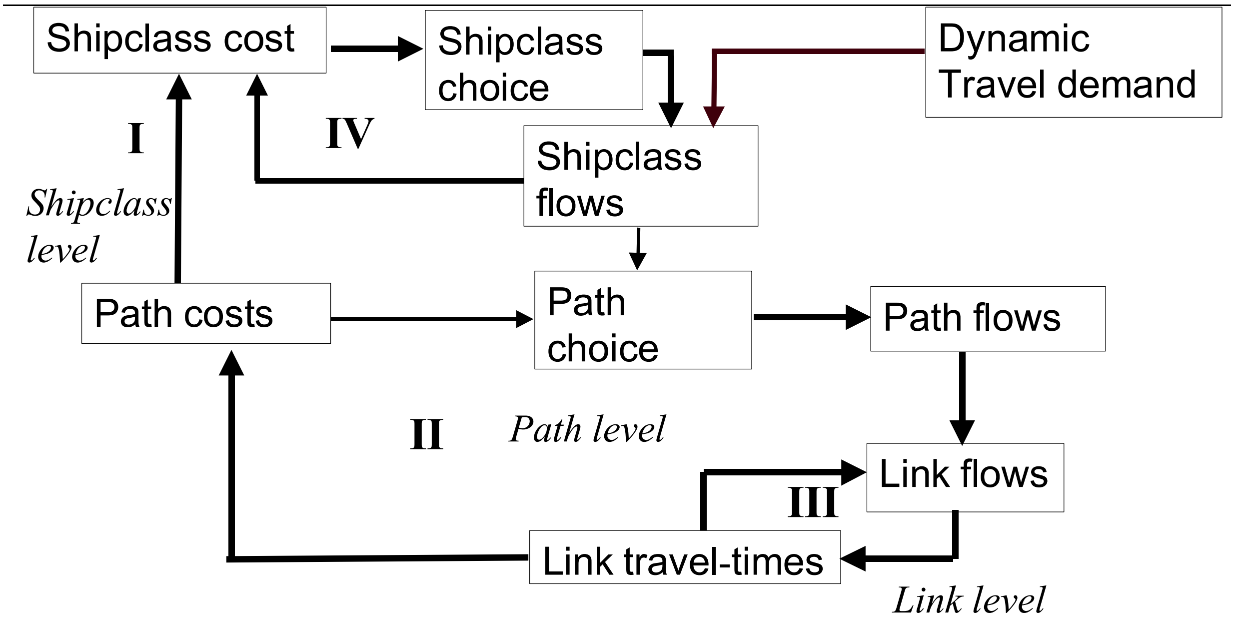

Simultaneous choice of shipclass and route in a dynamic network equilibrium framework

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A Variational Inequality formulation for combined shipclass and route choice

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The Netherlands has a finely meshed system of inland waterways that connects most industrial areas and serves as a gateway to industrial areas in Germany. As a result, inland shipping accounts for about 18% of the tonnage in goods transport; road transport accounts for 81% (internal), and rail transport for about 2% (total) or 1% (internal). A model system (called the BVMS) has been developed to analyse the impact of policy measures on freight traffic aspects such as shipclass and traffic flow. The BVMS is a dynamic model system with explicit consideration of queuing at bottlenecks such as locks and quays.

In this paper we present an alternative formulation for a specific aspect of this model system: the simultaneous choice of shipclass and route. It turns out that this problem can be formulated as a route-choice problem in a *supernetwork* in which shipclass choice and route choice can be modelled simultaneously. This paper presents an equilibrium formulation of simultaneous choice of ship class and route, unencumbered by the practical constraints that arise in the context of the BVMS. The equilibrium formulation can be recast as a Variational Inequality problem and solved using existing software for multiclass dynamic equilibrium assignment.

Keywords

Freight modelling, Variational Inequality, Supernetwork, International Conference on Travel Behaviour Research, IATBR

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1. Introduction

The Netherlands has a finely meshed system of inland waterways that connects most industrial areas, and serves as a gateway to key industrial areas in Germany. As a result, inland shipping accounts for about 18% (national + international) or 22% (national) of the tonnage in goods transport; road transport accounts for 46% (total) or 81% (internal), and rail transport for about 2% (total) or 1% (internal) (source: Statistics Netherlands; see Table 2).

1.1 Literature review

A general framework for freight mode-choice models can be found in [Roberts et al. (1977)]. A recent overview of strategic freight network planning models can be found in [Friesz (2000)]. A modelling approach to the inland shipping problem based on linear programming was proposed in [Khoshyaran (2000)]. A proposal to develop a model for inland shipping along the lines of a classical transportation model based on a demand-supply approach, the “Masterplan” for the BVMS was proposed in [Tavasszy et al. (1998)]. The approach and principles proposed in this proposal were taken as a starting point for model development.

