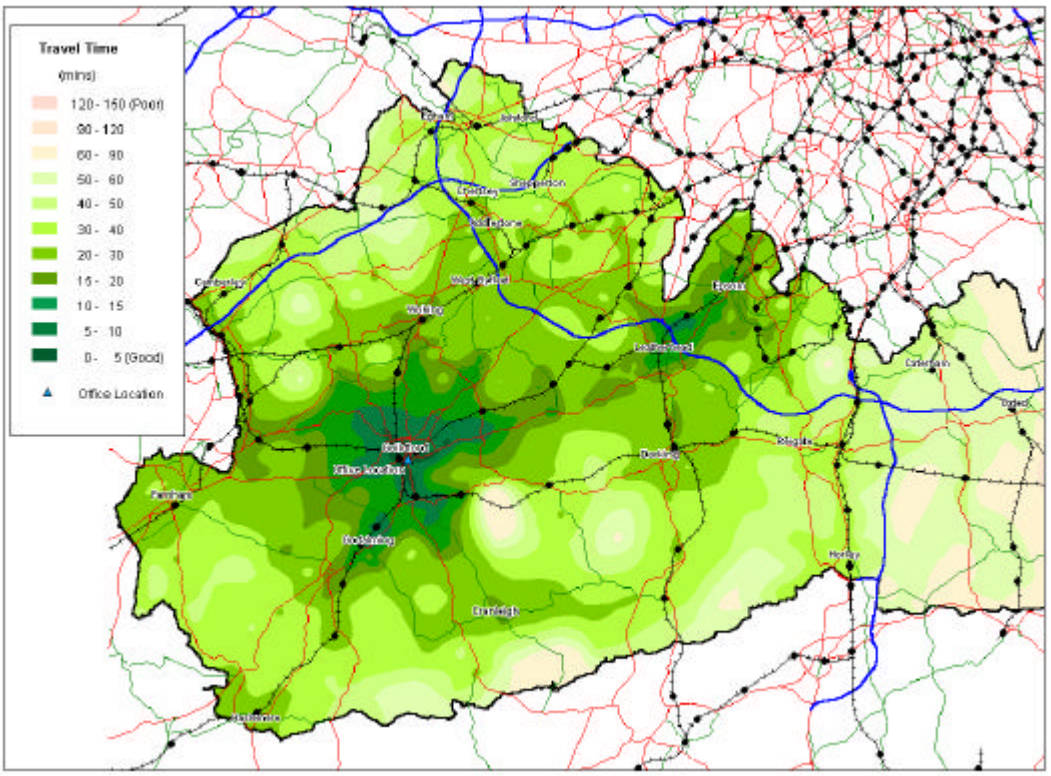

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Abstract

There exists a substantial body of evidence, derived both from theoretical and empirical studies, which indicates that the utility that an individual derives from participating in an activity is dependent upon the timing of the activity. Such dependencies can arise from a range of factors. For instance work based activities with requirements for employees to interact with clients, colleagues and customers either in meetings or in the office/shop, yield a higher level of utility during the opening times of the office/shop than at other times of the day. Individual work related productivity may also be greater at certain times of the day than at others, possibly for instance due to fewer distractions, giving rise to increased levels of utility at these times of the day. Likewise, in the case of shopping activities, the range, quality and cost of goods available may vary by time of day (as stocks are depleted and replenished at non-compensating rates).

Economists and time use researchers have independently explored the relationship between the utility of an activity and the amount of time allocated to it and found systematic relationships that vary according to the type of activity concerned and the context in which it is carried out (e.g., a shopping activity undertaken as a main purpose as opposed to one embedded in a larger work-related schedule).

Both conceptually and logically, the choice of the timing and duration of activities are closely interrelated. However few researchers have studied this relationship and, in particular, existing scheduling and time allocation models fail to treat activity timing and activity duration decisions in a holistic manner, at best making strong separability assumptions concerning the timing and duration dimensions. The relationship between timing and duration decisions coupled with temporal variations in travel time, travel cost, financial endowment, costs of goods and services, the marginal rate of utility of an activity etc, necessitates a more sophisticated treatment of time allocation and departure time choice. This gap is not only of theoretical importance, but also increasingly of practical policy significance, especially in light of recent concerns over the welfare and equity aspects of transport and land use policies with associated temporal pricing regimes, as reflected in the ongoing debate on social inclusion and environmental justice.

The objective of this paper is to present a utility theoretic model of joint activity timing and duration decision making, which addresses a number of the limitations associated with existing scheduling and time allocation models. The approach uses a continuous time formulation when modelling time allocation and choices of departure time and the temporally related activity start time. This continuous time formulation is used to develop the model in terms of the rate of flow of utility arising from participation in a series of linked activities. The disutility arising from travel time, access/egress time, route delay, facility wait time, activity wait time, late start time and travel time variability together with the utility derived from the consumption of a generalised good, complete the remaining components of the utility function. Each of these utility terms comprising the model formulation are either functions of the departure time, the duration of the activity or both, with the utility of activity participation being also a function of the intensity of activity participation. Temporal constraints are simultaneously built into the utility formulation with an additional financial constraint developed, incorporating savings, financial endowment arising from activity participation in addition to the costs of goods and services, the cost of travel etc. In a manner analogous with the utility formulation a number of the terms present in the financial constraint are also functions of the departure time, the activity duration or the intensity of activity participation or all three.

The paper is divided into a number of sections. Section one provides a brief introduction and

overview of the existing literature on activity scheduling and time allocation modelling approaches, highlighting their strengths and weaknesses. This review covers the domain of travel behaviour research and related work in economics and economic geography. Section two details the development of the theoretical model of joint activity timing and activity duration choice and explores the properties of the model, in particular how the theoretical model provides valuable insight into the nature of activity scheduling decisions within activity schedules. The section concludes by outlining the structure of an algorithm, which can be used to operationalise the time allocation and activity scheduling process developed earlier in the section. Section three of the paper outlines the development of an estimatable empirical form of the theoretical model that acknowledges that past and present activity/travel behaviour decisions affect both existing and expected future behavioural outcomes. The section concludes with a brief outline of how the empirical model can be utilised to develop a disaggregate accessibility measure. Section four details a case study in which the empirical model is utilised to assess and quantify individual accessibility within a public transport network encompassing rural, urban and suburban areas. Section five discusses the results of the case study whilst section six outlines the conclusions and discusses possible directions for future research and potential applications of the framework.

Keywords

Activity scheduling, departure time choice, time allocation, marginal/temporal utility profiles, spatial interaction, accessibility.

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1. Introduction

The importance of time on individual travel and activity behaviour, has been recognised by a number of researchers. Within the realm of time-space geography, Hägerstrand 1970, Lenntorp 1976, Burns 1979, Jones et al 1983, Miller 1999, Ashiru et al and others have discussed the importance of time constraints on individual travel behaviour. More generally Pas 1998 notes that '*…time could play a unifying role in the development of an integrated theory of travel behaviour…*', whilst Kitamura et al 1990, observe that '*…the relationship between the time spent on travel and the time spent on the activity has not been examined yet…*'.

There exists a considerable body of evidence, derived both from theoretical and empirical studies, which indicates that the value that an individual derives from participating in an activity is dependent upon the timing of the activity. Economists and time use researchers have explored the relationship between the utility of an activity and the amount of time allocated to it and found systematic relationships that vary according to the type of activity concerned and the context in which it is carried out. Kitamura et al 1996 found that individuals working for several days per week allocate a larger proportion of their daily out-of-home time to discretionary activities, than those working fewer days. Kitamura et al 1996 also show that the greater the commuting time the larger is the proportion of time that is allocated to in-home discretionary activities. Kraan 1996 and Bhat & Misra 1997 show that the time allocated to discretionary activities is affected by household and individual socio-economic and demographic characteristics.

Despite the conceptual and logical interrelationship between the choice of timing and the duration of activities, few researchers have studied this relationship. In particular, the majority of existing scheduling and time allocation models separate activity timing and activity duration considerations. Small 1982, Hendrickson & Plank 1984 and others consider departure time choice within the context of a micro-economic based discrete choice framework, whilst Bhat & Steed 2002 consider departure time choice through a non-parametric baseline hazard. Ettema et al 1995 utilise a hazard duration model to model activity duration choices, whilst Kitamura et al 1996 and Yamamoto & Kitamura 1997 utilise a discrete continuous choice model of time allocation for discretionary activities.

Becker's 1965 work on time and cost allocation and the later work of De Serpa 1971, Evans 1972, Ghez & Becker 1975, based upon Lancaster's 1966 economic consumer choice theory, has played a significant role in the development of time allocation theory. These researchers were amongst the first to recognise the relationship between time allocation, time as a commodity and goods purchased in the market place and their interrelated roles in the production

of commodities and the derivation of utility. The later, related household time allocation work of Winston 1982 (reviewed by Axhausen & Gärling 1992), extended Becker's framework to incorporate information on the timing and intensity of activities.

Small's 1982 work on departure time choice, with its schedule delay terms has provided valuable insight into the nature of activity scheduling behaviour, however when this model is applied to activity schedules with multiple activities it does, as noted by Polak & Jones 1994, become '*..rather cumbersome…*'. Temporal utility profiles are a conceptually appealing alternative to the use of schedule delay penalties and essentially consider personal preferences and external constraints as jointly generating temporal variations in the rate at which an individual derives utility from undertaking activities. Temporal utility profiles have been utilised by a number of researchers most notably Koenker 1979 in his investigation of optimal peak load pricing strategies and Winston 1982 in his analysis of individual household production. In Winston's model, the utility of an activity at a given instance in time, is a function of the individual's satisfaction gained whilst performing the activity and the intensity with which the activity is performed, both of which Winston hypothesised to be a function of the time of day and the duration of the activity.

In the transport sector a number of researchers have begun to consider the interrelationship between departure time choice and activity duration. Supernak 1990 presented a graphical framework, based upon temporal utility profiles, representing how an individual's perceived accumulation of utility when undertaking in-home and out-of-home activities varies with time of day. Supernak's graphical framework was unique in that it represented the first attempt within the arena of travel behaviour research to consider the dynamic interplay between the marginal utility of an activity and the disutility associated with travel and the effect of this on both activity choice and activity duration. Supernak 1992 subsequently modifies his graphical framework and proceeds to treat the activity engagement process as essentially a decision under uncertainty. Supernak argues that the activity engagement/disengagement process is based upon the utility that will be gained at a certain point in the future, and since this future utility cannot be predicted with certainty it must be treated as a random variable. Kitamura & Supernak 1997 subsequently outline a mathematical formulation of Supernak's 1992 activity choice under uncertainty and present some empirical evidence in support of the model. Polak & Jones 1994, represented the first formal implementation of temporal utility profiles in the field of travel behaviour. In their mathematical model of joint activity timing and activity duration choice Polak & Jones analyse travellers home based tour re-timing behaviour under road user charging in London. More recently Ettema & Timmermans utilise temporal utility profiles within a mathematical model conceptually similar to that of Polak & Jones, but reformulate the model in terms of scheduling costs associated with late and early departure from the origin and the schedule disutility at the destination. Lam & Yin 2001 outline a conceptual activity

based dynamic traffic assignment model, which incorporates temporal utility profiles of activities as the basis for activity choice behaviour.

However despite the acknowledgement of the inter-linkage between activity scheduling, time allocation and travel disutility these various frameworks neglect consideration of a number of factors which play an important role in individual activity duration and activity time choices. Most notable amongst these is travel time uncertainty which has been recognised by Polak 1987, Bates et al 2001, Noland & Polak 2002, and others as playing an important role in individual departure time choice decisions. An equally important factor influencing activity timing and activity duration choices is route delay, e.g. wait time penalties, modal interchange penalties, parking and other non-travel related time spent in transit between upstream and downstream activity locations.

Facility wait time, activity wait time and late start time, (respectively the schedule penalty incurred as a consequence of arrival at an activity location before its formal opening time, before the formal earliest start time of the activity and after the formal latest start time of the activity) are important scheduling penalties that influence individual activity scheduling and activity duration choices. Route delay, facility wait, activity wait and late start time scheduling penalties are particularly important for public transport users, who due to the discrete nature of public transport services, are faced with making additional scheduling decisions (Bates et al 2001). These additional scheduling decisions effect route choice (the degree of service interchange), the trade-off between early arrival, late arrival and activity duration in a manner not generally faced by users of walk, cycle or car transport modes, where departure time choice is continuous and thus can be chosen to increase the chances of arriving at an activity location on time.

