\right| = \left|\varepsilon_1^Z + \varepsilon_2^Z\right|$$

$$\left|\varepsilon_3^Y\right| = \left|\varepsilon_1^Z + \varepsilon_2^Z + \varepsilon_3^Z\right|$$

...

$$\left|\varepsilon_k^Y\right| = \left|\sum_{i=1}^{k} \varepsilon_i^Z\right|$$

As: $\left|\sum_{i=1}^{k} \varepsilon_i^Z\right| \le \sum_{i=1}^{k} \left|\varepsilon_i^Z\right|$ $\Rightarrow$

$$\left|\varepsilon_1^Y\right| \le \left|\varepsilon_1^Z\right|$$

$$\left|\varepsilon_2^Y\right| \le \left|\varepsilon_1^Z\right| + \left|\varepsilon_2^Z\right|$$

$$\left|\varepsilon_3^Y\right| \le \left|\varepsilon_1^Z\right| + \left|\varepsilon_2^Z\right| + \left|\varepsilon_3^Z\right|$$

...

$$\left|\varepsilon_k^Y\right| \le \sum_{i=1}^{k} \left|\varepsilon_i^Z\right|$$

- Summing up the left- and the right-hand-side terms and rearranging:

$$SAE^Y \le \sum_{k=1}^{N} (N - k + 1) \cdot \left|\varepsilon_k^Z\right| = \sum_{k=1}^{N} \left|\varepsilon_k^Z\right| + \sum_{k=1}^{N} (N - k) \cdot \left|\varepsilon_k^Z\right|$$

Error propagation through *SAE* (2)

$$SAE^{Y} \leq \sum_{k=1}^{N} (N-k+1) \cdot \left|\varepsilon_k^Z\right| = \sum_{k=1}^{N} \left|\varepsilon_k^Z\right| + \sum_{k=1}^{N} (N-k) \cdot \left|\varepsilon_k^Z\right|$$

- ✓ Recalling the definition of SAE^Z and the one of convolution:

$$SAE^{Y} \leq SAE^{Z} + \left(k * \left|\varepsilon_k^Z\right|\right)[N]$$

- ✓ is the sought relationship between the error statistic calculated on the variable *z*, SAE^Z, and the same statistic calculated on its cumulative variable *y*, SAE^Y

Error propagation through *SSE* (1)

- ✓ Let's assume the *Sum of the Squared Errors*, *SSE, instead*:

$$SSE^Z = \sum_{k=1}^{N} \left(\epsilon_k^Z\right)^2 \qquad SSE^Y = \sum_{k=1}^{N} \left(\epsilon_k^Y\right)^2$$

- ✓ Following the same steps as before:

$$\begin{aligned}
\left(\varepsilon_1^Y\right)^2 &= \left(\varepsilon_1^Z\right)^2 &&= \left(\varepsilon_1^Z\right)^2 \\
\left(\varepsilon_2^Y\right)^2 &= \left(\varepsilon_1^Z + \varepsilon_2^Z\right)^2 &&= \left(\varepsilon_1^Z\right)^2 + \left(\varepsilon_2^Z\right)^2 + 2\left(\varepsilon_1^Z\varepsilon_2^Z\right) \\
\left(\varepsilon_3^Y\right)^2 &= \left(\varepsilon_1^Z + \varepsilon_2^Z + \varepsilon_3^Z\right)^2 &&= \left(\varepsilon_1^Z\right)^2 + \left(\varepsilon_2^Z\right)^2 + \left(\varepsilon_3^Z\right)^2 + 2\left(\varepsilon_1^Z\varepsilon_2^Z\right) + 2\left(\varepsilon_1^Z\varepsilon_3^Z\right) + 2\left(\varepsilon_2^Z\varepsilon_3^Z\right) \\
\left(\varepsilon_4^Y\right)^2 &= \left(\varepsilon_1^Z + \varepsilon_2^Z + \varepsilon_3^Z + \varepsilon_4^Z\right)^2 &&= \left(\varepsilon_1^Z\right)^2 + \left(\varepsilon_2^Z\right)^2 + \left(\varepsilon_3^Z\right)^2 + \left(\varepsilon_4^Z\right)^2 + 2\left(\varepsilon_1^Z\varepsilon_2^Z\right) + 2\left(\varepsilon_1^Z\varepsilon_3^Z\right) + 2\left(\varepsilon_1^Z\varepsilon_4^Z\right) + 2\left(\varepsilon_2^Z\varepsilon_3^Z\right) + 2\left(\varepsilon_2^Z\varepsilon_4^Z\right) + 2\left(\varepsilon_3^Z\varepsilon_4^Z\right) \\
&\cdots \\
\left(\varepsilon_k^Y\right)^2 &= \cdots \\
&\cdots
\end{aligned}$$