The state of the goods transport market in The Netherlands is described in [De wit and Van Gent (1996)]. A sketch of the inland shipping model currently under development is given in [Catalano and Van der Zijpp (2001)]. This article builds on the model proposed in [Lindveld and Catalano (2000)].

A simulation-based approach to inland-shipping traffic assignment was presented in [Van der Zijpp and Bliemer (1999)]. This approach was used in a real-world implementation of a model system for inland shipping known as the BVMS (“BinnenVaart Model Systeem”) project commissioned by the AVV.

In [Nagurney and Dong (2002)] we find an overview of how to use supernetworks to incorporate various decisions (e.g. commuting versus telecommuting) that do not correspond to physical routes.

1.2 Approach taken in this paper

Model systems have been developed to analyse the impact of policy measures on freight traffic aspects such as mode split and traffic flows. Such models always reflect the practical constraints under which they are built. In this paper we will focus on modelling a specific aspect of the inland shipping system (the combined shipclass, shipper, route-choice model). It turns out that this model can only be formulated as a flow problem on a suitably defined supernetwork using the variational inequality formalism. This paper aims to give an equilibrium formulation of simultaneous choice of shipclass and route, unencumbered by practical constraints such as imposed by consulting practice.

1.3 Structure of this paper

The structure of this paper is as follows: In section 2 the characteristics of the object system, inland shipping in The Netherlands, are sketched in order to extract its salient features. On basis of these features and the use of a supply-demand interaction model, the characteristics of the object system are mapped onto model properties, and a system diagram of the object system is proposed in section 3. In section 4 a mathematical formulation of the user-equilibrium model of object system is presented in terms of a Variational Inequality Problem (VIP).

2. Characteristics of Inland Shipping in The Netherlands

Ships come in 6 size *classes* (10 if tug-pushed dumb barges are counted) as shown in Table 2, and Table 3, and 4 *types* (motorships, tankers, tug-pushed dumb barges, and containerships).

The market shares (in terms of the amount of tonnes of freight transported in 2000) tonnes of the transport modalities are shown in Table 2. From this table it is clear (without investigating the market share in terms of tonne*kilometers) that for internal goods transport, road transport has the largest market share (81.3%) followed by inland shipping (17.8%). In terms of tonnes transported all other modes (e.g. rail) are negligible. For this reason there is some interest in modelling the inland shipping transport system.

Table 1 : Goods Transport in The Netherlands

Goods transport in The Netherlands (2002 [mln. Tonnes])		**Total**		**Internal**	
Modality	**Subdivision**	**Mln tonnes**	**%**	**Mln tonnes**	**%**
Shipping		739.6	**50.8**	102.0	**17.8**
	Sea	424.5	29.1	0.0	0.0
	Inland	315.1	21.6	102.0	17.8
Road		584.6	**40.1**	464.7	**81.3**
	Own	153.4	10.5	144.6	25.3
	professional	431.1	29.6	320.2	56.0
Rail		28.1	**1.9**	5.2	**0.9**
Air freight		1.3	**0.1**	0.0	**0.0**
Pipeline		104.0	**7.1**	0.0	**0.0**
Totaal		1456.8	**100.0**	572.0	**100.0**

Source: Statistics Netherlands (2002)

Table 2 shows the distribution of the number of active ships (in 2002), the total tonnage per shipclass (in 2002), and the average freight price (1997 prices [EUR/(Ton*Km)]).

It is clear that shipclasses 0 and I provide a negligible contribution to the total transport capacity, but a non-negligible contribution to the number of ships. It is also clear that shipclasses IV, Va, and >Va amount to 47% of the total number of ships, but provide over 70% of the total capacity. The largest ships are found on the Rhine (which connects Rotterdam with the Ruhrgebiet industrial area in Germany); the medium-size ships are found on the major waterways, and the smallest ships are found everywhere.

Table 2 : Fleet composition and average freight price (source:)

				Total Tonnage 2002		Avg. price 1997	
Tonnage	**Ship class**	**# ships (2002)**	**%**	**[Tonnes * 1000]**	**%**	**[0.01 EUR/(Ton*Km)]**	**%**
21-250	<1	123	3	20	0	4.43	100
250-400	I	313	8	106	2	4.43	100
400-650	II	635	17	343	8	3.39	77
650-1000	III	981	26	799	18	2.99	67
1000-1500	IV	861	23	1,024	23	2.76	62
1500-3000	Va	787	21	1,756	39	2.62	59
3000+	>Va	111	3	405	9	2.44	55
Total		3811		4,453			

Source: Statistics Netherlands (1997, 2002)

From Table 2 we can also see the effect of economies of scale: the freight price per tonne * km of the largest ship is about 55% of that of the smallest ship. As ships demand a certain minimum fee that increases by shipclass, the cheapest shipclass for a given consignment is the smallest shipclass that can carry it.

