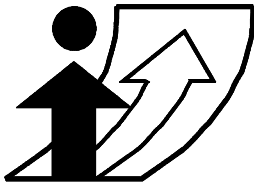


Modeling activity duration and travel choice from a common microeconomic framework

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Modeling activity duration and travel choice from a common microeconomic framework

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Abstract

Travel utility in discrete choice modeling is a (truncated) conditional indirect utility function (CIUF) whose specification can be derived from a consumer behavior framework that includes time assigned to activities besides good consumption, which means that the derivation of a CIUF requires the implicit existence of conditional demands for goods as well as for activity times. In this paper we apply this approach to a DeSerpa like framework, explicitly obtaining those demand models as well as the CIUF. Thus, information on time assigned to activities could be used to estimate conditional time assignment models that involve the same set of parameters as the mode choice model. The microeconomic specifications of the resulting activity duration models and the CIUF (which are far from being simple) are discussed, including the explicit calculation of all time values and extensions to consider other constrained activities.

Keywords

Consumer theory, time allocation, discrete choice, demand models.

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1. Introduction

Travel utility in discrete choice modeling is a (truncated) conditional indirect utility function (CIUF) whose specification can be derived from a consumer behavior framework that includes time assigned to activities besides good consumption. This was made evident in the pioneering article by Train and McFadden (1978), where the specification of modal utility using travel cost divided by the wage rate was first justified using a goods/leisure consumer framework with most of the properties of a model like the one proposed by Becker (1965), used later by Jara-Díaz and Farah (1987) for further developments.

The link between discrete travel choice models and the underlying consumer behavior framework was further reinforced in the literature on the value of travel time savings, particularly after Truong and Hensher (1985) and Bates (1987), who used the framework developed by DeSerpa (1971) in order to show that the ratio between the marginal utilities of travel time and travel cost had a correspondence with multipliers within time assignment models that include technical constraints.

By 1994, we showed that the derivation of a CIUF required the implicit existence of conditional demands for goods as well as for activity times (Jara-Díaz, 1994, 1998), a framework that was first applied by Jara-Díaz and Guevara (2002) for the joint estimation of a mode choice model and a labor supply model. In this article we expand that framework to the whole set of goods and activities, taking into consideration that travel (mode) choice and activity demand models come from a common microeconomic framework such that their full specifications are linked through common parameters. We show that estimating both type of models from the same population makes it possible to obtain very rich information regarding individual preferences and all the values of time as defined in the literature.

The paper is organized as follows. First, we present a fairly general consumer behavior model that includes time assignment to activities, following DeSerpa (1971), from which a discrete travel choice model can be derived, necessarily obtaining time assignment and goods consumption models as intermediate steps. The different concepts of value of time are then presented. Using a Cobb-Douglas form for direct utility, the activity-consumption model is solved *conditional* on the mode chosen for some trip (e.g. work), from which analytically explicit conditional solutions for goods, $X^*(w, c_t, T_t)$, and activities, $T^*(w, c_t, T_t)$, are obtained as functions of the wage rate w , travel cost c_t and travel time T_t . These solutions are replaced back in the direct utility function, obtaining $U(T^*, X^*)$, which is the CIUF usually called modal utility that commands mode choice. Thus, an explicit system of equations representing a set of

activity duration and goods consumption models (including labor supply) is obtained. As these are all derived from the same framework, they are shown to share common parameters. Then we postulate, as suggested by Jara-Díaz (1998), that information on time assigned to activities could be used to estimate conditional time assignment models that involve the same set of parameters as the mode choice model. The microeconomic specifications of the resulting activity duration models and the CIUF (which are far from being simple) are discussed, including the calculation of time values and extensions to consider other constrained activities within a fairly general framework.

2. A model system for travel, activity times and goods consumption

Let us consider the following model after DeSerpa(1971)

$$\text{Max } U(T, X) \quad (1)$$

$$wT_w - \sum_{k \in K} P_k X_k - c_t \geq 0 \leftarrow \lambda \quad (2)$$

$$\tau - \sum_{i \in I} T_i - T_w - T_t = 0 \leftarrow \mu \quad (3)$$

$$T_t - T_t^{MIN} \geq 0 \leftarrow \kappa_t \quad (4)$$

where U is the utility function, X , P and T are vectors of goods consumed, goods prices and time assigned to activities respectively, T_w corresponds to variable work, w is the wage rate, c_t is travel cost, τ is the length of the period considered, T_t^{MIN} corresponds to a exogenous minimum travel time restriction. I and K and are the sets of all activities (but work and travel) and all goods respectively. Finally, λ , μ and κ_t are Lagrange multipliers.