- ✓ Summing up the terms on the two sides and rearranging:

$$\begin{aligned}
SSE^Y &= \sum_{k=1}^{N} (N-k+1)\cdot\left(\varepsilon_k^Z\right)^2 + 2\sum_{k=1}^{N-1}\sum_{i=k+1}^{N}(N-i+1)\cdot\left(\varepsilon_k^Z\cdot\varepsilon_i^Z\right) \\
&= \sum_{k=1}^{N}\left(\varepsilon_k^Z\right)^2 + \sum_{k=1}^{N}(N-k)\cdot\left(\varepsilon_k^Z\right)^2 + 2\sum_{k=1}^{N-1}\varepsilon_k^Z\cdot\sum_{i=k+1}^{N}(N-i+1)\cdot\varepsilon_i^Z
\end{aligned}$$

Error propagation through *SSE* (2)

$$SSE^{Y} = \sum_{k=1}^{N} (N-k+1) \cdot \left(\varepsilon_k^Z\right)^2 + 2\sum_{k=1}^{N-1} \sum_{i=k+1}^{N} (N-i+1) \cdot \left(\varepsilon_k^Z \cdot \varepsilon_i^Z\right)$$

$$= \sum_{k=1}^{N} \left(\varepsilon_k^Z\right)^2 + \sum_{k=1}^{N} (N-k) \cdot \left(\varepsilon_k^Z\right)^2 + 2\sum_{k=1}^{N-1} \varepsilon_k^Z \cdot \sum_{i=k+1}^{N} (N-i+1) \cdot \varepsilon_i^Z$$

