Incorporating and valuing induced demand within the official UK cost-benefit analysis framework

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Basic UK Approach

Basis of Cost-Benefit Analysis (CBA) using the change in Consumer Surplus (ΔCS) established in the UK around 1970

In most circumstances ΔCS may be approximated by the ”Rule of a Half”:

$$\Delta CS \approx -\tfrac{1}{2} \Sigma_{ij} (T'_{ij} + T_{ij})(C'_{ij} - C_{ij})$$

where T is travel (between i and j), C is (generalised) cost, and the prime (’) denotes the ”after” position (”Do-Something”).

It has been used predominantly for evaluating capital infrastructure in urban studies and inter-urban highways

Simplifications adopted

For various institutional and political reasons, work on UK urban studies declined after the mid 1970s

Highway Investment continued to use CBA but with the key simplification that travel demand was fixed, ie

$T'_{ij} = T_{ij}$

Hence, $\Delta CS = -\Sigma_{ij} T_{ij} (C'_{ij} - C_{ij})$

Simplifications adopted

This conveniently allows highway benefits to be calculated on a **link** basis:

$$\Delta CS = -\Sigma_a (V'_a \, c'_a - V_a \, c_a)$$

where V is flow on link a, and c is the link cost.

Evaluation software was focussed on link formulae

Challenges

By 1990, the assumption of fixed demand for interurban highway was being seriously challenged.

However, UK Dept. of Transport resisted change because:

- modelling changes in demand would be difficult in practice
- no easy way to modify link-based software

Change

Position finally changed with the publication of the influential SACTRA Committee report on ”Trunk Roads and the Generation of Traffic” 1994:

(¶15.24) *We recommend that variable matrix economic evaluations are undertaken for schemes as the cornerstone of the economic appraisal in every case, except where it can be shown that the trip matrix will not vary as a result of the scheme being appraised*

More recently, there has been a (long delayed) re-awakening of interest in ”multi-modal evaluation”

At the same time, there have been new guidelines ”New Approach to Appraisal”, though in terms of **methodology** these are largely cosmetic.