The majority of research undertaken to date on departure time choice has predominantly concentrated upon a single disjointed outbound journey usually from home to work and has tended to neglect the implicit linkage between re-timing of the outward journey and the effect of this re-timing on other journeys and activities present within the activity schedule. This interrelationship has been difficult to analyse, however several authors have started to consider aspects of this interrelationship most notably Polak & Jones 1994 and Ettema & Timmermans.

In common with departure time choice models the majority of time allocation models consider a single activity with the more advanced approaches considering a simple three-leg tour. However it is generally agreed by researchers, Ben-Akiva & Lerman 1979, Hanson 1980, Kitamura et al 1990, Supernak 1990, Ashiru et al and others that, activity schedules, the sequential linking of trips and activities, serve to increase an individual's overall accessibility, than is possible with schedules composed of a single activity. Moreover Mahmassani et al

1997 in their analysis of trip chaining, scheduling and route choice travel behaviour of commuters, present empirical evidence that suggests that activity schedules with more than one out-of-home activity are more likely to lead to re-scheduling than schedules with a single out-of-home activity.

As previously discussed the interrelationship between activity scheduling, time allocation decisions and individual accessibility has not been considered by many researchers despite its theoretical and practical importance for transport-land use scheme and policy development. In the remainder of this paper we present a utility theoretic model of joint activity scheduling and duration choice which addresses a number of the limitations present within existing scheduling and time allocation models. In particular it incorporates the interrelationship between the re-timing, the change in duration of one or more activities and the effect of these changes on all other activities present within the activity schedule. In addition the model presented incorporates travel time variability, route delay and facility wait, activity wait and late start time schedule penalties formerly neglected by researchers. An algorithm is presented which can be used to operationalise the theoretical model and an empirical model is subsequently developed. Several activity based accessibility measures are developed from the theoretical and empirical models one of which is applied in a simple case study. The results of the case study are discussed followed by a discussion of conclusions and directions for future research.

2. Theoretical Activity Scheduling & Time Allocation Model

In this section we develop a utility theoretic model of joint activity timing and duration choice, which addresses a number of the limitations associated with existing scheduling and time allocation models. The section outlines a number of interesting properties of the mathematical model, in particular how the theoretical model provides valuable insight into the nature of activity scheduling and time allocation decisions. The section concludes with an algorithm, that can be used to operationalise the theoretical time allocation and activity scheduling model developed earlier in the section.

2.1 Overview of the Theoretical Framework

Figure 1 represents a generalised graphical representation of the utility theoretic framework. The figure reflects a subset of the (n) activities present within the activity schedule in which the (a-1)th to (a+1)th activities present within an activity schedule are presented. In the graphical representation of the utility theoretic framework a continuous clock time formulation is used (in contrast to the discrete/segmented series of time intervals utilised by some re-

searchers), in which time is defined relative to a fixed point set at midnight, with a total available cumulative time budget of T minutes, where T is typically set to 24 hours, namely a time window straddled by the need for sleep.

Figure 1 indicates that the a-th activity commences at time t_{sa} and ends at time t_{ea}. The figure also indicates that the travel time, route delay, facility wait time, activity wait time and the access/egress time are functions $f_1(.)$, $f_2(.)$, $f_3(.)$, $f_4(.)$, $f_5(.)$ respectively of the departure time t_{ea-1} from the (a-1)th activity location, which is also the time at which the (a-1)th activity ends.

The functions $f_1(.)$ to $f_5(.)$ (later generalised to $f_x(.)$ in the paper) inclusive denote the manner in which the travel time, route delay, facility wait time, activity wait time and the access/egress time vary with time during the course of the day. The amount of time T_a that an individual spends in undertaking the a-th activity is temporally related to the start time of the activity t_{sa} and the end time of the a-th activity t_{ea} i.e. $T_a = t_{ea} - t_{sa}$.

Since travellers can exert varying levels of control over their arrival time through their departure time, the departure time from an activity location will form the basis of the following discussions. It is worth noting that there are alternative combinations of options available for the analysis.

In the following utility theoretic model it is assumed that the individual considers a deterministic utility function which is dependent upon, i) activity participation utility reflected in temporal utility profiles, ii) disutilities associated with travel time, route delay, facility wait time, activity wait time, late start time, access/egress time and travel time variability, and iii) consumption of a generalised good or service. Temporal constraints are simultaneously built into the utility formulation with an additional financial constraint developed, incorporating savings, financial endowment arising from activity participation in addition to the cost of travel and the costs of goods and services.

In addition it is also assumed that the individual has perfect knowledge of all the transport options available to him/her and is able to maximise utility. It is further assumed that the utility derived from the various sources are time separable and additive in nature. In addition the individual is assumed to have previously decided which activities he/she would like to undertake and in which particular sequence. The implications of these assumptions are considered later within the paper, respectively within section 2.4 and within the summary and conclusions.

2.2 Marginal Utility & Intensity-Duration Profiles

Utility functions that vary over time form the basis of the utility theoretic model. These enable time of day of choices to be reflected more efficiently within activity schedules encompassing

more than one departure time choice variable, than the classic early/late start schedule delay penalties utilised by Small 1982 during his analysis of the departure time choice for work related journeys, as discussed in section 1.

The rate at which an individual derives utility from undertaking an activity in the home or out of the home is known to vary by time of day. Observation of personal behaviour may reveal that the value one derives from an activity rises during the course of undertaking the activity, approaches a maximum, possibly remains at this level for a period of time before beginning to decline as one looses interest in the activity in question. The rate of ascent and descent of the utility profile and the location of the point of maximum utility are functions of the characteristics of the individual, the nature of the activity, and how the activity is placed in each daily schedule.

A bell shaped temporal utility curve has been proposed by Joh et al 2002 and extended by Ettema & Timmermans. This describes how the utility associated with an activity varies over time. The bell shaped curve is characterised by a warming up period in which the marginal utility gradually increases until a maximum or optimum is reached after which the marginal utility steadily decreases to its minimum value. This is an intuitively appealing profile whose characteristics are similar to a number of different activity types including the hypothetical example previously considered. A particular advantage that the bell shaped profile possesses over the other profiles is that it has a smooth continuous curve with no discontinuities and thus can be differentiated and integrated at all instances in time. The benefit of these latter properties for practical implementation of marginal utility profiles will be outlined later within the paper.

A number of researchers most notably Winston 1982, Axhausen & Gärling 1992, Lam & Yin 2001 have noted that the utility an individual derives from activity participation is a function of the time of day at which the activity is performed, the duration of the activity and intensity with which the activity is performed.

Figure 2 depicts a number of alternative intensity-duration profiles including negative exponential, negative power, negative Gaussian and linear (descending). These respective profiles all depict the same behavioural characteristic of the intensity of an activity, namely as the duration available for an activity is reduced the intensity with which the activity is undertaken increases and as the activity duration is increased then the intensity of the activity decreases.

2.3 Utility Theoretic Model

Incorporating the assumptions previously discussed, a generalised utility theoretic model of the following form is proposed.

$$MaxU = \sum_{a=1}^{n} U_a(t_{sa}, T_a, I_a) + \sum_{X=1}^{\Delta} U_X\left(\sum_{a=1}^{n} f_X(t_{ea})\right) + U_G\left(\sum_{a=1}^{n} G_a\right) \quad (1)$$

Subject to the following financial F and time T constraints:

$$F = S + \sum_{a=1}^{n} W_a(t_{sa}, T_a, I_a) - \sum_{a=1}^{n} P_a(t_{sa}, T_a, I_a) - \sum_{a=1}^{n} C(t_{ea}) \quad (2)$$

$$T = \sum_{a=1}^{n} T_a + \sum_{a=1}^{n}\left(\sum_{X=1}^{5} f_X(t_{ea})\right) \quad (3)$$

Where,

a A subscript denoting the a-th activity within the activity schedule.

t_{sa} The time at which the a-th activity commences.

t_{ea} The time at which the a-th activity ends (alternatively referred to as the departure time).

T_a The duration of the a-th activity.

I_a The intensity with which the a-th activity is undertaken.

u_a The rate of flow of utility or the marginal utility of the a-th activity, which is a function of the start time of the activity.

U_a The total utility derived from the a-th activity, which is a function of the start time of the activity, the duration of the activity and the intensity with which the activity is performed.

X A subscript denoting the disutilities in the model which are described by function f_x, all of which are a function of the departure time t_{ea}.

Δ The total number of disutilities assumed within the model. The disutilities considered are those associated with travel time (T), route delay (D), facility wait time (w), activity wait time (W), access/egress time (E), late start time (L) and travel time variability (V) which are respectively functions f_1, f_2, f_3, f_4, f_5, f_6 and f_7 of the departure time t_{ea} (where $\Delta = 7$).

G_a The total goods consumed during the course of undertaking the a-th activity, which is assumed to be independent of the time of day.

U_G The total utility derived from the consumption of a generalised good during the course of undertaking the activity, which is assumed to be independent of the departure time.

S The total savings, unearned income or dividends of the individual/person type under consideration, which is assumed to be independent of the time of day.

C The total cost of travel at time t_{ea} from the location of the a-th activity to the location of the (a+1)th activity location, which is a function of the departure time.

W_a The total financial endowment gained from undertaking the a-th activity, which is a function of the start time of the activity, the duration of the activity and the intensity with which the activity is undertaken.

w_a The wage rate or rate of financial endowment received by the individual as a consequence of undertaking the a-th activity.

P_a The total cost of the goods consumed during the course of undertaking the a-th activity, which is a function of the start time of the activity.

p_a The unit cost of the goods consumed during the course of undertaking the a-th activity, which is a function of the start time of the activity, the duration of the activity and the intensity with which the activity is undertaken.

T The total available time budget of the individual.

The utility theoretic framework outlined in equations 1-3 inclusive can be considered to be an expansion of the classical activity timing/time allocation framework discussed by Becker 1965, Small 1982, Polak & Jones 1994, Ettema & Timmermans amongst others, to incorporate temporal utility profiles, activity duration and activity intensity. In its broadest sense the framework has parallels with the unified theory of travel and activities proposed by Jara-Diaz 1998a, 1998b.

Considering the rate of flow of utility $u_a(.)$, the financial endowment $w_a(.)$ and the cost of goods $p_a(.)$ associated with participation in an activity a, at time t_{sa} of intensity I_a, it can be shown that if the activity commences at time t_{sa} and lasts for a duration T_a ending at a time of $t_{ea} = t_{sa} + T_a$ then from figure 1 it follows that:

$$U_a(t_{sa}, T_a, I_a) = \int_{t_{sa}}^{t_{ea}} u_a(t)dt \tag{4}$$

$$W_a(t_{sa}, T_a, I_a) = \int_{t_{sa}}^{t_{ea}} w_a(t)dt \tag{5}$$

$$P_a(t_{sa}, T_a, I_a) = G_a \int_{t_{sa}}^{t_{ea}} p_a(t)dt \tag{6}$$

The effect of a change in the timing of an activity can be expressed as:

$$\frac{\partial U_a(t_{sa}, T_a, I_a)}{\partial(t_{sa})} = u_a(t_{ea}) - u_a(t_{sa}) \tag{7}$$

$$\frac{\partial W_a(t_{sa}, T_a, I_a)}{\partial(t_{sa})} = w_a(t_{ea}) - w_a(t_{sa}) \tag{8}$$

$$\frac{\partial P_a(t_{sa}, T_a, I_a)}{\partial(t_{sa})} = G_a\left(p_a(t_{ea}) - p_a(t_{sa})\right) \tag{9}$$

Where $u_a(.)$, $w_a(.)$ and $p_a(.)$ are the respective time of day varying functions of rate of flow of, utility, financial endowment and cost of goods and services.

Equations 7-9 inclusive imply that a change in the timing of an activity will effect the utility, financial endowment and cost of goods and services associated with the activity, at a rate which is a linear function of the respective differences between the marginal utility, marginal financial endowment and the marginal cost of goods at the end and the beginning of the activity.