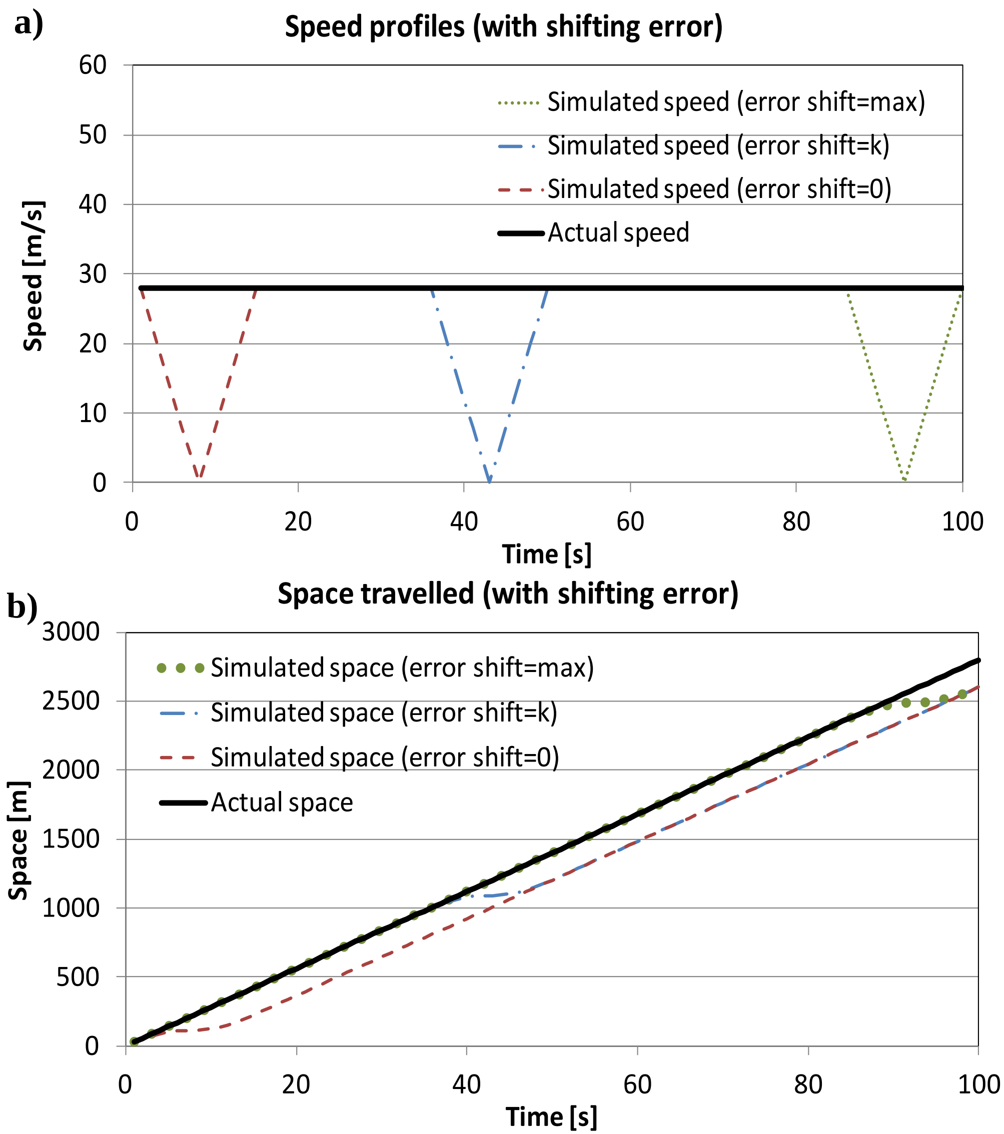
- Then, recalling the definition of the SSE^Z and the one of convolution, we can express the SSE^Y as a function of ε_k^Z and k:

$$SSE^{Y} = SSE^{Z} + \left(k * \left(\varepsilon_k^Z\right)^2\right)[N] + 2\sum_{k=1}^{N-1} \varepsilon_k^Z \cdot \sum_{i=k+1}^{N} (N-i+1) \cdot \varepsilon_i^Z$$

Physical interpretation (1)