In this model utility depends on consumption of all goods and on the time assigned to all activities (including work and travel time, unlike Becker, 1965. See also Evans, 1972). There are income (2), time (3) and exogenous or technical (4) constraints. For a given mode choice, the solutions for the endogenous variables are conditional on the wage rate (w) and on both minimum travel time (T_t^{MIN}) and travel cost (c_t).

The interpretation of the Lagrange multipliers within the framework of non-linear programming, establishes that they correspond to the variation of the objective function evaluated at

the optimum due to a marginal relaxation of the corresponding restriction. Thus, λ is the *marginal utility of income*, μ is the *marginal utility of time as a resource* and κ_i is the *marginal utility of diminishing the travel time constraint*. These multipliers are helpful to define the three concepts of the value of the time identified by DeSerpa (1971). These are: a) the value of time as a resource, that values monetarily the relaxation of the total restriction of time, μ/λ ; b) the value of assigning time to a specific activity, $(\partial U/\partial T_i)/\lambda$; and the value of saving time in a specific constrained activity (travel in this case), κ_i/λ , that values monetarily the change in utility due to a reduction in T_i^{MIN} .

Manipulating first order conditions, The following relations can be obtained

$$\frac{\mu}{\lambda} = w + \frac{\partial U/\partial T_w}{\lambda} \quad (5)$$

$$\frac{\kappa_j}{\lambda} = \frac{\mu}{\lambda} - \frac{\partial U/\partial T_j}{\lambda} = w + \frac{\partial U/\partial T_w}{\lambda} - \frac{\partial U/\partial T_j}{\lambda} \quad (6)$$

As shown by Bates (1987) and Jara-Díaz (2002a), the value of saving time in travel can be estimated directly from a discrete choice model as the ratio between the marginal utilities of time and cost. Now we will show how all value of time concepts can be estimated empirically.

Following Jara-Díaz and Guevara (2002), let us now consider a Cobb-Douglas utility function

$$\text{Max} \quad U = \Omega T_w^{\theta_w} T_t^{\theta_t} \prod_{i \in I} T_i^{\theta_i} \prod_{k \in K} X_k^{\eta_k} \quad (7)$$

and constraints (2) to (4). First order conditions can be obtained for all activities and goods. These are

$$\frac{\partial U}{\partial T_i} = \mu = \frac{\theta_i}{T_i} U \quad \forall i \in I \quad (8)$$

$$\frac{\partial U}{\partial T_w} + \lambda w - \mu = \frac{\theta_w}{T_w} U + \lambda w - \mu = 0 \quad (9)$$

$$\frac{\partial U}{\partial T_t} - \mu + \kappa_t = \frac{\theta_t}{T_t} U - \mu + \kappa_t = 0 \quad (10)$$

$$\frac{\partial U}{\partial X_k} - \lambda P_k = \frac{\eta_k}{X_k} U - \lambda P_k = 0 \quad \forall k \in K \quad (11)$$

$$(T_t - T_t^{MIN}) \kappa_t = 0 \quad (12)$$

From equation (11) the expenditure on good k can be obtained. Adding over all goods, defining B as the summation over all goods exponents and using equation (2) in its active form, we get

$$\frac{\lambda}{U} = \frac{B}{(wT_w - c_t)} \quad (13)$$

Similarly, solving for T_i from equation (8), adding over all activities belonging to set I and using constraint (3) we obtain

$$\frac{\mu}{U} = \frac{A}{(\tau - T_w - T_t^{Min})} \quad (14)$$

where A is defined as the summation over all activity exponents. Using equations (9), (13) and (14) we get a quadratic equation for time assigned to work, i.e.

$$\frac{\theta_w}{T_w} + w \frac{B}{(wT_w - c_t)} - \frac{A}{(\tau - T_w - T_t^{Min})} = 0 \quad (15)$$