Substituting equations 4-6 inclusive into equations 1 and 2 leads to the following utility formulation where it should be noted that the temporal constraint reflected in equation 3 has been subsumed within the upper and lower limits of the integral terms present in equations 10 and 11 below. In addition the number of disutility terms has been defined as 7.

$$MaxU = \sum_{a=1}^{n}\left(\int_{t_{sa}}^{t_{ea}} u_a(t)dt\right) + \sum_{X=1}^{7} U_X\left(\sum_{a=1}^{n} f_X(t_{ea})\right) + U_G\left(\sum_{a=1}^{n} G_a\right) \tag{10}$$

Subject to the following financial F constraint:

$$F = S + \sum_{a=1}^{n}\left(\int_{t_{sa}}^{t_{ea}} w_a(t)dt\right) - \sum_{a=1}^{n} G_a\left(\int_{t_{sa}}^{t_{ea}} p_a(t)dt\right) - \sum_{a=1}^{n} C(t_{ea}) \tag{11}$$

Formulating the Lagrangian of equations 10 and 11 leads to:

$$L = \sum_{a=1}^{n}\left(\int_{t_{sa}}^{t_{ea}} u_a(t)dt\right) + \sum_{X=1}^{7} U_X\left(\sum_{a=1}^{n} f_X(t_{ea})\right) + U_G\left(\sum_{a=1}^{n} G_a\right) - \lambda\left(S + \sum_{a=1}^{n}\left(\int_{t_{sa}}^{t_{ea}} w_a(t)dt\right) - \sum_{a=1}^{n} G_a\left(\int_{t_{sa}}^{t_{ea}} p_a(t)dt\right) - \sum_{a=1}^{n} C(t_{ea})\right) \tag{12}$$

From figure 1 it follows that:

$$t_{sa} = t_{ea-1} + \sum_{X=1}^{5} f_X(t_{ea-1}) \tag{13}$$

$$t_{sa+1} = t_{ea} + \sum_{X=1}^{5} f_X(t_{ea}) \tag{14}$$

$$\frac{\partial t_{sa}}{\partial t_{ea-1}} = 1 + \sum_{X=1}^{5} f_X^{/}(t_{ea-1}) \tag{15}$$

$$\frac{\partial t_{sa+1}}{\partial t_{ea}} = 1 + \sum_{X=1}^{5} f_X^{/}(t_{ea}) \tag{16}$$

Differentiating the Lagrangian outlined in equation 12 with respect to the departure time choice decisions associated with the (a-1)th and a-th activities (i.e. t_{ea-1} and t_{ea} respectively) and utilising equations 13-16 inclusive, together with a slightly modified form of equations 7-9 inclusive, it can be shown that the first order conditions defining the optimal departure time are characterised by the following expression:

$$u_{a-1}(t_{ea-1}) + \left[\sum_{X=1}^{7} U_X f_X^{/}(t_{ea-1})\right] - \lambda\left[w_{a-1}(t_{ea-1}) - G_{a-1}p_{a-1}(t_{ea-1}) - C^{/}(t_{ea-1})\right] = \left[1 + \sum_{X=1}^{5} f_X^{/}(t_{ea-1})\right]\left(u_a(t_{sa}) + \lambda\left(G_a p_a(t_{sa}) - w_a(t_{sa})\right)\right) \tag{17}$$

$$u_a(t_{ea}) + \left[\sum_{X=1}^{7} U_X f_X^{/}(t_{ea})\right] - \lambda\left[w_a(t_{ea}) - G_a p_a(t_{ea}) - C^{/}(t_{ea})\right] = \left[1 + \sum_{X=1}^{5} f_X^{/}(t_{ea})\right]\left(u_{a+1}(t_{sa+1}) + \lambda\left(G_{a+1} p_{a+1}(t_{sa+1}) - w_{a+1}(t_{sa+1})\right)\right) \tag{18}$$

Equations 17 and 18 reveal a particularly interesting aspect of the activity scheduling process, namely that at the optimum departure time from an activity location the rate of flow of utility derived from the last minute spent in undertaking the current/upstream activity in the activity schedule, is equal to the rate of flow of utility derived from the first minute of the next/downstream activity, net of the time of day variations of the utilities of travel time, route delay, facility wait time, activity wait time, egress/access time, late start time, travel time variability, and financial endowment, cost of goods and services and the cost of travel. The scheduling process involves the substitution of a minute of an activity with a lower marginal utility with an activity possessing a higher marginal utility net of the time of day variations of the various disutility components and the financial endowment, cost of goods and services and the cost of travel.

The above expressions are analogous of the long appreciated aspect of activity behaviour that travel is only undertaken when the net utility derived from downstream activity participation and travel exceeds the utility derived from an existing activity. Winston 1982, Supernak 1990, 1992, Polak & Jones 1994 and Ettema & Timmermans have identified similar properties concerning the marginal utility profiles at the start and end of adjacent activities within a schedule.

If we consider the simple case of the bell shaped temporal utility profiles of two home based activities outlined in figure 3 (e.g. watching a sporting event on television and eating a meal with the family), for which the travel time, route delay, facility wait time, activity wait time, late start time, access/egress time, the travel time variability and associated disutilities are assumed to be independent of the time of day. If it is assumed that the travel cost, financial endowment and the cost of goods and services are also independent of the time of day and that no goods are consumed and no financial endowment derived from the activities. If it further assumed that both activities cannot be undertaken simultaneously then from equations 17 and 18 and figure 3, the individual will switch from watching the sports event to eating a meal with the family at point in time of 16:45 hours where the marginal utility profiles associated with the two activities intersect and at the point where the upstream activity of watching the sporting event ceases to be more attractive than the downstream activity of eating a meal. Figure 3 also reveals that in the absence of an alternative activity the individual will cease eating a meal with his/her family and resume watching the sporting event at 20:00 hours.

Whilst this is a simple example of the activity scheduling process it does depict an important aspect of the scheduling and time allocation process. As the temporal and financial endowment and cost assumptions detailed above are relaxed, then the scheduling of the upstream and downstream activities is undertaken in a manner such as to preserve the marginal utility conditions detailed in equations 17 and 18.

2.4 Activity Scheduling & Time Allocation Algorithm

In the remainder of the section we outline the structure of an algorithm which can be utilised to operationalise the time allocation and activity scheduling process discussed earlier and depicted in equations 17 and 18. Figure 4 represents a simplified flow chart of the activity scheduling and time allocation algorithm.

The algorithm commences with an initialisation of variables and arrays which is followed by the definition of the start and end times of the time window, which intrinsically defines the available total time budget (T). The total time budget is then segmented into a number of time slices (n) each of equal magnitude. If the day is utilised as the available time budget, then utilising one minute of time gives 1,440 time slices. The algorithm then proceeds to determine whether all activity schedules have been analysed. If the answer is no, the algorithm then determines the current activity (a) within the schedule. Having determined the current activity, the location of this activity is determined (k_a). In the case of discretionary activities or spatial interaction choices this will almost certainly involve the analysis of alternative activity locations (k_a). The start time of the current activity is determined, which in the case of the first activity will be midnight or zero minutes (refer to horizontal axis of figure 3). The algorithm then ascertains whether there is a downstream activity (a+1) to be considered within the schedule. If the answer is yes, the location of this activity location is determined which as in the case of the current activity can exceed unity (k_{a+1}). During the next step in the algorithm the net rate of flow of utility (nu_{ka}) (i.e. the left-hand side of equation 18) is calculated for all time slices for the current activity location (k_a). During the next step in the algorithm the net rate of flow of utility (nu_{ka+1}) (i.e. the right-hand side of equation 18) is determined for all time slices for the downstream activity location (k_{a+1}). The time slice (te_{ka}) at or temporally downstream of the current activity location, at which the first order optimality conditions detailed in equation 18 is satisfied is identified. If the second order optimality conditions are satisfied within the Lagrangian detailed in equation 12 at the prospective optimal departure time (i.e. $d^2L/d^2t_{ea} < 0$), then this departure time is in fact a global optimum for the activity. It is possible to introduce a tolerance at this stage within which if satisfied optimality conditions can be assumed to exist. This time slice represents the point in time at which the individual ceases to undertake the current activity (a) and commences to travel to the downstream activity location (k_{a+1}). If the second order optimality conditions are not satisfied at the time slice in question, then proceed temporally downstream of the time slice in question and identify the first time slice at which both the first and second order optimality conditions are satisfied. It is possible to reduce the number of computational calculations undertaken by considering only the time slices at or downstream of the current activity start time. However this will reduce

the accuracy of the numerical differentiation exercise (refer to later discussions). The duration of the current activity can now be determined from the simple expression $T_{ka} = te_{ka} - ts_{ka}$.

The algorithm then proceeds to determine the total utility (U_{ka}) and the net total utility (NU_{ka}) of the current activity. This is followed by a calculation of the net cumulative utility (CNU_{ka}) of all activities which have been undertaken to date within the schedule. All the disutility components associated with departure from the current activity location (k_a) to the downstream activity location (k_{a+1}) at time (te_{ka}) are determined. The corresponding start time of the downstream activity (ts_{ka+1}) is determined. The downstream activity (a+1) now becomes the current activity and similar substitutions are made for the location of the current activity, and the start time of the current activity. As before the algorithm determines whether the end of the activity schedule has been reached and if not proceeds to repeat the process for all remaining downstream activities. If the end of the activity schedule has been reached, then in the instance of the daily time budget, midnight is defined as the end time of the last activity. Given this assumption the activity duration and associated total utility, net total utility and net cumulative utility can be determined. At this stage the net cumulative utility of all activities undertaken within the schedule is equated to the utility of the schedules and details of the activity schedule are stored for later analysis. The algorithm determines whether all activity schedules have been considered and if not proceeds to repeat the process for all remaining activity schedules. If all activity schedules have been considered then the algorithm proceeds to identify and report on details of the activity schedule within the subset of schedules with the greatest utility of schedules U(schedule), at which point the algorithm is terminated.

The key challenge when attempting to implement this algorithm is obtaining appropriate temporal functional forms for the first order derivatives for the disutility terms $f_x^{/}$ present in equations 17 and 18 and the second order derivatives for the disutility terms $f_x^{//}$ present within the Lagrangian depicted in equation 12. In reality these disutility terms will be a complex function of departure time which will vary according to the relative locations of the current and downstream activities and the associated route of travel between these respective locations. In such instances numerical differentiation, Curtis et al 1984, can be utilised to obtain numerical estimates for these first and second order derivatives of the disutility terms. An implicit assumption inherent within the algorithm is that the order of activities to be performed within the schedule has been pre-defined. It is for instance possible that an activity schedule with an entirely different sequence may yield the optimal utility of the schedule.

3. Empirical Activity Scheduling Model

The theoretical model outlined in section 2 provides some valuable insight into the nature of the activity scheduling and time allocation process, however as discussed earlier and noted by Polak & Jones 1994 challenges arise when attempts are made to operationalise the model. In this section we outline the development of an empirical form of the theoretical model developed in section 2 which implicitly acknowledges that past and present activity/travel behaviour decisions affect both existing and expected future behavioural outcomes.

3.1 Empirical Model

Reconsidering equations 1 and 2 together with the Lagrangian outlined in equation 12 and utilising a second order Taylor's series approximation for a function of three variables outlined in equation 19 the Lagrangian can be expressed in a modified form shown in equation 20:

$$\begin{aligned} f(x,y,z) = {} & f(a,b,c) + \frac{\partial f(a,b,c)}{\partial x}(x-a) + \frac{\partial f(a,b,c)}{\partial y}(y-b) + \frac{\partial f(a,b,c)}{\partial z}(z-c) \\ & + \frac{1}{2!}\left[\frac{\partial^2 f(a,b,c)}{\partial x^2}(x-a)^2 + \frac{\partial^2 f(a,b,c)}{\partial y^2}(y-b)^2 + \frac{\partial^2 f(a,b,c)}{\partial z^2}(z-c)^2\right] \\ & + \frac{2}{2!}\left[\frac{\partial^2 f(a,b,c)}{\partial x \partial y}(x-a)(y-b) + \frac{\partial^2 f(a,b,c)}{\partial y \partial z}(y-b)(z-c) + \frac{\partial^2 f(a,b,c)}{\partial z \partial x}(z-c)(x-a)\right] + ... \end{aligned} \tag{19}$$

$$\begin{aligned} U(schedule) = {} & \sum_{a=1}^{n} U_a(t_{sa}^*, T_a^*, I_a^*) + \sum_{a=1}^{n} \alpha_a \Delta t_{sa} + \sum_{a=1}^{n} \lambda_a \Delta t_{sa}^2 + \sum_{a=1}^{n} \beta_a \Delta T_a + \sum_{a=1}^{n} \mu_a \Delta T_a^2 \\ & + \sum_{a=1}^{n} \eta_a \Delta I_a + \sum_{a=1}^{n} \gamma_a \Delta I_a^2 + \sum_{a=1}^{n} \kappa_a \Delta t_{sa} \Delta T_a + \sum_{a=1}^{n} \nu_a \Delta T_a \Delta I_a + \sum_{a=1}^{n} \omega_a \Delta I_a \Delta t_{sa} \\ & + \sum_{X=1}^{7} U_X\left(\sum_{a=1}^{n} f_X(t_{ea})\right) + U_G\left(\sum_{a=1}^{n} G_a^*\right) + \lambda \sum_{a=1}^{n} C(t_{ea}) + \lambda\left[\sum_{a=1}^{n} G_a P_a(t_{sa}^*, T_a^*, I_a^*) - \sum_{a=1}^{n} W_a(t_{sa}^*, T_a^*, I_a^*) - S\right] \end{aligned} \tag{20}$$