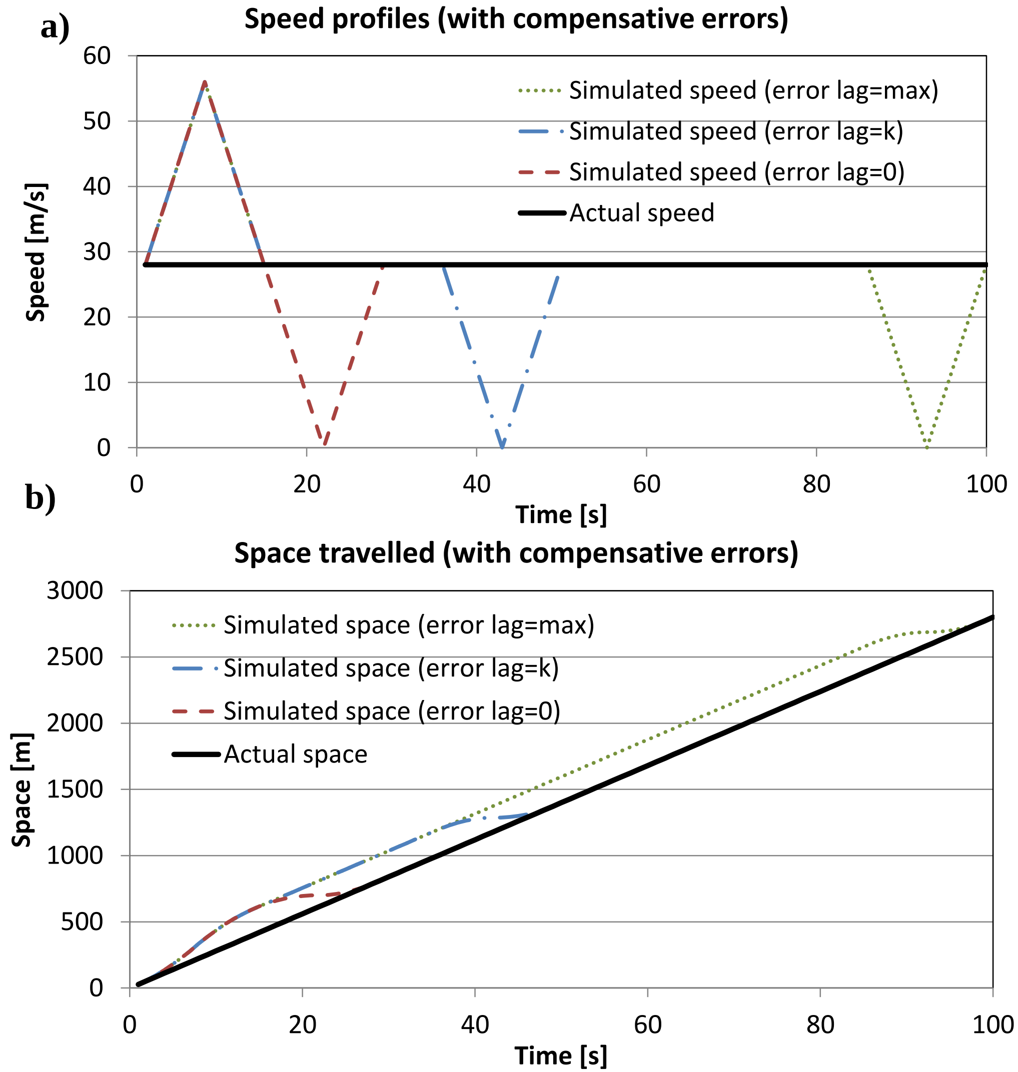
- a) shows the **actual speed profile** (*black solid line*) and **three simulated profiles** (*dashed/dotted lines*) that present an error that is the same as for magnitude but shifted in time;

- b) shows the four corresponding integral functions that is the travelled spaces



Physical interpretation (2)
'compensatory errors'

- Speed profiles and space travelled with positive and negative residuals of equal magnitude.
- a) shows **three simulated speed profiles** with different lag widths between the two compensatory errors (dashed/dotted lines) and the **actual speed profile** (black solid line);
- b) shows the corresponding travelled spaces.