The available shipclasses are listed in Table 3 with their tonnage. Shipclasses 0-5 are single-hull ships, shipclasses 6-10 consist of either a tug or a motorship plus one or more dumb barges. The shipclasses are listed in order of increasing demands on the waterways: waterways that can accommodate shipclass n can also accommodate shipclasses 0,1, …, n-1.

Table 3 : Overview of ship classes

Nr	Shipclass	Tonnage	Length	Width	Draught Loaded	Empty	Heigth
			[M]	[M]	[M]	[M]	[M]
0	<1	21-250	12-15	4.3	-	1.2	5
1	I	250-400	39	5.1	2.2	1.2	5
2	II	400-650	55	6.6-7.2	2.5	1.4	6
3	III	650-1000	67	8.2	2.5	1.5	6.3
4	IV	1000-1500	85	9.5	2.8	1.6	6.7
5	Va	1500-3000	110	11.4	3.5	1.8	6.7-8.8
6	Vb	3200-6000	175	11.4	4	1.8	8.8
7	VIa	3200-6000			4	1.8	8.8
8	VIb	6400-12000			4	1.8	8.8
9	VIc	9600-18000			4	1.8	8.8
10	VId	9600-18000			4	1.8	8.8

Source: Statistics Netherlands (2002)

Only the Rhine accommodates all shipclasses; routes between Rotterdam and Antwerp and between Amsterdam and the Rhine can accommodate shipclasses up to and including shipclass 9. The limiting factors are draught, width and length rather than height.

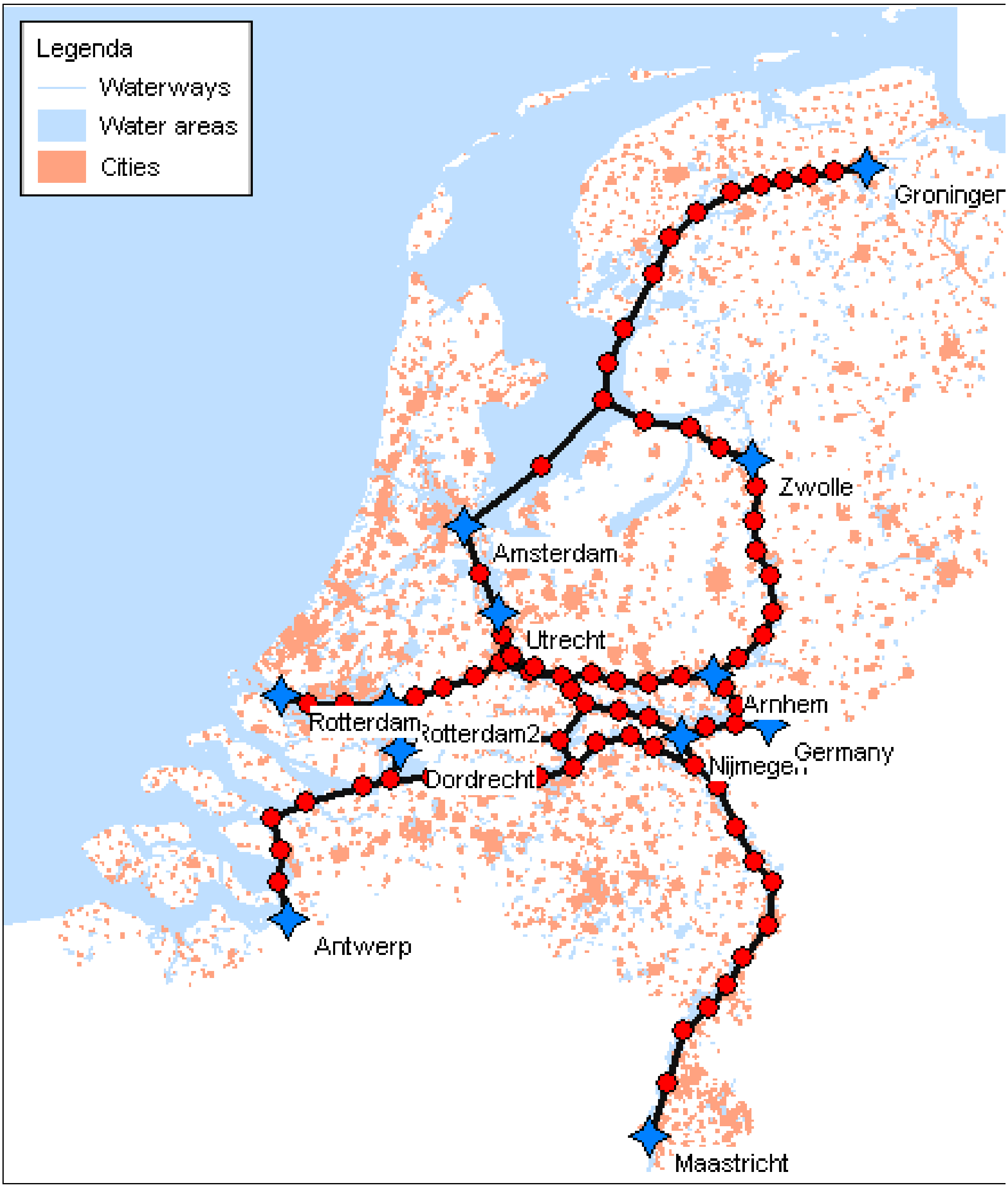
This leads to a network of main waterways approximately as shown in Figure 1; note that the waterways network for national shipping has a finer meshwidth.