Where,

$$\Delta t_{sa} = t_{sa} - t_{sa}^* \tag{21}$$

$$\Delta T_a = T_a - T_a^* \tag{22}$$

$$\Delta I_a = I_a - I_a^* \tag{23}$$

$$\alpha_a = \frac{\partial U_a}{\partial t} - \lambda \frac{\partial W_a}{\partial t} + \lambda G_a \frac{\partial P_a}{\partial t} \tag{24}$$

$$\beta_a = \frac{\partial U_a}{\partial T} - \lambda \frac{\partial W_a}{\partial T} + \lambda G_a \frac{\partial P_a}{\partial T} \tag{25}$$

$$\eta_a = \frac{\partial U_a}{\partial I} - \lambda \frac{\partial W_a}{\partial I} + \lambda G_a \frac{\partial P_a}{\partial I} \tag{26}$$

$$\lambda_a = \frac{1}{2!}\left(\frac{\partial^2 U_a}{\partial t^2} - \lambda \frac{\partial^2 W_a}{\partial t^2} + \lambda G_a \frac{\partial^2 P_a}{\partial t^2} \right) \tag{27}$$

$$\mu_a = \frac{1}{2!}\left(\frac{\partial^2 U_a}{\partial T^2} - \lambda \frac{\partial^2 W_a}{\partial T^2} + \lambda G_a \frac{\partial^2 P_a}{\partial T^2} \right) \tag{28}$$

$$\gamma_a = \frac{1}{2!}\left(\frac{\partial^2 U_a}{\partial I^2} - \lambda \frac{\partial^2 W_a}{\partial I^2} + \lambda G_a \frac{\partial^2 P_a}{\partial I^2} \right) \tag{29}$$

$$\kappa_a = \frac{2}{2!}\left(\frac{\partial}{\partial t}\frac{\partial U_a}{\partial T} - \lambda \frac{\partial}{\partial t}\frac{\partial W_a}{\partial T} + \lambda G_a \frac{\partial}{\partial t}\frac{\partial P_a}{\partial T} \right) \tag{30}$$

$$\nu_a = \frac{2}{2!}\left(\frac{\partial}{\partial T}\frac{\partial U_a}{\partial I} - \lambda \frac{\partial}{\partial T}\frac{\partial W_a}{\partial I} + \lambda G_a \frac{\partial}{\partial T}\frac{\partial P_a}{\partial I} \right) \tag{31}$$

$$\omega_a = \frac{2}{2!}\left(\frac{\partial}{\partial I}\frac{\partial U_a}{\partial t} - \lambda \frac{\partial}{\partial I}\frac{\partial W_a}{\partial t} + \lambda G_a \frac{\partial}{\partial I}\frac{\partial P_a}{\partial t} \right) \tag{32}$$

Where (t_{sa}^*, T_a^*, I_a^*) and (t_{sa}, T_a, I_a) are the timing, duration and intensity of an activity observed in an existing activity schedule and in a modified activity schedule respectively.

Equation 20 can be considered to be analogous to the schedule delay penalty terms utilised by Small 1982 and others but has been developed through use of marginal utility profiles and denotes the effect on the utility of the activity schedule of a change in the timing, duration and intensity of one or more activities present within the activity schedule. Equation 20 also shows that past and present activity and travel behaviour decisions effect both existing and expected future behavioural outcomes. Equation 20, whilst denoting the utility of the entire activity schedule, can be used to assess the effect on individual activity and travel behaviour of changes (temporally based or otherwise) of a range of factors, such as the cost of travel, financial endowment, the cost of goods and services, goods consumption, activity intensity (e.g. the retention of additional or fewer staff, the improved use of information technology etc) changes to the disutility components encompassing travel time, route delay, facility wait time, activity wait time, late start time, access/egress time and travel time variability.

3.2 Activity Based Accessibility Measure

In the remainder of the section we outline how the empirically based utility of schedules model discussed earlier in the section can be utilised to develop an absolute and a differentially based accessibility measure that can be used in assessing overall accessibility levels and

differential changes in accessibility as a consequence of joint activity timing and activity duration scheduling choices. These behavioural changes can arise in response to a range of factors including changes to the land use-transport environment, changes in an individual's activity schedule time constraints, changes in the opening/closing times of facilities, changes in the intensity of an activity, changes in the cost of travel, financial endowment and cost of goods and services etc.

Equations 20 whilst representing the utility of the entire activity schedule can, according to Weibull (1976, 1980), Miller 1999 and Ashiru et al, be considered also to be a measure of the accessibility of the entire activity schedule, if a standard utility maximisation framework is utilised.

The effect on the utility of the schedule of a change in activity scheduling and time allocation as previously discussed can be denoted by equation 33. In such an instance equation 33 can be considered to be a measure of the change in the utility of the schedule or the change in accessibility.

$$\begin{aligned}\Delta U(schedule) = &\left[\sum_{a=1}^{n} \alpha_a \Delta t_{sa}\right] + \left[\sum_{a=1}^{n} \lambda_a \Delta t_{sa}^2\right] + \left[\sum_{a=1}^{n} \beta_a \Delta T_a\right] + \left[\sum_{a=1}^{n} \mu_a \Delta T_a^2\right] + \left[\sum_{a=1}^{n} \eta_a \Delta I_a\right] \\ &+ \left[\sum_{a=1}^{n} \gamma_a \Delta I_a^2\right] + \left[\sum_{a=1}^{n} \kappa_a \Delta t_{sa} \Delta T_a\right] + \left[\sum_{a=1}^{n} \nu_a \Delta T_a \Delta I_a\right] + \left[\sum_{a=1}^{n} \omega_a \Delta I_a \Delta t_{sa}\right] \\ &+ \left[\sum_{X=1}^{7} \Delta U_X \left(\sum_{a=1}^{n} f_X(t_{ea})\right)\right] + \lambda\left[\Delta\left(\sum_{a=1}^{n} C(t_{ea})\right)\right]\end{aligned} \tag{33}$$

Where,

$$\Delta U_X\left(\sum_{a=1}^{n} f_X(t_{ea})\right) = \left[U_X\left(\sum_{a=1}^{n} f_X(t_{ea})\right) - U_X\left(\sum_{a=1}^{n} f_X(t_{ea}^*)\right)\right] \tag{34}$$

$$\Delta\left(\sum_{a=1}^{n} C(t_{ea})\right) = \left(\sum_{a=1}^{n} C(t_{ea}) - \sum_{a=1}^{n} C(t_{ea}^*)\right) \tag{35}$$

The formulation of the utility of schedule model outlined in equation 33 enables the individual to tradeoff between activity timing, duration and intensity when adjusting to changes in travel cost, travel time, route delay, access/egress time, facility wait time (responsive to changes in the opening/closing times of facilities), activity wait time, late start time (the last two terms are activity schedule time constraint terms and are thus responsive to changes in flexible working policies), travel time variability, financial endowment and unit cost of goods. Table 1 contains details of a number of properties possessed by the various components of equation 33.

It is worth noting that equations 20-33 inclusive encompass a diverse range of marginal utility profiles, financial endowment regimes, intensity profiles, goods pricing mechanisms, travel cost pricing regimes and disutility mechanisms and can be simplified to reflect actual timing, pricing, endowment and disutility mechanisms as well as the limitations in the availability of data.

It should be noted that it is possible to utilise the Taylor's series approximation to encompass the temporal disutility terms (e.g. travel time, route delay, facility wait time, activity wait time, late start time, access/egress time and travel time variability). In such cases a second order Taylor's series approximation for a function of two variables would significantly increase the complexity of equations 20 and 33.

4. Activity Scheduling Case Study

In the following section we outline the application of the empirically based accessibility measures outlined previously in section 3 to a public transport network located in Surrey, England, a region bordering southwest London with a population of over 1 million residents and an area with one of the highest recorded car availability rates of any part of the UK. The public transport network in Surrey, depicted in figure 5, is characterised by low frequency bus services linking urban centres and a radial rail network linking Surrey to southeast England and London. In addition Heathrow and Gatwick airports, the UK's two busiest airports, are both located immediately adjacent to Surrey's borders. A number of major strategic roads pass through Surrey including the M25, one of Europe's busiest motorways.

In the following case study the empirically based utility formulations and accessibility measures outlined in equations 20 and 33 are simplified to reflect:

- A simple three legged tour.
- Activity utility, financial endowment and cost of goods and services are independent of the intensity of the activity.
- Cost of bus travel is a linear function of distance travelled whilst the cost of rail travel is a function of both the distance travelled and the time of day.
- No goods or services are consumed in pursuit of any activities and thus no costs are incurred by the individual.
- The various disutility terms are linear functions of the magnitude of the disutility.
- Financial endowment is only derived from work related activity.

It is further assumed that the activity marginal utility profiles are characterised by a bell shaped curve defined in equation 36. This is a modified version of the bell shaped profile originally proposed by Joh et al 2002 and later extended by Ettema & Timmermans and has been extended to facilitate spatial interaction choices (i.e. destination selection) at the same time as activity duration and timing choices. This is achieved by the incorporation within the temporal utility function of details on the attractiveness and opening and closing times of the activity location.

$$u(t)_{l_a k_a} = \frac{s_{l_a} e_{l_a} \left(a_{k_a}^{a_{l_a}} \left(t_{ck_a} - t_{ok_a} \right)^{b_{l_a}} \right)}{\left(\exp\left(e_{l_a} \left(t - \left(q_{l_a} + t_{sk_a} t_{l_a} \right) \right) \right) \right) \left(1 + \exp\left(-e_{l_a} \left(t - \left(q_{l_a} + t_{sk_a} t_{l_a} \right) \right) \right) \right)^{(s_{l_a}+1)}} \qquad (36)$$

Where,

t Time of day (clock time).

t_{sk_a} The time at which the activity a undertaken at location k_a is started.

$u(t)_{l_a k_a}$ The marginal utility of activity participation (alternatively known as the rate of flow of utility) of individual type l undertaking an activity a at location k_a.

θ_{l_a} An activity specific parameter for individual type l which positions the marginal utility function on the time axis.

σ_{l_a} An activity specific parameter for individual type l which defines the symmetrical or asymmetrical nature of the marginal utility profile and affects the position of the saturation point.

If $\sigma_{l_a} < 1$ then the temporal location of the saturation point is before θ_{l_a} in which case the warming up period of the activity is of greater duration than the cooling down period of the activity.

If $\sigma_{l_a} > 1$ then the temporal location of the saturation point is after θ_{l_a} in which case the warming up period of the activity is of shorter duration than the cooling down period of the activity.

If $\sigma_{l_a} = 1$ then the temporal location of the saturation point is identical to θ_{l_a} in which case the warming up period of the activity is of equal duration to the cooling down period of the activity.

ε_{l_a} An activity specific parameter for individual type l which defines the steepness of the marginal utility function in the vicinity of the saturation point.

τ_{l_a} An activity specific parameter for individual type l which defines the degree to which the activity marginal utility is dependent upon the duration of the activity (relative clock time) as opposed to the clock time (absolute clock time).

If $\tau_{l_a} = 0$ then the activity marginal utility function is only dependent upon clock time.

If $\tau_{l_a} = 1$ then the activity marginal utility function is dependent upon the duration of the activity and thus an activity of fixed duration would give rise to a marginal utility which is independent of the clock time.

If $\tau_{l_a} > 0$ and $\tau_{l_a} < 1$ then the activity marginal utility function is dependent upon both the duration of the activity timing of the activity.

a_{k_a} Attractiveness of the activity location (k_a). A finite non-negative real number, representing the relative attractiveness of the activity/opportunity location under consideration.

α_{l_a} A parameter defining the marginal utility of the attractiveness of the activity undertaken at the opportunity/facility location for individual *l*.

β_{l_a} A parameter defining the marginal utility of the activity participation time associated with the activity undertaken at the opportunity/facility location for individual *l*.

t_{ok_a} The time at which the activity location (k_a) opens.

t_{ck_a} The time at which the activity location (k_a) closes.