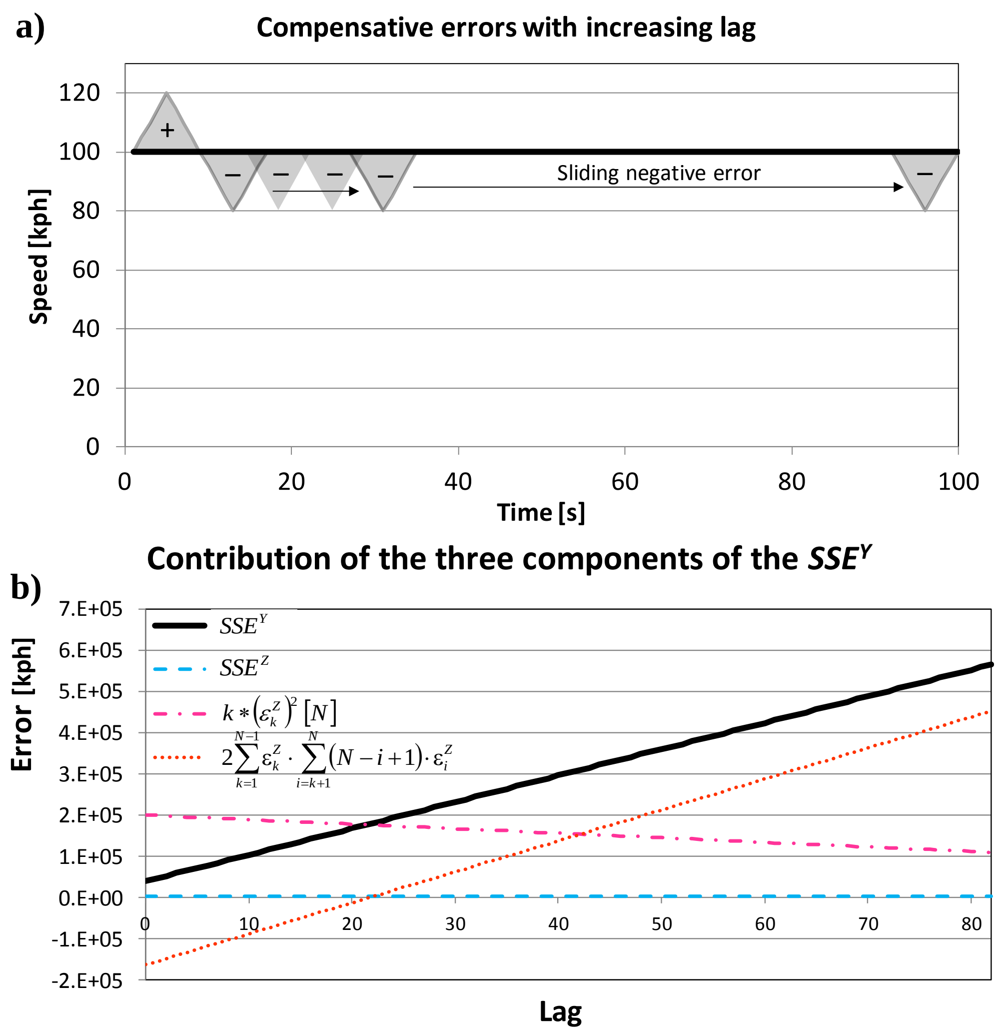
Physical interpretation (3)

- ✓ Negative residual drifting apart from the positive one

SSE^Z is **insensitive to the lag width**

$(N-\kappa)*\left(\varepsilon^Z[\kappa]\right)^2$ **weights more the early errors than the late ones** (i.e. convolution term)

$2\sum_{k=1}^{N-1}\varepsilon_k^Z \cdot \sum_{i=k+1}^{N}(N-i+1)\cdot\varepsilon_i^Z$ **sharply increases with lag**



Specification for car-following models (1)

- Let's assume:
  - $z_k = v_k =$ **speed** of the follower vehicle at time k
  - $y_k = x_k =$ follower vehicle **trajectory** i.e. **space travelled** at time k
- Given a general CF model *(Wilson, 2008)*:

$$a_k = f\left(s_k, v_k, \Delta v_k, \beta\right)$$

- CF model are usually solved with simple integration schemes *(Treiber and Kanagaraj, 2015)*:

$$\begin{cases} v_{k+1} = v_k + a_k \cdot \Delta t & & \text{(a)} \\ x_{k+1} = x_k + v_{k+1} \cdot \Delta t & \text{Eulerian} & \text{(b)} \\ or & & \\ x_{k+1} = x_k + \dfrac{v_{k+1} + v_k}{2} \cdot \Delta t & \text{Ballistic} & \text{(c)} \end{cases}$$

Specification for car-following models (2)

- ✓ Following the same steps as before we obtain:
- Forward Euler ➔ $$\varepsilon_k^X = \Delta t \cdot \sum_{i=1}^{k} \varepsilon_i^V$$
- Ballistic update ➔ $$\varepsilon_k^X = \Delta t \cdot \left(\sum_{i=1}^{k-1} \varepsilon_i^V + \frac{\varepsilon_k^V}{2} \right)$$
- ✓ That yield similar relationships as before:
- Euler ➔ $$SSE^X = \Delta t^2 \cdot \left[SSE^V + \left(k * \left(\varepsilon_k^Z\right)^2\right)[N] + 2\sum_{k=1}^{N-1} \varepsilon_k^V \cdot \sum_{i=k+1}^{N} (N - i + 1) \cdot \varepsilon_i^V \right]$$
- Ballistic ➔ $$SSE^X = \Delta t^2 \cdot \left[\frac{SSE^V}{4} + \left(k * \left(\varepsilon_k^Z\right)^2\right)[N] + 2\sum_{k=1}^{N-1} \varepsilon_k^V \cdot \sum_{i=k+1}^{N} \left(N - i + \frac{1}{2} \right) \cdot \varepsilon_i^V \right]$$