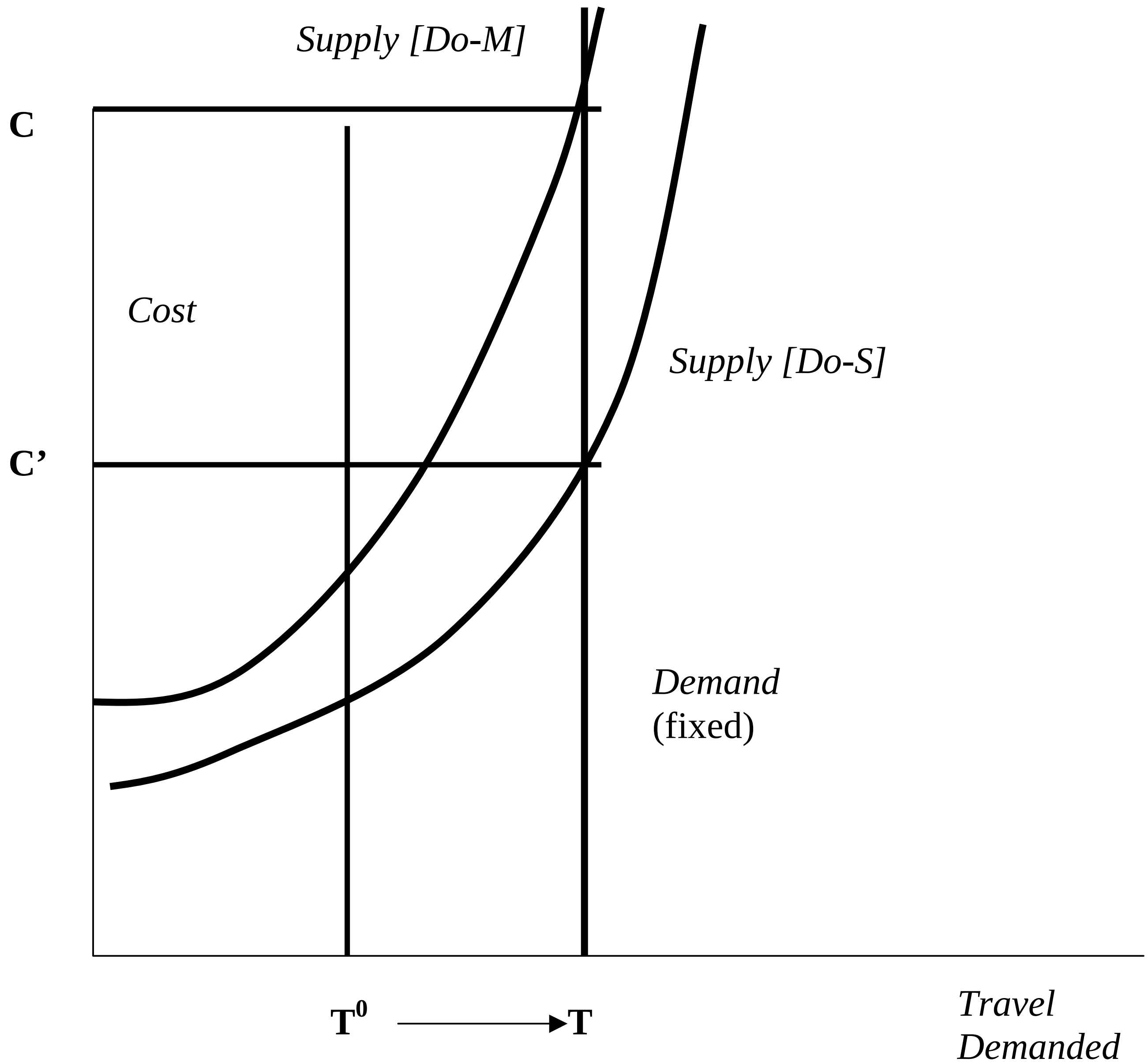
Current issues

Moreover, although many of the principles had been ”forgotten”, the shift to non-fixed Demand does not actually raise any new issues for the calculation of ΔCS

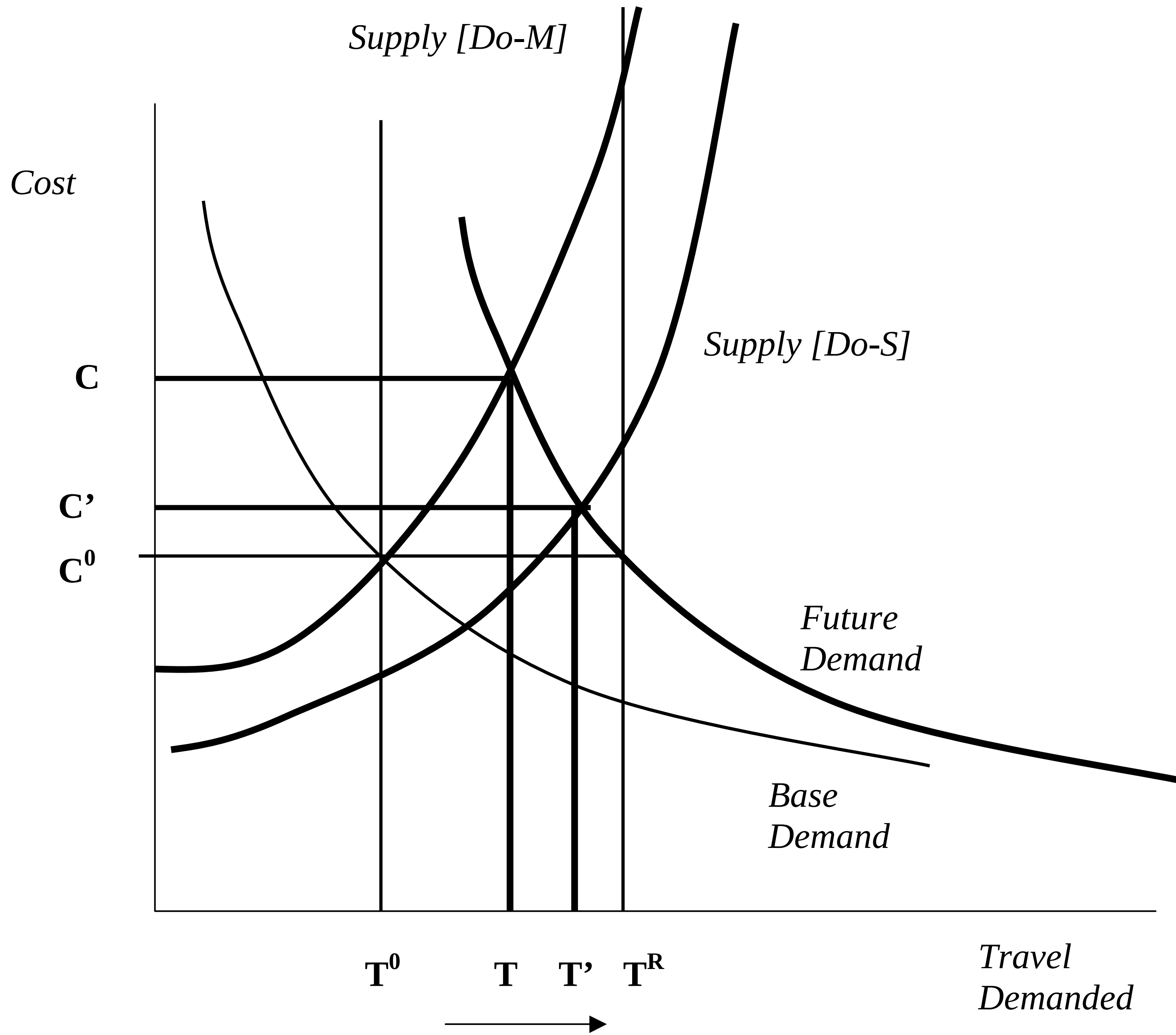
The linear approximation remains valid in most cases