Details of a hypothetical home-based tour are outlined in table 2 for a working adult undertaking a mandatory home-work-home tour. The model parameters associated with the bell shaped marginal utility profile utilised in the case study, are based in part on the empirical findings of Ettema & Timmermans, who examined a number of properties of the bell shaped marginal utility profile associated with a number of home based tours. Figure 6 depicts the bell shaped marginal utility profile and model parameters associated with all three activities within the hypothetical tour presented in table 2.

Equation 37 is a simplified version of the utility formulation outlined in equation 20 with a number of linear functions introduced for the various disutility terms:

$$\begin{aligned} U(schedule) = & \sum_{a=1}^{n} U_a(t_{sa}^*, T_a^*) + \sum_{a=1}^{n} \boldsymbol{a}_a \Delta t_{sa} + \sum_{a=1}^{n} \boldsymbol{l}_a \Delta t_{sa}^2 + \sum_{a=1}^{n} \boldsymbol{b}_a \Delta T_a + \sum_{a=1}^{n} \boldsymbol{m}_a \Delta T_a^2 + \sum_{a=1}^{n} \boldsymbol{c}_a \Delta t_{sa} \Delta T_a \\ & - \left[\boldsymbol{q}\left(\sum_{a=1}^{n} (t_{k_a}) \right) + \boldsymbol{m}\left(\sum_{a=1}^{n} D_{k_a} \right) + \boldsymbol{g}\left(\sum_{a=1}^{n} w_{k_a} \right) + \boldsymbol{h}\left(\sum_{a=1}^{n} W_{k_a} \right) + \boldsymbol{n}\left(\sum_{a=1}^{n} L_{k_a} \right) + \boldsymbol{r}\left(\sum_{a=1}^{n} E_{k_a} \right) + \boldsymbol{y}\left(\sum_{a=1}^{n} V_{k_a} \right) \right] \\ & + \boldsymbol{l}\left[\sum_{a=1}^{n} C(t_{ea}) - \sum_{a=1}^{n} w_a T_a \right] \end{aligned} \qquad (37)$$

The bell shaped marginal utility function detailed within equation 36 is utilised in defining the activity related model parameters α_a, β_a, λ_a, μ_a, χ_a previously defined within equations 24, 25, 27, 28 and 30. The hypothetical model disutility parameters, in part based upon the empirical

findings of Small 1982, are depicted in table 3 and vary in accordance with the relative importance of the various disutility components to the individual and reveal that $\nu>\psi>\mu>\rho>\theta>\gamma>\eta$. That is the individual considers time spent waiting to commence an activity to be less onerous than time spent waiting for a facility to open and time spent waiting for a facility to open over travel time and travel time over access/egress time. The parameter values further indicate that the individual prefers access/egress time over route delay and route delay over travel time variability and travel time variability over late start time. It is assumed that for the purposes of the case study that these parameters do not vary by activity. In reality it is likely that these disutility coefficients will vary according to the nature of the upstream and downstream activities.

A detailed spatially referenced transport network of Surrey, encompassing public transport and walk modes of travel is utilised in the case study, with existing land use facilities modelled within a point based spatially referenced framework. The utility of schedules formulation detailed in equations 20, 33 and 37 can be maximised by the use of a numerical technique. In this case study the Hooke and Jeeves pattern search algorithm is utilised, which whilst being applicable to minimisation problems, can be equally applied to maximisation problems by simply reversing the sign of the utility of schedules formulation.

5. Case Study Results

Figure 7 denotes the total journey time in minutes from the home location under consideration to the office at the relevant departure time from the home. Figure 8 denotes the total journey time in minutes from the office to the home location at the relevant office departure time. Figures 7 and 8 jointly indicate that total journey time to and from the office is lowest for home locations situated close to the office, which is located within the town of Guildford. Total journey times (encompassing access/egress time, onboard travel time and route delay), along bus corridors into and out of Guildford are lower on the edge of the town than for alternative locations within the county. Figures 7 and 8 also indicate that total journey time by rail is lower for homes located in the vicinity of the railway stations of Dorking, Epsom and Leatherhead. These locations benefit from lower overall journey times than other railway locations, ostensibly due to the faster non-stop railway services serving Guildford, available from these stations. The figures in addition show that home locations in the vicinity of the railway stations of Farnham, Haslemere, Godalming, West Byfleet and Woking also benefit from relatively shorter overall journey times than alternative locations. It is evident from figures 7 and 8 that the overall journey time profiles to/from the office location are not identical, highlighting the variation in public transport service provision by time of day.

Figure 9 depicts the utility of schedules or maximum accessibility of the hypothetical tour detailed in table 2 for public transport travel. The figure indicates that where overall journey times are lowest there are relatively high corresponding levels of accessibility as defined by the utility of schedules formulation utilised in the current case study. Accessibility is greatest for home locations situated in the vicinity of Guildford and along the bus corridors serving the town. In addition home locations in the vicinity of the railway corridors serving the railway stations of Dorking, Epsom, Godalming, Haslemere, Leatherhead, West Byfleet and Woking tend to have higher levels of accessibility than locations outside of these railway corridors. The results also indicate that accessibility for homes located in the eastern and southern parts of the county tends to be lowest, where in the main public transport service provision to and from Guildford is poorest.

Figure 10 depicts the duration of the first home-based activity in minutes for all parts of the county. If it is assumed that the first home activity commences at midnight, then figure 10 can also be interpreted as the departure time in minutes after midnight from the home to the office location. The figure indicates that for homes located in the vicinity of Guildford greater time is allocated to the first home-based activity than more distant locations and for such locations up to 8.2 hours is allocated to this activity. Since the duration of the first activity can also be considered to be the departure time from the home location to the office it is evident that owing to the lower journey times an individual with a home located in the vicinity of Guildford is able to embark on the journey to work at a later point in time of up to 08:11 hours. A similar pattern of time allocation is also evident in home locations situated close to the principal bus corridors and also for those locations in the vicinity of the railway corridors serving the railway stations of Dorking, Epsom, Farnham, Haslemere, Godalming, Leatherhead, West Byfleet and Woking. In these locations individuals are, due to the lower overall journey time from the home to the office, able to allocate a greater proportion of the available daily time budget to the first home activity, thus embarking on the journey to work later in the day. In the case of Cranleigh, Dorking, Farnham, Godalming, Haslemere, West Byfleet and Woking individuals depart for work at around 07:45 hours whilst in the more accessible parts of Epsom and Leatherhead individuals depart from home at 08:06 hours. In contrast individuals living in the southeast of the county spend as little as 6.75 hours at home embarking on the journey to work 1.5 hours earlier than individuals living in the more accessible parts of Guildford.

Figures 11 and 12 respectively denote the departure time in minutes from the office to the home location and the duration of the work related activity. The figures indicate a complex pattern of time allocation and activity scheduling behaviour. The figures for instance show that in locations in which the journey times from the office to home are high, such as the eastern part of the county, an individual may embark on the journey home from work at around

17:00 hours. At locations where journey times from the office to the home are neither high or low (e.g. between 20 and 40 minutes journey time) individuals in the main embark on the homeward journey later in the day at between 18:15 and 19:05 hours. Figure 12 indicates that for these locations a greater amount of time, between 10.1 and 11.1 hours, is allocated to the work related activity. Figure 12 shows that for home locations in the vicinity of Guildford in which the greatest amount of time is allocated to the first home activity, a lower proportion of time within the daily time budget is allocated to the work activity than for alternative home locations. Figures 11 and 12 respectively indicate that individuals living in these locations will embark on the homeward journey at between 17:55 and 18:15 hours, having previously allocated between 9.75 and 10.1 hours to the work activity.

Figure 13 details the duration of the final home activity and indicates that for home locations in the vicinity of Guildford and Leatherhead up to 6.1 hours is allocated to the activity in contrast to alternative locations, where as little as 3.8 hours is allocated to this activity. The figure also indicates that in the southeast of the county where total journey times to and from the office location are high up to 6.1 hours is allocated to the final home activity. Figure 11 and 12 show that individuals living in these locations have scheduled less time for the work activity and depart earlier from the office location on the homeward journey than other locations within the county.

The pattern of activity scheduling and time allocation depicted in figures 7-13 is quite complex and shows that the travel and schedule disutility components, the utility derived from activity participation, the cost of travel and financial endowment jointly influence activity and travel behaviour. In particular it is evident that earlier time allocation and activity timing decisions directly influence downstream activity scheduling and time allocation decisions.

6. Summary & Conclusions

The theoretical and empirical activity scheduling and time allocation models and related accessibility measures outlined in this paper represent a significant advance on existing time allocation/activity scheduling models and accessibility measures. These existing time allocation and activity schedule models have in the main tended to make strong separability assumptions concerning the timing and duration dimensions of activity behaviour. In addition these existing models have rarely considered activity schedules, in the main considering a single disjointed activity or trip episode. In particular the incorporation of travel time, route delay, facility wait time, activity wait time, late start time, access/egress time, travel time variability disutility components, facilitates a more robust, realistic and detailed analysis of activity and travel behaviour. The holistic treatment of time allocation and activity scheduling within the

paper is conceptually and theoretically appealing since such activity/travel behavioural decisions in reality seldom occur independently of each other.

The simple case study presented within the paper highlights the strength of the theoretical and empirical model formulations and reveals that within a real world context even the simplest of activity schedules require a complex activity scheduling and time allocation process be undertaken. This activity scheduling and time allocation process has been shown to be dependent upon a range of factors including the marginal utility of activities, temporal variations in a number of disutility components, the cost of travel, cost of goods and services, financial endowment and knowledge of the performance and choices available within the integrated land use transport environment.

6.1 Future Research

Despite the potential advantages of the proposed activity scheduling and time allocation models, there are a number of areas which can be identified for further research. One of these involves assessing the performance of the time allocation and activity scheduling models on individual activity/travel behaviour resulting from a range of policy interventions including new/improved transport networks, changes in the reliability of transport networks, new/improved land use facilities, extended facility opening, improved scheduling of transport and land use services, flexible working practices, concessionary travel/fare schemes, the cost of travel, salary/taxation changes, new/improved forms of service delivery, changes in activity intensity, changes in the cost of goods and services.

An additional area of future research may involve introducing improvements to the structure of the activity utility formulation and the related marginal utility function in order to better reflect the interrelationship between the duration of an activity and its timing.

In addition to the aforementioned areas of research, another potential research avenue may involve the examination of how limitations in the cognitive ability of human beings to collect, store, analyse and optimise significant volumes of information prior to making their activity scheduling and time allocation choices will influence the selected activity schedule and the resulting level of accessibility. A future area of possible research could involve modifying the theoretical and empirical utility model formulations to reflect an individual subjected to bounded rationality, in which cognitive constraints on the decision making process exist.

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Table 1: Properties of the Change in Utility of Schedules Empirical Model Formulation

No.	Behavioural Properties of Equation 33
1	The first and second pair of square brackets reflect the effect on the utility of the entire activity schedule of a change in the timing of one or more activities.
2	The third and fourth pair of square brackets reflect the effect on the utility of the entire activity schedule of a change in the duration of one or more activities.
3	The fifth and sixth pair of square brackets reflect the effect on the utility of the entire activity schedule of a change in the intensity of one or more activities.
4	The seventh pair of square brackets reflect the effect on the utility of the entire activity schedule of a combination of a change in the timing and the duration of one or more activities.
5	The eighth pair of square brackets reflects the effect on the utility of the entire activity schedule of a combination of a change in the duration and the intensity of one or more activities.
6	The ninth pair of square brackets reflects the effect on the utility of the entire activity schedule of a combination of a change in the timing and the intensity of one or more activities.
7	The tenth pair of square brackets reflects the effect on the utility of the entire activity schedule of a change in the disutility (e.g. travel time, route delay, facility wait time, activity wait time, late start time, access/egress time and travel time variability.
8	The eleventh pair of square brackets reflects the effect on the utility of the entire activity schedule of a change in the cost of travel between one or more pair of activity locations.