Solving this quadratic equation for T_w yields

$$T_w^* = \frac{(B + \theta_w)(\tau - T_t^{Min}) + (A + \theta_w) \frac{c_t}{w} \pm \sqrt{\left((B + \theta_w)(\tau - T_t^{Min}) + (A + \theta_w) \frac{c_t}{w} \right)^2 - 4\theta_w (\tau - T_t^{Min}) \frac{c_t}{w}}}{2(A + B + \theta_w)} \quad (16)$$

In order to investigate whether equation (16) has two roots or only one is valid, we can solve equation (15) for $\theta_w = 0$, which yields

$$T_w^* = \frac{B}{A + B} (\tau - T_t^{Min}) + \frac{A}{A + B} \frac{c_t}{w} \quad (17)$$

This represents the optimal work time for an individual that extracts neither utility nor disutility from work (a goods-leisure trade-off model). Now we can explore the general expres-

sion (16) as $\theta_w = 0$ approaches zero. With the minus sign T_w^* approaches zero, while with the plus sign expression (17) is recovered. This shows that only the plus sign should be considered in equation (16).

Defining

$$\alpha = \frac{(A + \theta_w)}{2(A + B + \theta_w)}; \quad \beta = \frac{(B + \theta_w)}{2(A + B + \theta_w)}; \quad \gamma_j = \frac{\theta_j}{(A + B + \theta_w)} \quad \forall j \in I \wedge j = t \quad (18)$$

equation (16) can be written as

$$T_w^* = \beta(\tau - T_t^{Min}) + \alpha \frac{c_t}{w} + \sqrt{\left(\beta(\tau - T_t^{Min}) + \alpha \frac{c_t}{w} \right)^2 - (2\alpha + 2\beta - 1)(\tau - T_t^{Min}) \frac{c_t}{w}} \quad (19)$$

Equation (19) is a model for the labor supply of individuals who are characterized by direct preferences implicitly represented by α and β , which are the parameters to be estimated. In this model, travel time, travel cost and the wage rate are the exogenous variables and T_w is dependent variable.

Having solved for T_w , we can solve for the optimal time assigned to the remaining activities as well. To do this, note that from equations (8) and (14) we can get

$$T_i = \frac{\theta_i}{A} (\tau - T_w^* - T_t^{Min}) \quad \forall i \in I \quad (20)$$

It is relevant to observe that the Cobb-Douglas form for utility has a property that is reflected in our result (20), but in a slightly different way. In our model of time assignment – goods consumption, this property states that the time freely assigned to an activity is a proportion of the available time, which in this case is the total minus the time assigned to work and travel. The difference here is that the decision on time assignment depends on the mode characteristics (time and cost) through T_w .

Analogously, from equations (11) and (13) the optimal (conditional) consumption of every good can be obtained as

$$X_k = \frac{\eta_k}{P_k B} (wT_w^* - c_t) \quad \forall k \in K \quad (21)$$

Again, equation (21) shows that the expenditure in every good is a proportion of available income. Just as in the unconstrained activities case, consumption decisions are linked to mode choice (travel time and cost) through the optimal work time and cost directly.

Having solved explicitly for time assignment to activities and optimal consumption conditional on mode choice, we can obtain an explicit expression for the conditional indirect utility function (CIUF) that represents modal utility. This is obtained by replacing the optimal values (functions) from equations (19), (20) and (21) in (7), which yields

$$V_t = \left(\frac{\Omega}{A^A B^B} \prod_{k \in K} \left(\frac{\eta_k}{P_k} \right)^{\eta_k} \prod_{i \in I} (\theta_i)^{\theta_i} \right) (wT_w^* - c_t)^B (\tau - T_w^* - T_t^{Min.})^A T_w^{*\theta_w} T_t^{Min.\theta_t} \quad (22)$$

As the problem is invariant to monotonic transformations of utility, we can normalize by taking root $(A+B+\theta_w)$ in equation (22). Using definitions (18) this yields

$$V_t = \hat{\Omega} (wT_w^* - c_t)^{1-2\alpha} (\tau - T_w^* - T_t^{Min.})^{1-2\beta} T_w^{*2\alpha+2\beta-1} T_t^{Min.\gamma_t} \quad (23)$$

Equations (19), (20), (21) and (23) form a model system for activities time assignment, goods consumption and mode choice involving common parameters (α and β), goods specific parameters (η_k) and activity specific parameters (θ_i). This complete system improves over the one formulated and experimented by Jara-Díaz and Guevara (2002), which included the labor supply equation (19) and a linear version of the CIUF only.