The problems are all concentrated in obtaining a satisfactory equilibrium between Supply and Demand, for both Do-Minimum (DM) and Do-Something (DS)

Old assumptions (Exogenous growth from T^0 to T)



New assumptions (Exogenous growth from T^0 to T’)



Computational issues

First, we must select a functional form for the demand curve

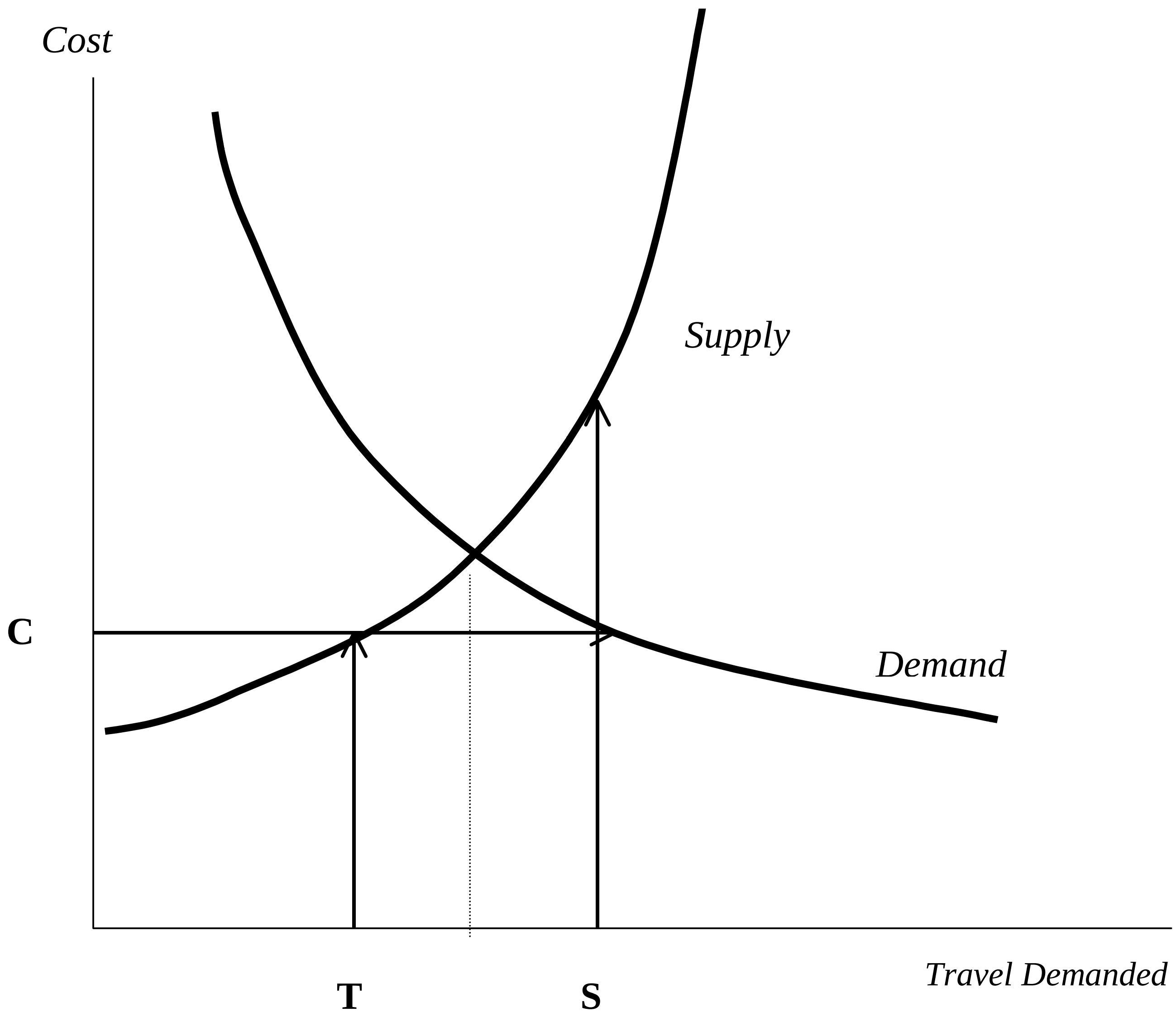
Assuming a base year demand in equilibrium, we must allow for exogenous growth **assuming unchanged costs**