Table 2: A Hypothetical Home-Work-Home Activity Tour

No.	Activity Description	Earliest Start Time	Latest Start Time
1	Home, prepare for the day.	00:00	08:30
2	Undertake paid employment at the office.	08:00	10:00
3	Home, prepare for the evening.	17:00	19:00

Table 3: Utility of Schedules Model Parameters

Symbol	Description	Value
θ	Travel time disutility coefficient	0.106
μ	Route delay disutility coefficient.	0.175
γ	Facility wait time disutility coefficient.	0.065
η	Activity wait time disutility coefficient.	0.060
ν	Late start time disutility coefficient.	0.254
ρ	Access/egress time disutility coefficient.	0.125
ψ	Travel time variability disutility coefficient.	0.215
w_a	Endowment/wage rate (£ per hour).	5.000
λ_a	Value of money parameter.	-0.110

Figure 1: Graphical Representation of the Utility Theoretic Framework

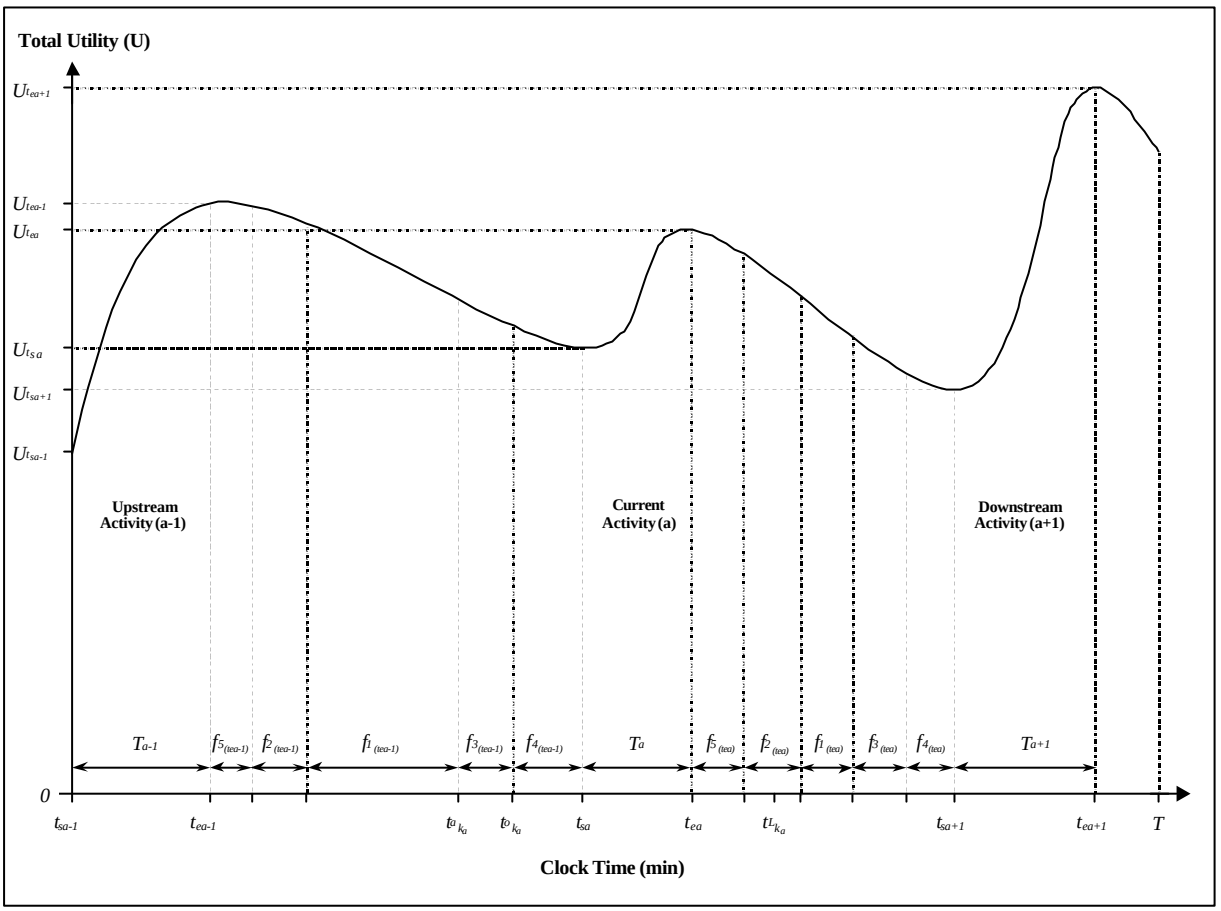


Figure 2: Examples of Activity Intensity-Duration Profiles

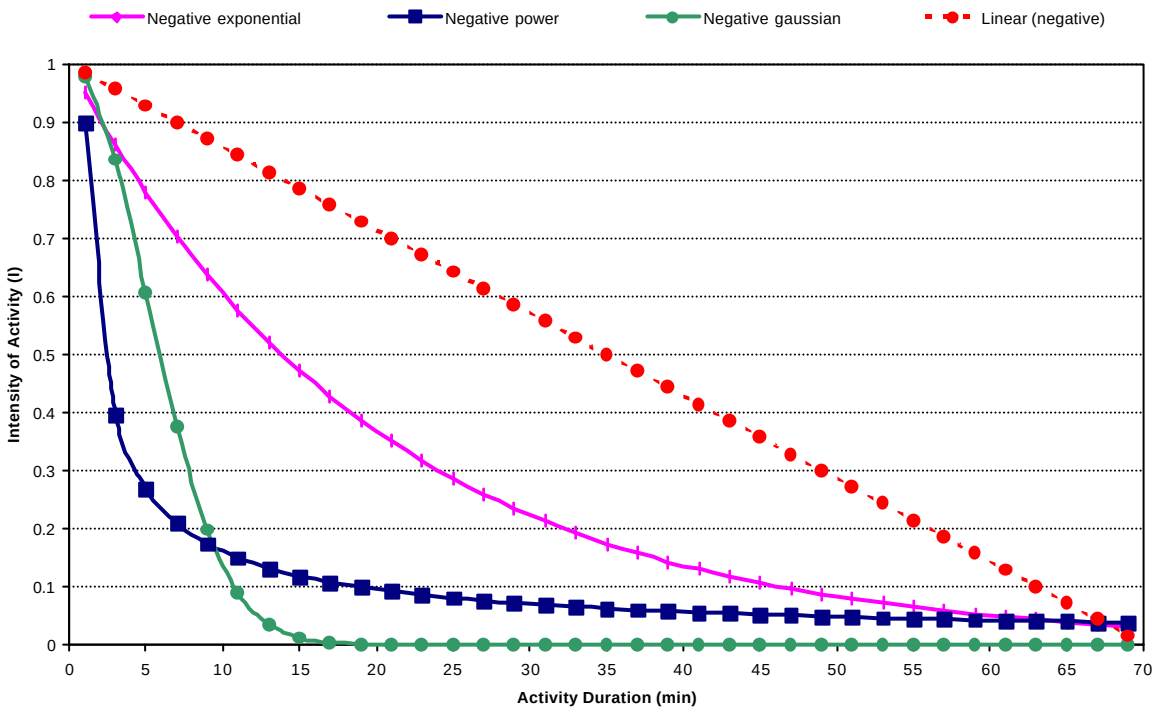


Figure 3: The Optimal Scheduling of Two Home Based Activities

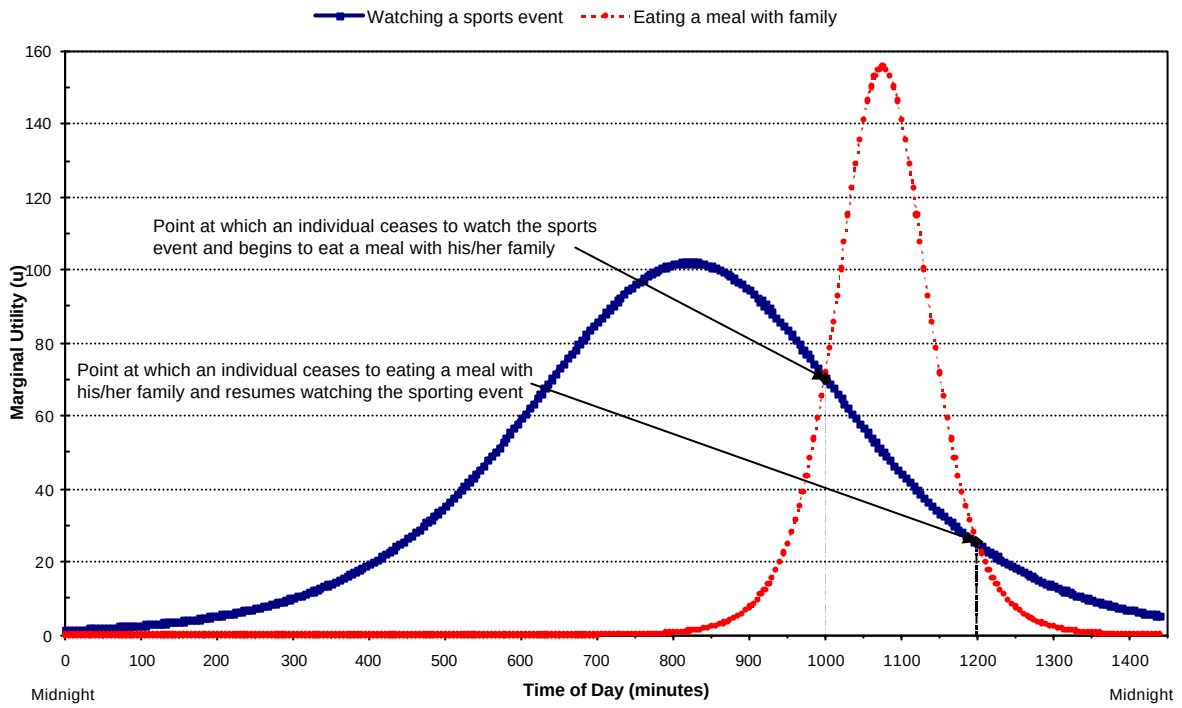


Figure 4: Activity Scheduling & Time Allocation Algorithm

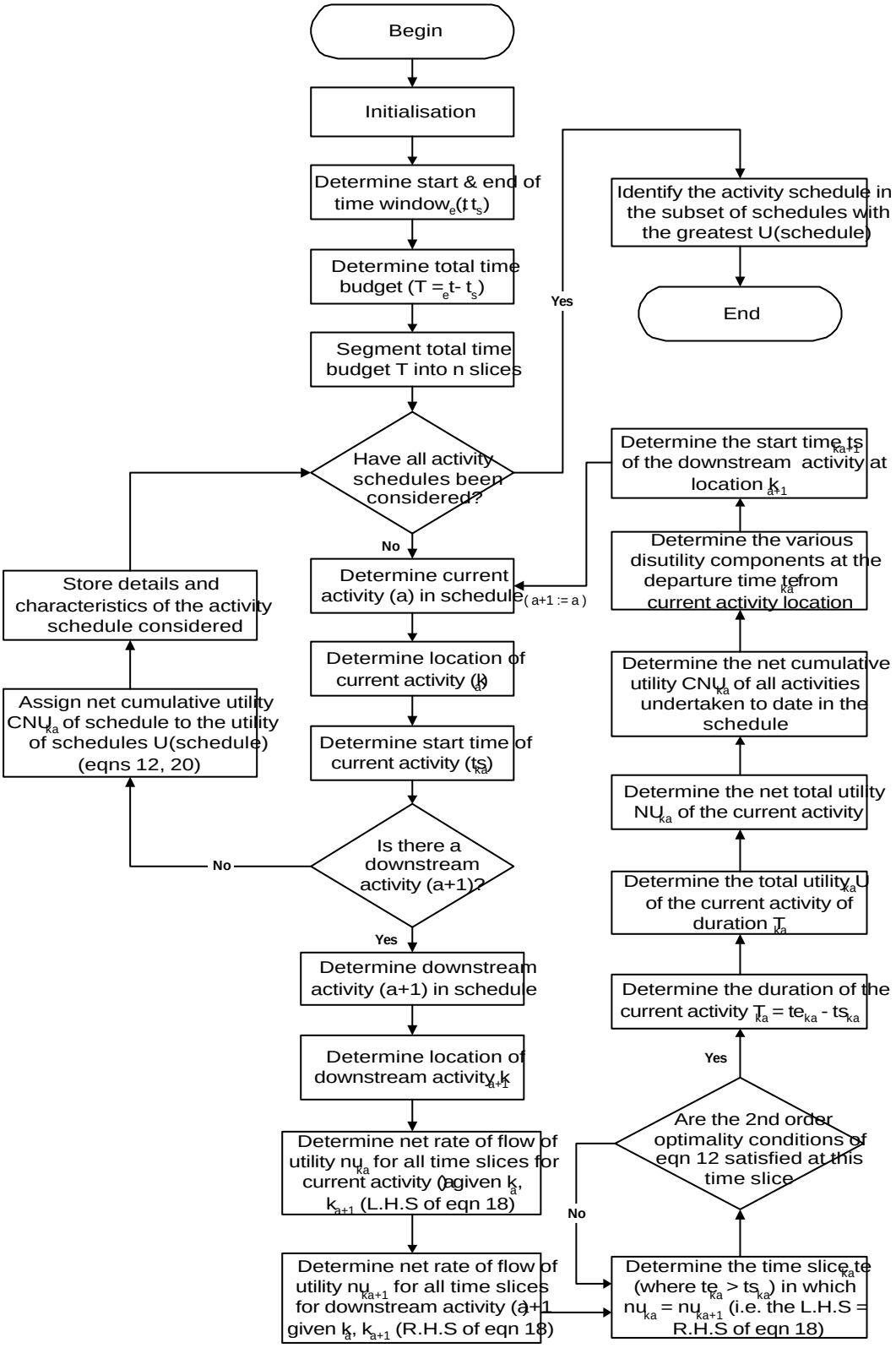


Figure 5: Surrey Local Bus Network - AM Peak Hourly Bus Frequencies (Mon-Fri)