Using a complete system of models as the one described above is particularly useful not only for the efficient estimation of parameters, but also to calculate the different concepts of time values presented earlier, directly from the results. These are the value of time as a resource (value of leisure), the value of assigning time to a specific activity, and the value of saving time in a specific constrained activity (travel in this case). First, from (13), (14) and (18) the value of leisure can be calculated as

$$\frac{\mu}{\lambda} = \frac{1-2\beta}{1-2\alpha} \frac{(wT_w^* - c_t)}{(\tau - T_w^* - T_t^{Min.})} \quad (24)$$

From (10), (13) and (18), the value of assigning time to travel is

$$\frac{\partial U / \partial T_t}{\lambda} = \frac{\theta_t}{T_t} \frac{U}{\lambda} = \frac{\gamma_t}{1-2\alpha} \frac{(wT_w^* - c_t)}{T_t^{Min.}} \quad (25)$$

From equations (10), (13) and (14), the value of κ_t/λ depends on the ratios A/B and θ_t/B , which yields the value of saving travel time as

$$\frac{\kappa_t}{\lambda} = \frac{\mu}{\lambda} - \frac{\theta_t}{T_t} \frac{U}{\lambda} = \frac{1-2\beta}{1-2\alpha} \frac{(wT_w^* - c_t)}{(\tau - T_w^* - T_t^{Min})} - \frac{\gamma_t}{1-2\alpha} \frac{(wT_w^* - c_t)}{T_t^{Min}} \quad (26)$$

Finally, from equations (9), (13) and (18) the value of assigning time to work is

$$\frac{\partial U / \partial T_w}{\lambda} = \frac{2\alpha + 2\beta - 1}{1 - 2\alpha} \frac{(wT_w^* - c_t)}{T_w^*} \quad (27)$$

Note that the value of saving each of the usual components of travel time (in vehicle, waiting and walking) can be calculated as a relatively simple extension of these results. If other modal time (e.g. waiting) was considered, a new minimum time constraint (analogous to equation 4) should be added. This would generate a new multiplier (κ_w) and an activity parameter in utility (θ_e). Thus, new first order conditions would be added, analogous to equations (10) and (12). Within a new equation resembling (26) a parameter γ_e (analogous to γ_t) would appear, and the subjective value of saving waiting time (κ_e/λ) would be defined. By construction, the sum of the minimum times ($T_e^{Min} + T_t^{Min}$) would replace the minimum travel time (T_t^{Min}) in equation (14).

3. The complete and general system of equations

The system derived in the previous section can be extended beyond the travel activity, to encompass other restricted activities as well. To begin with, note that equation (20) shows that unconstrained activities (those that are freely assigned more time than the minimum) must have positive marginal utilities (positive θ_t), otherwise they would not be undertaken. Besides, every unpleasant activity (negative θ_t) will be assigned the exogenous minimum, because the sign of its marginal utility is constant. This does not mean that an activity that is assigned the minimum time is necessarily unpleasant, because the optimal time assignment could be less than the exogenous minimum. The treatment of the constrained activities is similar to that of travel time within the model. Let I and R be the sets of all unrestricted and restricted activities respectively. Then equation (14) can be generalized to

$$\frac{\mu}{U} = \frac{A}{\left(\tau - T_w - \sum_{r \in R} T_r^{Min} \right)} \quad (28)$$

Analogously, exogenous minimum consumption levels (fixed expenses) or non work income can be included in the model as well. Fixed income and fixed expenses can be included in a way that is similar to that of travel cost, and can be added (or subtracted) without altering the model or the first order conditions. Let J be the set of goods whose consumption has a minimum (active), letting K denote the unrestricted goods and I_f the fixed income. The equation (13) turns into

$$\frac{\lambda}{U} = \frac{B}{\left(wT_w + \left(I_f - \sum_{j \in J} P_j X_j^{Min.} \right) \right)} \quad (29)$$