Then we find the equilibrium separately for DM and DS

This involves significant computational problems

The measure of ΔCS is highly sensitive to the accuracy of the equilibrium

Computational issues



Possible solution algorithms

Algorithm	Computing effort/iteration	Number of iterations
Cobweb		
Averaging Schemes		
Optimisation		

Implementation issues

There remain concerns and uncertainties about the form of the Demand curve

The modelling of exogenous growth in a multi-modal context is not fully developed

Limited availability of suitable techniques for obtaining equilibrium

Limited understanding of level of convergence necessary for reliable ΔCS

Development of an algorithm

Key questions:

- efficiency
- definition of objective function (VDM)
- tractability (programming)
- generality
- availability

Basic approach

To solve complicated systems of equations we can often formulate an optimisation problem which by design has the same solution.

This allows us to use constrained optimisation techniques

Beckmann *et al* (1956) proposed an ‘objective function’ (to be minimised) consistent with Wardrop equilibrium

Equilibrium point (T^*,C^*) can be obtained by maximising the area between demand and supply curves.

Basic approach

Taking the negative, we have

$$\min \; z(T) = \int_0^T [\, f_S(W) - f_D^{-1}(W)\,]\, dW$$

where $f_D^{-1}(T)$ is the inverse demand curve, representing the price C at which the demand would reach level T.

Subject to certain conditions, this one-dimensional approach can be extended to the case where the elements T, f and W are matrices.

We therefore focus on how to evaluate the demand and supply integrals

Basic approach: Supply curve

The supply function gives matrix of O-D costs associated with demand T.

For the "all or nothing" case, we define

$$\varepsilon_{ija} = \begin{cases} 1 & \text{,if the trip from i to j uses link a,} \\ 0 & \text{otherwise.} \end{cases}$$

With congested assignment and multiple routes, ε gives the proportion of i-j movements using link a.

We treat ε as an $[a \bullet ij]$ matrix: (interface between Demand and Supply)

Basic approach: Supply curve

"Assignment" subsumes a number of “modules”:

- definition of the set of paths for each i-j movement
- choice between paths, yielding ε.
- loads on links (matrix product $V = \varepsilon . T$).
- capacity restraint - adjust link speeds
- true supply process: link costs $c = f(V)$
- skim O-D costs (matrix product $C = \varepsilon^T . c$).

where V and c are link-based vectors

“supply curve” is outcome of a series of assignments of different matrices T, each yielding a corresponding cost matrix C

Basic approach: Supply curve

We confine ourselves to "standard" case of link costs independence

The supply integral is the line integral

$$\int_{\mathbf{0}}^{\mathbf{T}} \mathbf{C}_s^T(\mathbf{w}).d\mathbf{w} \quad = \quad \int_{\mathbf{0}}^{\mathbf{T}} [\mathbf{c}^T(\mathbf{w}).\boldsymbol{\varepsilon}].d\mathbf{w}$$

changing the variable of integration from T to V, we have:

$$\int_{\mathbf{0}}^{\mathbf{V}} \mathbf{c}^T(\boldsymbol{\omega}).d\boldsymbol{\omega} \quad = \quad \sum_a \int_0^{V_a} c_a(x) \; dx$$

for separable link cost functions

This is identical to the integral used in the fixed demand case.