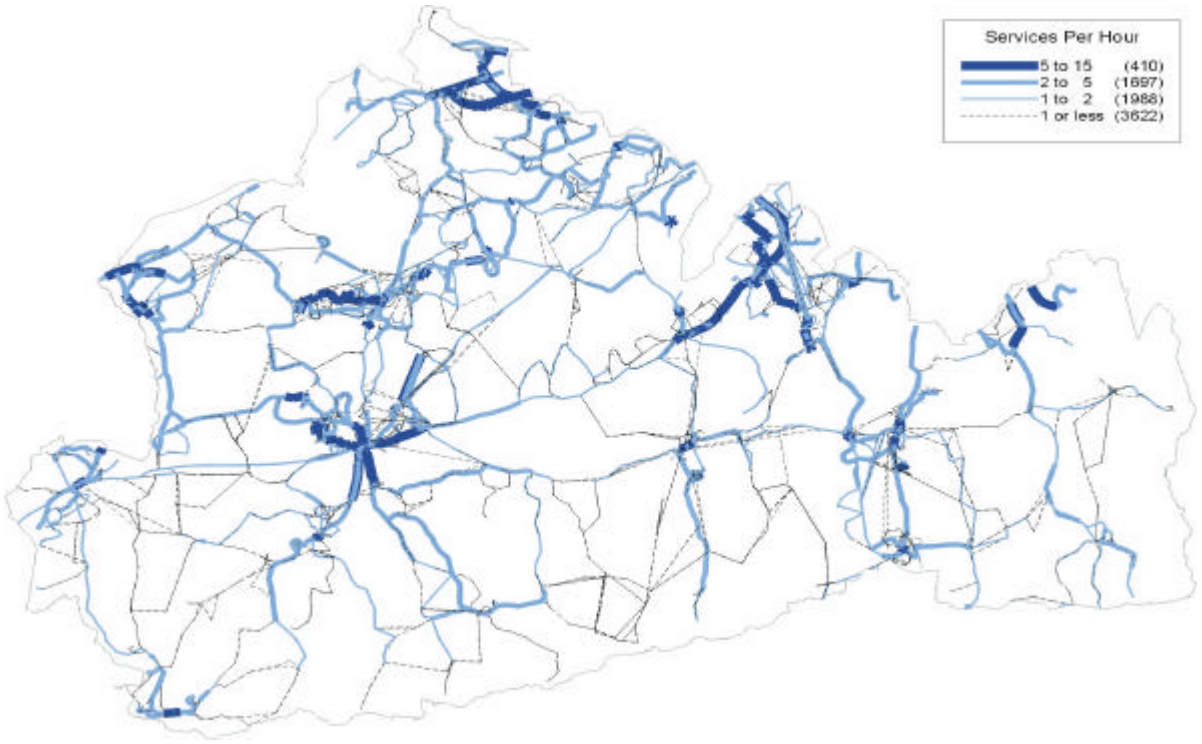
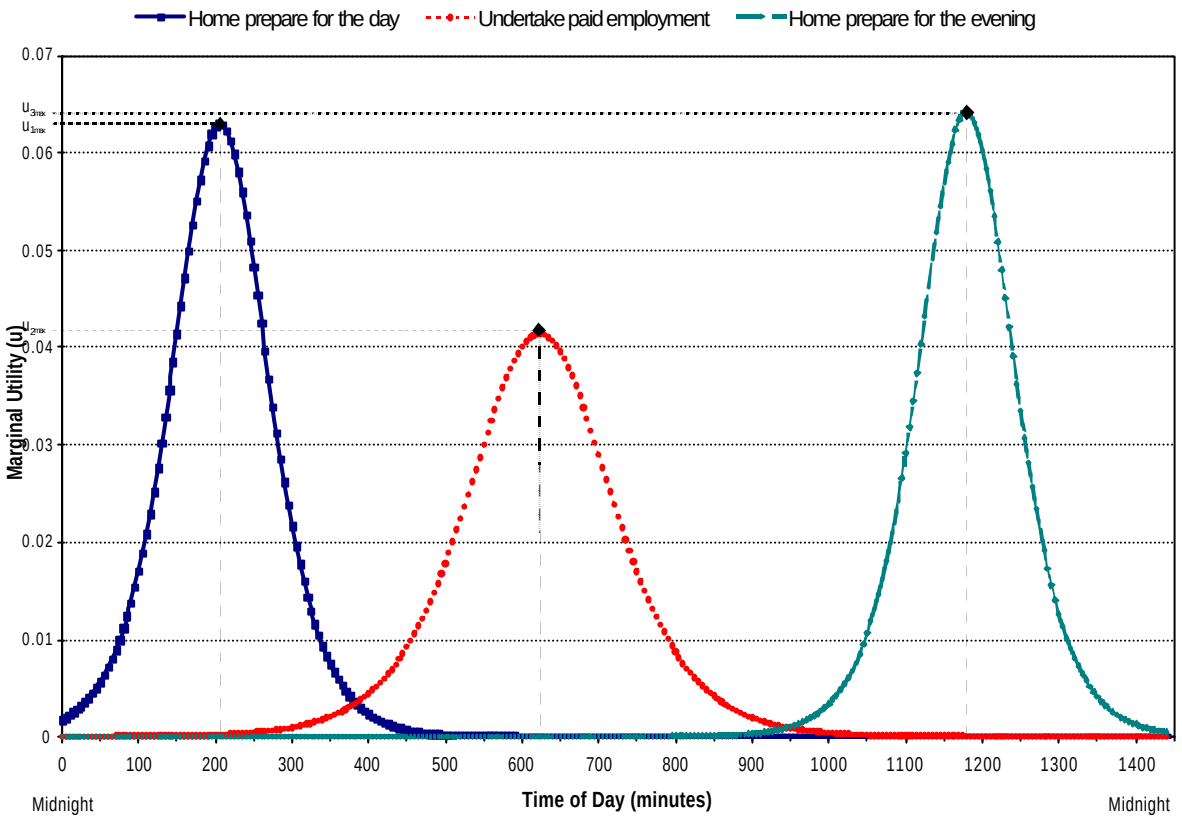


Figure 6: Marginal Utility Profiles of Activities Undertaken During a Home-Based Tour



Parameter \ Activity	Home - Morning	Work	Home - Evening
s_{l_a}	1.00	1.00	1.00
e_{l_a}	0.024	0.016	0.024
q_{l_a}	206	623	1,179
t_{bk_a}	0 *	0 *	0 *
t_{l_a}	0.000	0.500	0.000
a_{k_a}	0.0550	0.0015	0.0550
a_{l_a}	0.550	0.450	0.544
b_{l_a}	0.600	0.800	0.600
$t_{ck_a} - t_{ok_a}$	720	720	720

* A value has been assumed for graphical presentation.

Figure 7: Public Transport Travel Time from the Home to the Office

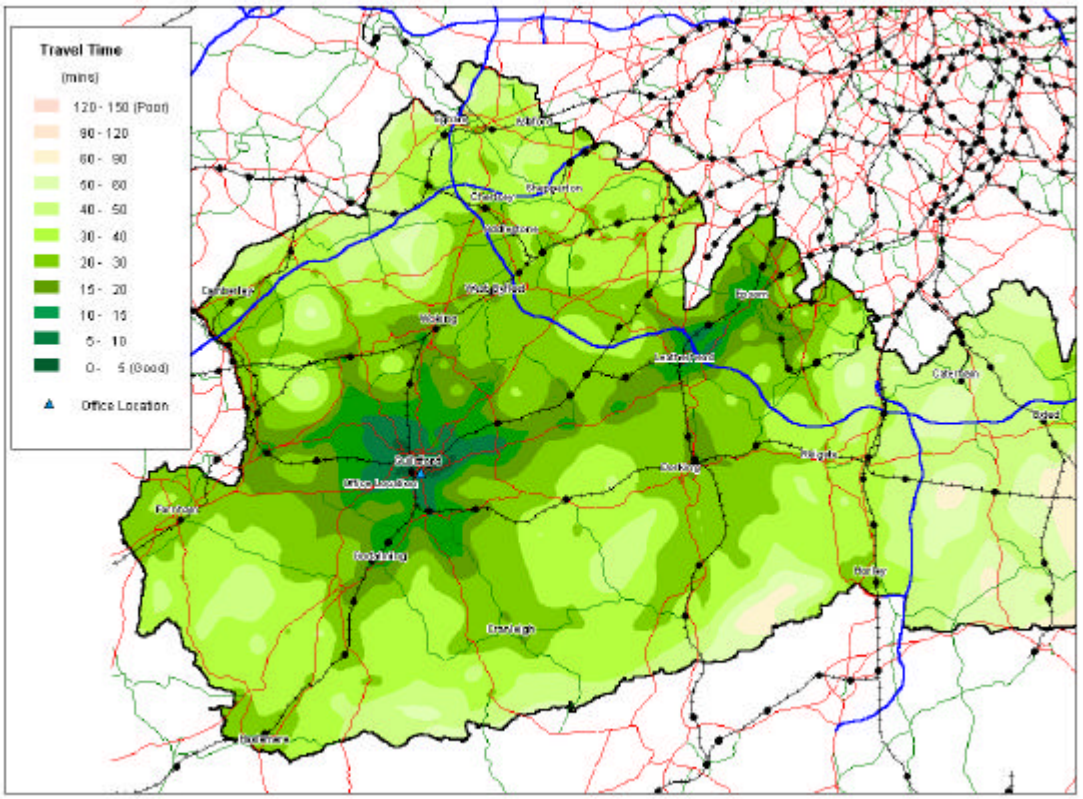


Figure 8: Public Transport Travel Time from the Office to the Home

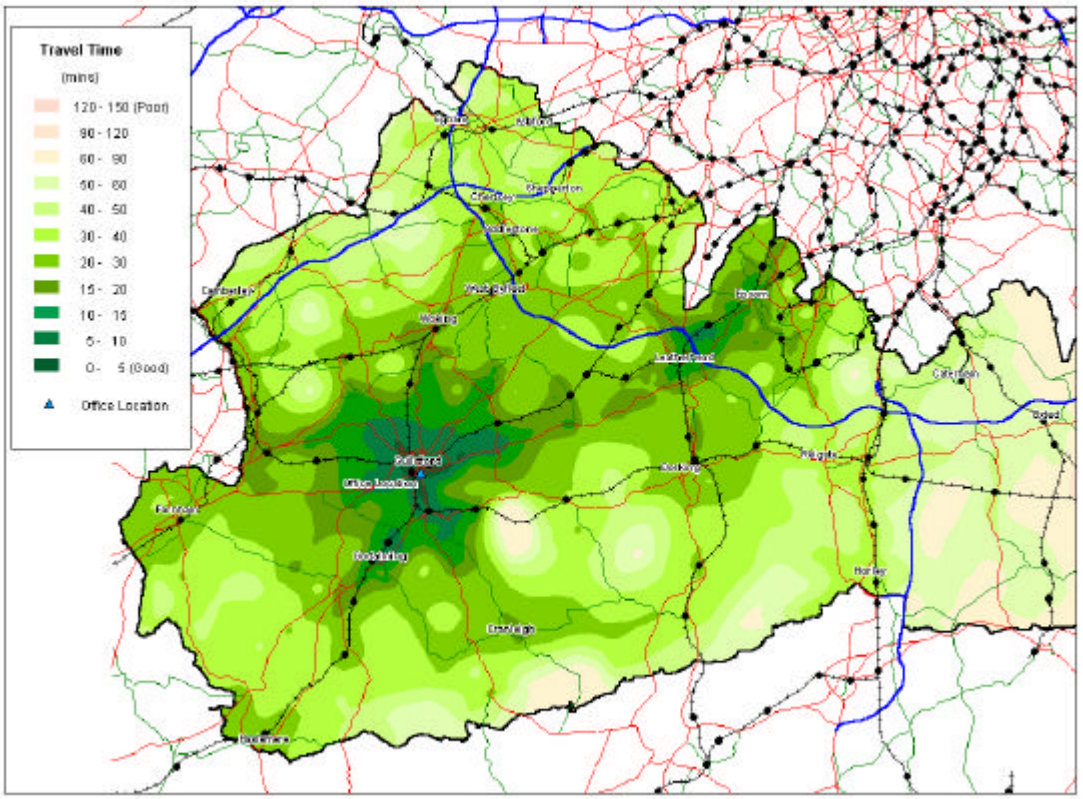


Figure 9: The Accessibility or Utility of Schedules of the Hypothetical Home-Based Tour