Noting that A and B should represent summations over the unrestricted variables, and defining

$$G_f = \left(\sum_{j \in J} P_j X_j^{Min.} - I_f \right), \quad T_f = \sum_{r \in R} T_r^{Min.} \quad \text{and} \quad \varphi_k = \eta_k / (A + B + \theta_w) \quad (30)$$

we get the general version of equation (19), the general model for labor supply (31), that generates the complete generalized system, i.e.

$$T_w^* = \beta(\tau - T_f) + \alpha \frac{G_f}{w} + \sqrt{\left(\beta(\tau - T_f) + \alpha \frac{G_f}{w} \right) - (2\alpha + 2\beta - 1)(\tau - T_f) \frac{G_f}{w}} \quad (31)$$

$$T_i^* = \frac{\gamma_i}{(1 - 2\beta)} (\tau - T_w^* - T_f) \quad \forall i \in I \quad (32)$$

$$X_k^* = \frac{\varphi_k}{P_k(1 - 2\alpha)} (wT_w^* - G_f) \quad \forall k \in K \quad (33)$$

$$V = \tilde{\Omega} (wT_w^* - G_f)^{1-2\alpha} (\tau - T_w^* - T_f)^{1-2\beta} T_w^{*2\alpha+2\beta-1} \prod_{r \in R} T_r^{Min \gamma_r} \prod_{j \in J} X_j^{Min. \varphi_j} \quad (34)$$

Note that because of the restrictions on consumption (2) and time (3) only up to $n-1$ time assignment or good consumption models can be estimated (with n the cardinal of the corresponding set of unrestricted activities or goods). On the other hand, one can formulate and estimate as many discrete choice models as restricted variables exist, unless one choice determines two or more variables simultaneously. In many cases one does not know exactly which activities (or goods) are restricted, which is something that can be explored empirically on R and J in definitions (30).

One of the advantages of the model system as derived here is that data can be accommodated to different degrees of aggregation in the variables, because adding activities (or goods) do not change the structure of the model. This can be observed directly from parameters A and B , which can be associated with those of leisure and a generalized good respectively in a fully aggregated goods-leisure-work-restricted activities model. In such a fully aggregated model, the derivation of the labor supply equation (the fundamental one) would not change, and equations (13) and (14) would be the first order conditions directly.

4. Conclusions and further research

We have shown that coupling microeconomically founded activity models and travel choice models can be quite rewarding from the viewpoint of the understanding of individual behavior. This had been explored in the pioneering work by Train and McFadden (1978) within a Becker (1965) like framework, in a fairly aggregated manner. By including work and travel as potential sources of direct utility, we have been able to obtain a system of models for activity time assignment, goods consumption and travel, within a fairly general consumer behavior framework including time. This has a number of advantages and poses a number of challenges as well.

From a conceptual viewpoint, making an explicit link between the CIUF and a complete consumer behavior model shows that flexibility within the modal utility has a microeconomic meaning. Expression (23) involves travel time playing two different roles, i.e. diminishing available time (through the optimal labor supply) and as a direct source of (dis)utility. On the other hand, the marginal utility of income (minus the partial derivative with respect to travel cost) is far from being constant or simple. This means that non-linear formulations of the CIUF should not be proposed on pure “flexibility gains” grounds but mostly on its underlying meaning. Further, we have been able to relate explicitly the parameters of the utility function with those of the CIUF, which greatly helps the interpretation of the results in terms of the different concepts of value of time and the marginal utilities of the different activities.

The joint estimation of the activity models, the goods consumption models and the discrete travel choice models poses some important additional challenges. One is to generate detailed information on activities and travel, specifically obtained to make experiments with this framework, including detailed data regarding the work contract. Also, information on consumption patterns will make the corresponding consumption models useful as well. On the analytic side, a second line to move on is to consider more complete microeconomic frameworks, as the one suggested by Jara-Díaz (2003) regarding the technical constraints, to gener-

ate new models that include novel dimensions regarding goods-activities production functions. Finally, there is an econometric challenge in the joint estimation of activity-travel models with a microeconomic basis; the stochastic structure of the activity model should be discussed further as part of this task. The extension of this approach to consider many constrained activities suggests that this could be the framework for a general system encompassing a series of continuous and discrete choices modeled simultaneously.

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