Basic approach: Demand function

Starting point

- All forms of demand function in current Guidance have property of ‘separability’ which means that demand for the highway movement i-j depends <u>only</u> the highway cost i-j

- This restriction defines ‘simple elasticity model’

- Some assignment packages include efficient algorithms for equilibrium with separable demand (SATEASY, EMME/2)

Basic approach: Demand function

Review of more complicated demand functions,within general sphere of random utility models :

- Constrained distribution models (Evans, 1976)
- Chapter 10 in Ortúzar & Willumsen (1994)
- Oppenheim (1995)

Demand integral can be represented in closed form at least for any hierarchical demand model based on a logit formulation with “utility” linear in generalised cost

Basic approach: Demand function

In all cases, the demand integral =

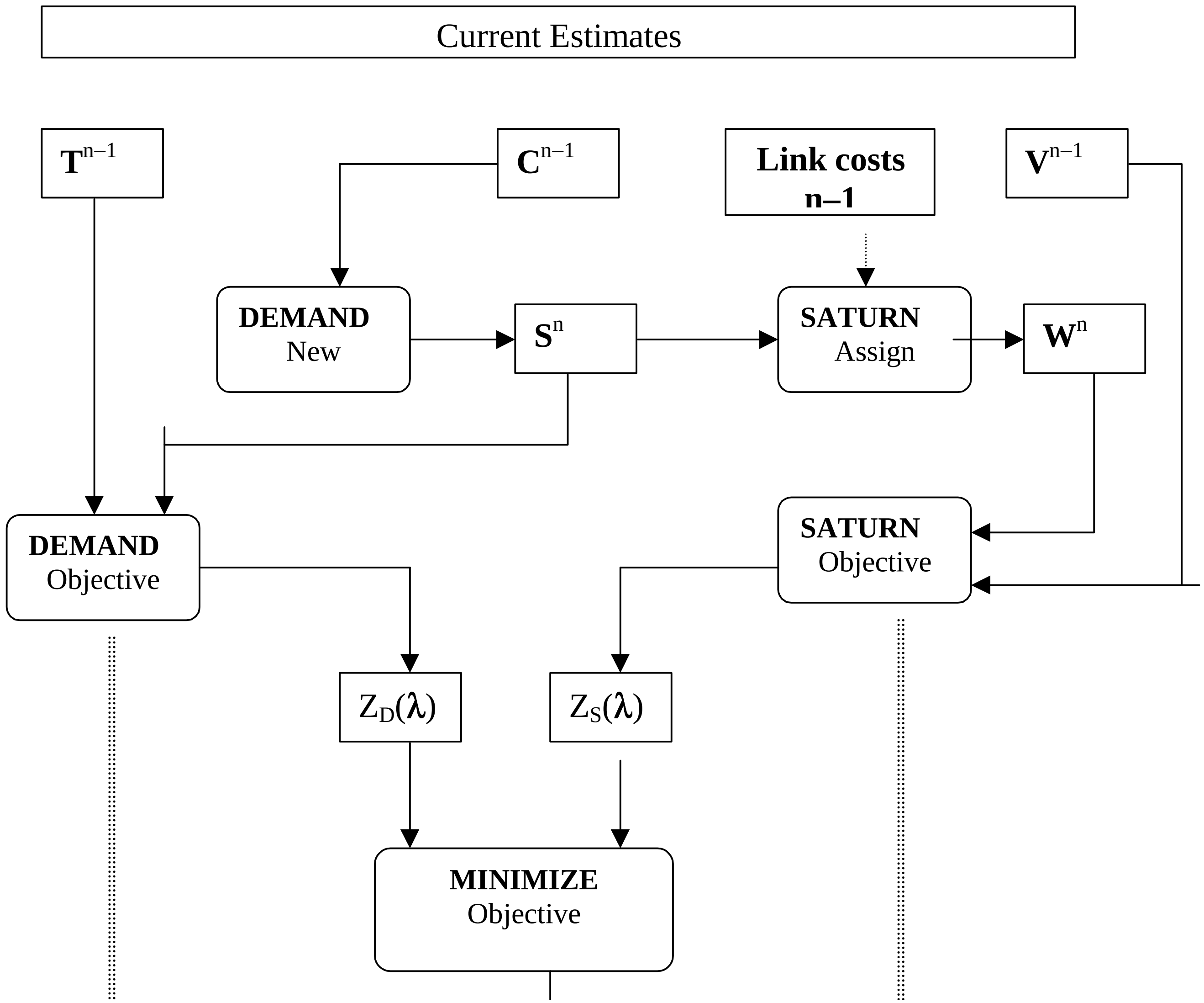
consumer surplus +

total consumption (in demand cost terms) -

correction for "residual consumption"

Hence it should be possible to extend equilibrium assignment packages to handle some or all of the stages of demand modelling.

Proposed algorithm



Proposed algorithm