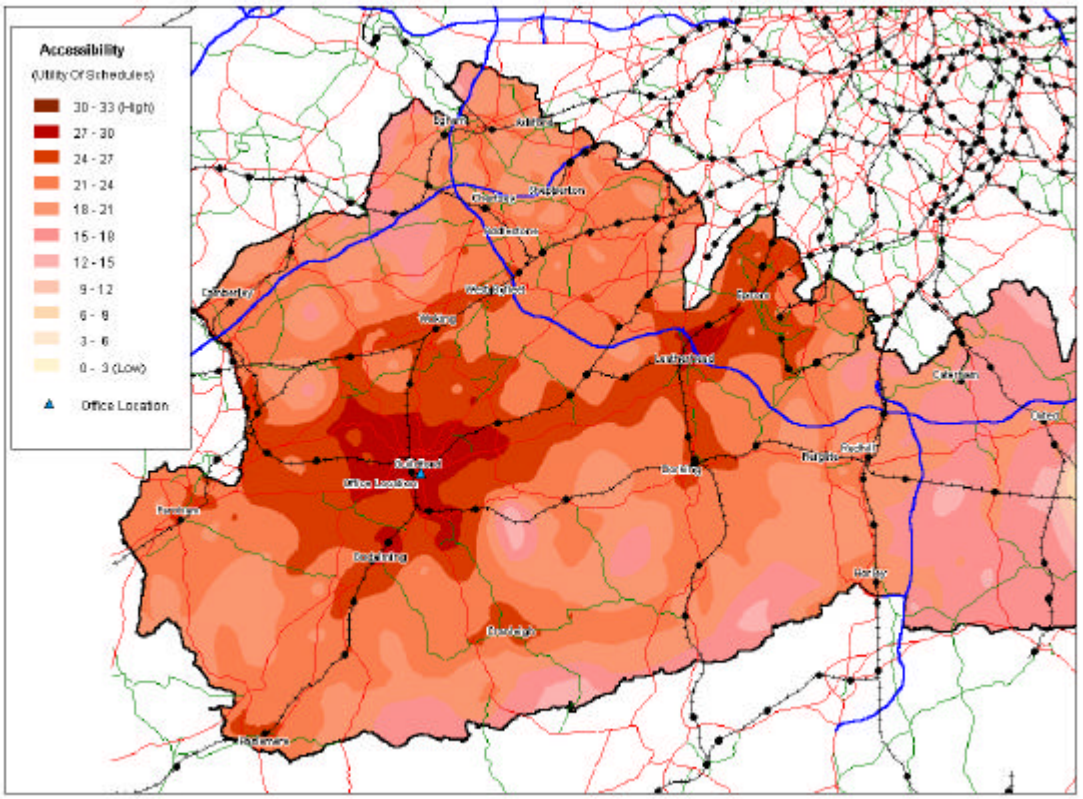


Figure 10: The Duration and Departure Time of the First Home-Based Activity

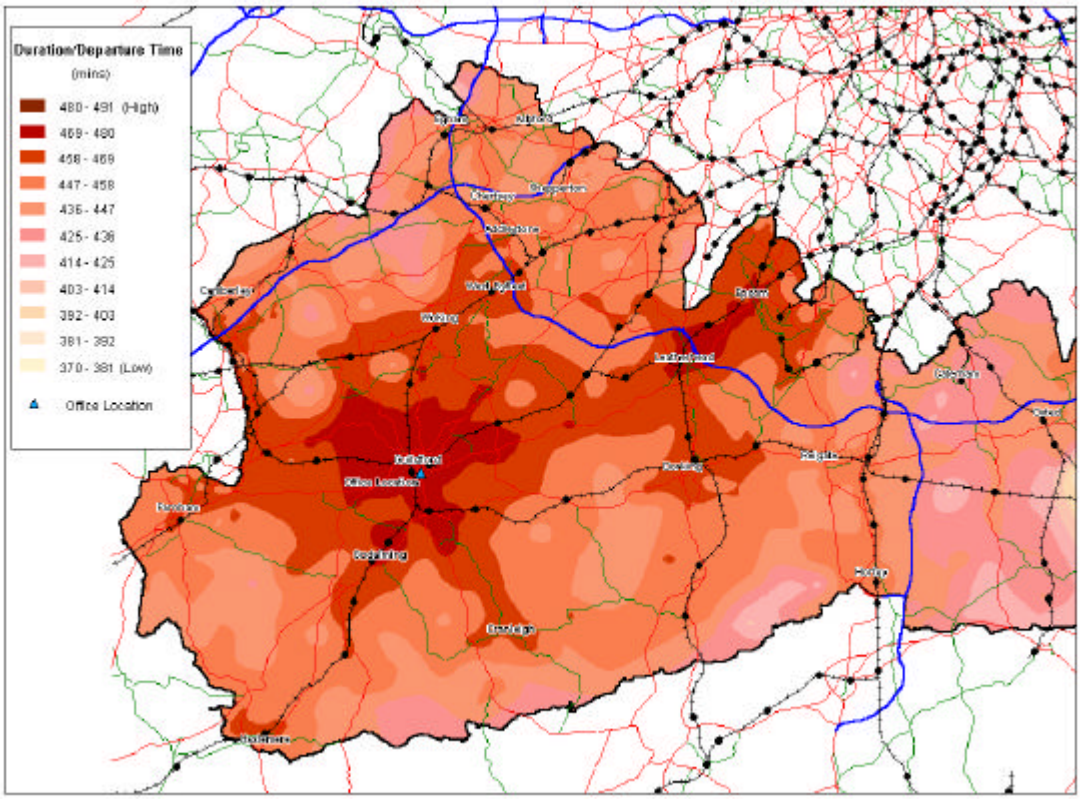


Figure 11: The Departure Time in Minutes from the Office to the Home

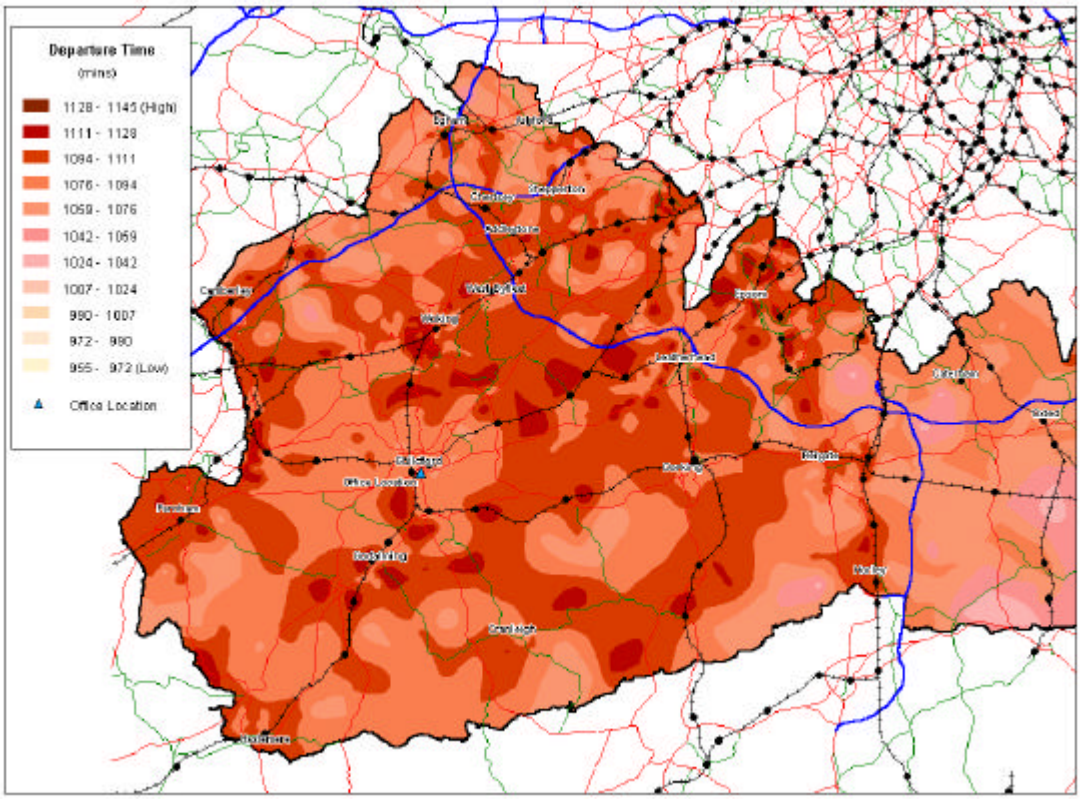


Figure 12: The Duration of the Work Activity

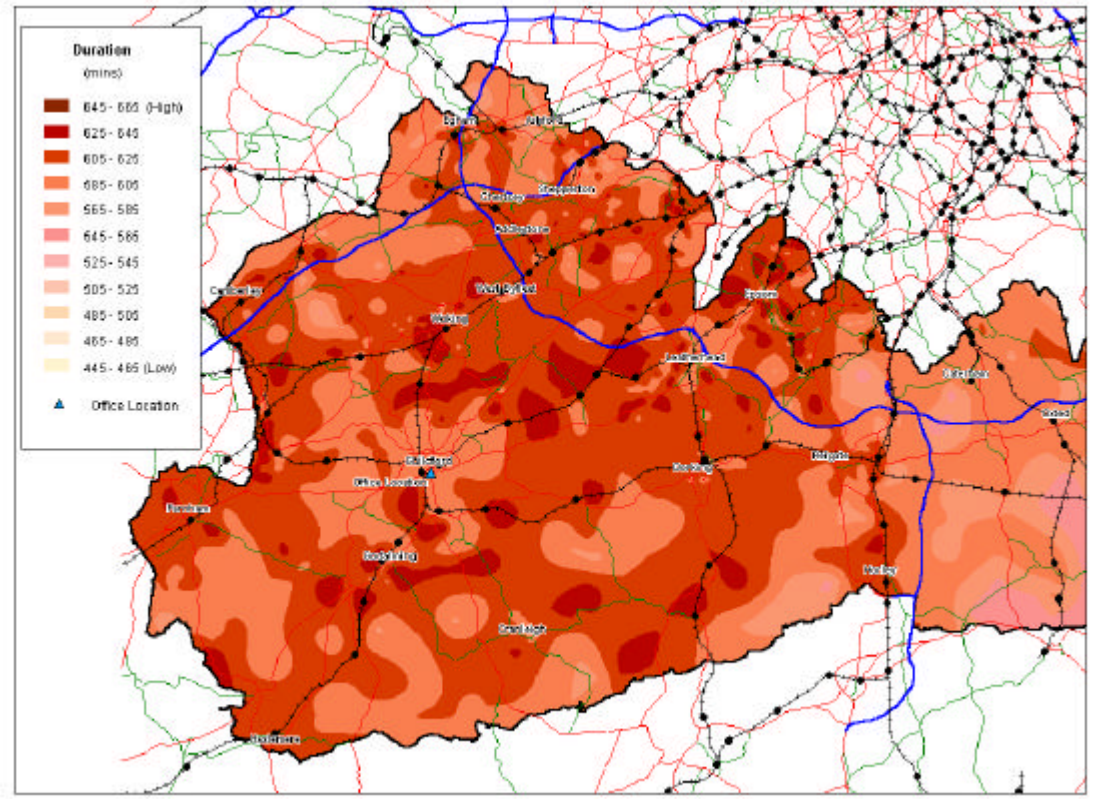


Figure 13: The Duration of the Second Home-Based Activity