Specification for macroscopic traffic flow models

- Let's assume:
- $\boldsymbol{z_k} = \boldsymbol{flow_k}$ = **flow rate** in interval k
- $\boldsymbol{y_k} = \boldsymbol{cum_k}$ = **cumulative number of vehicles** arrived until k
- We have:

$$Y = cum = \sum_{i=1}^{k} \left(flow_i \cdot \Delta t \right)$$

- Same as the forward Euler integration scheme
- The relationship between SSE^{cum} and SSE^{flow} is the same as the one between SSE^{X} and SSE^{V}
- In a single link, at each instant, the **sum of cumulative inflows and outflows** gives the number of vehicles accumulated in the link at that instant, i.e. the **link density**

Implications on model validation

- ✓ If a dynamic model is to be validated against a specific variable, it is **preferable** to calculate the statistic on the **cumulative of the variable** itself
- ✓ In this way model residuals dynamics are taken into account
- ✓ In case of car-following, validating models on the errors made on the **space travelled** implicitly takes into account the **speed errors too**, while the opposite is not true
- ✓ **Density** is more meaningful then **flows** (note: if a direct measurement of density is not available, as usual, cumulative flows may suffer from accumulation of measurement errors)
- ✓ two simulated flow profiles returning the same value for an error statistic might depict completely different evolutions of the traffic density on a link.

Implications on model calibration

- Calibrating a car-following model against **speed** or **spacing**, using the *Sum of Squared Errors* as measure of discrepancy, means minimizing the following objective functions, respectively:

$$\min\left\{SSE^{V}\right\}$$

$$\min\left\{SSE^{X}\right\} = \min\left\{SSE^{V} + \left(k * \left(\varepsilon_k^{Z}\right)^{2}\right)[N] + 2\sum_{k=1}^{N-1}\varepsilon_k^{V} \cdot \sum_{i=k+1}^{N}(N-i+1)\cdot\varepsilon_i^{V}\right\}$$

- Calibration on **speed**: optimal for speed and **indeterminate for spacing**

- Calibration on **spacing**: optimal for spacing and **suboptimal for speed**

Case study: car-following calibration (1)

- **Cross-validation** (Punzo and Simonelli, 2005):
  As either optimality is reached on speed or on space we can (only) evaluate the robustness of calibration results on one variable with respect to the other variable:

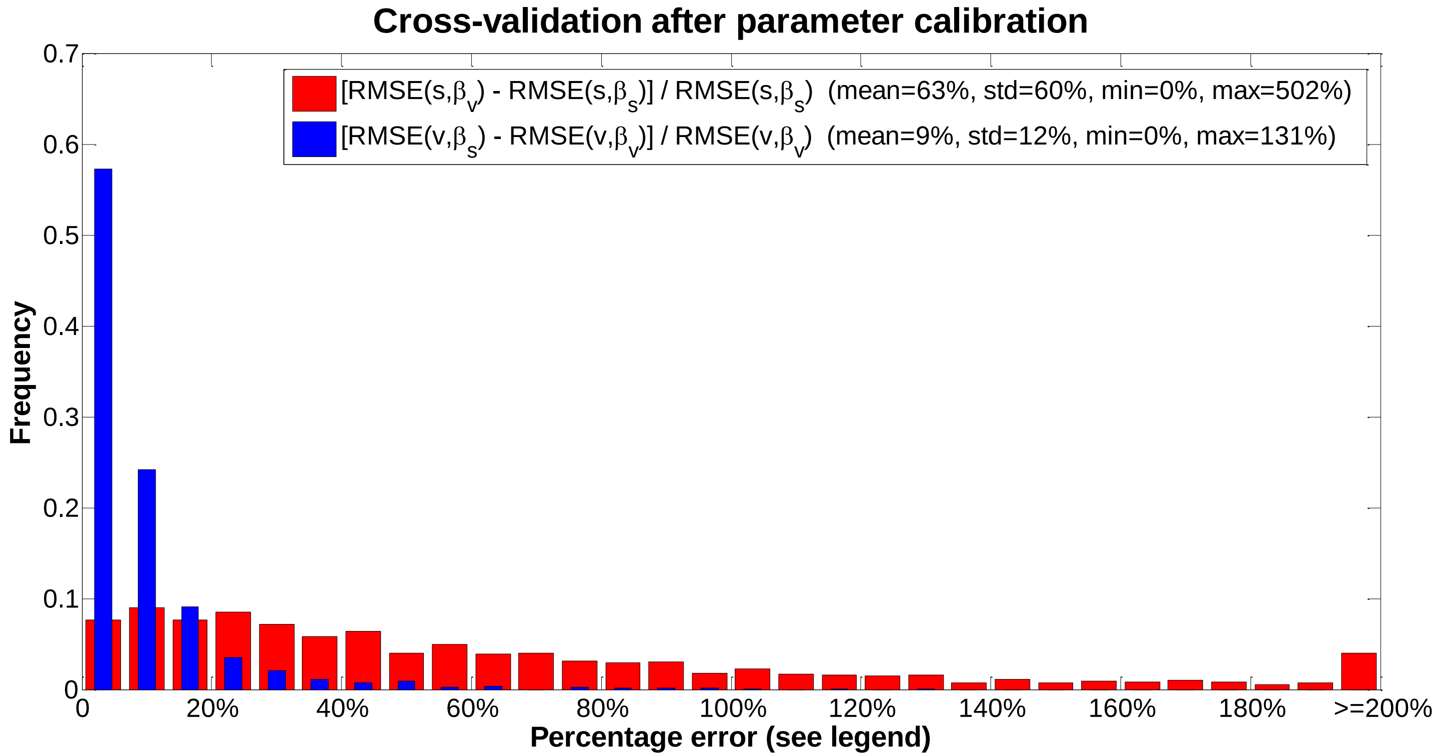
$$\frac{RMSE(s,\beta_V)-RMSE(s,\beta_S)}{RMSE(s,\beta_S)} \quad \text{(a)}$$

$$\frac{RMSE(v,\beta_S)-RMSE(v,\beta_V)}{RMSE(v,\beta_V)} \quad \text{(b)}$$

where $RMSE(i,\beta_j)$, with $i, j \in \{s, v\}$, is the Root Mean Square Error of the variable *i* obtained by running the model with the optimal parameters calibrated against the variable *j*.

Case study: car-following calibration (2)

✓ Calibration of the Intelligent Driver Model (IDM) (Treiber et al., 2000) against both speed and inter-vehicle spacing for all the 2037 trajectories in the I80-1 'reconstructed' NGSIM dataset (Montanino and Punzo, 2015):



Conclusions

- ✓ Global error statistics keep no memory of the occurrence times of model errors, or of their order
- ✓ this weakness may be solved by considering the convolution of model errors and time
- ✓ a convolution of this kind can be achieved by replacing a time-discrete variable by its cumulative in statistics as the *SAE* or the *SSE*
- ✓ a general model for the propagation of model residuals in the above error statistics from a variable to its cumulative has been developed
- ✓ The model yields mathematical relationships between the above error statistics applied to a variable and the same statistics applied to the cumulative of the variable itself
- ✓ the obtained relationships are model-independent
- ✓ *If direct measurements of a cumulative variable are available (i.e. no measurement errors accumulation), in general, it is more meaningful to validate and calibrate against this variable than on its derivative*

Main references

V. Punzo, M. Montanino, 2016. Speed or spacing? Cumulative variables, and convolution of model errors and time in traffic flow models validation and calibration. *Transportation Research Part B 91, 21-33*.

M. Montanino, V. Punzo, 2015. Trajectory data reconstruction and simulation-based validation against macroscopic traffic patterns. *Transportation Research Part B 80, 82-106*.

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MULTITUDE. COST Action TU0903 - Methods and tools for supporting the Use caLibration and validaTIon of Traffic simUlation moDEls. www.multitude-project.eu.

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