This network connects the ports of Rotterdam and Antwerp with the German hinterland, and connects Amsterdam with the Rhine. It also provides a connection for shipclasses 0-5 between Rotterdam and Maastricht in the South, and provides a connection for shipclasses 0-4 between Rotterdam and Groningen in the North-West. Although this network is very sparse some route-choice is still possible.

In addition to route choice one has shipclass choice: for most O-D relationships the maximum shipclass is limited, and smaller ships may sometimes use shorter routes. On top of this the water level of the rivers varies by season.

This is a complication not encountered in traditional transport models, which we will address in this paper.

Figure 1 : Main waterways in The Netherlands



2.1 Actors and architecture of the inland navigation model

In principle two to three actors are involved in the shipping process: the sender (who owns the shipment), the shipping office (which intermediates between senders and carriers), and the carrier (which controls the ship). The selection of the carrier and the determination of the freight price are determined in a competitive market with few senders, few shipping offices, and many independent carriers. Most of the carriers operate a single ship.

In the context of the model architecture, these actors are replaced by simple model components. The senders are collectively represented by an (exogenous) O-D matrix. The shipping offices and the market are represented by the modelling assumption that minimum cost carriers are selected. The carriers are collectively represented by an assignment module and an empty-ship module that accounts for the traffic of empty ships caused by the imbalance of the goods flows. The market pressure on the carriers is represented by the assumption of a stochastic assignment model.

2.1.1 The market structure

In principle there is an open market for shipping services, which however (see [de Wit et al. (1996)]) has some counter-intuitive characteristics that affect the modelling.

In the first place, inland shipping is a very competitive sellers' market due to the fact that the goods flow is generated by about 400 large to medium-sized companies, but is transported by about 3800 very small businesses, as most ships work on an owner/operator basis. As such owner/operators face fixed costs (such as insurance, depreciation, salaries) and as their ship is their only source of income, they are often forced to follow negative supply curves when prices are low. In addition several market niches can be characterised as oligopsony.

Shipping agents match transport demand and supply; some of which are private companies, some are "collectives" (cooperations of individual shippers to bundle acquisition), and some of which are ship-owning companies. Therefore the market is not completely transparent.

Since owner\operators live on their ships and often have little additional costs, they are usually able to underbid shipping companies with employed staff, forcing them into niche markets. Structural factors in this sector (low entry threshold, high exit threshold) make that for the past 20-30 years the market mechanism has driven prices down, but still has been unable to achieve a healthy balance between transportation supply and demand. Such factors should be taken into consideration in models of the shipping market.

2.1.2 The outlines of the model system architecture

The model system works with an exogenous matrix of freight transport demand differentiated into 26 NSTR groups; forecasting is done using a pivot point method. The exogenous yearly O/D freight matrices are converted into a season-dependant daily shipment list using empirical distributions of shipment size (i.e. the amount of goods allocated to a single ship). The consignment size (i.e. the total amount of goods that the sender has ready for shipping) is unobserved and could not be modelled.

The list of daily shipments is used to determine the shipclass used to carry the shipment, and the route used by the carrier. Shipclass choice is modelled explicitly, and is assumed to be driven by cost, timeliness and convenience from a logistical point of view. The shipclass choice model is based on observable characteristics such as transport cost and travel time that act as explanatory variables. As the inland shipping market is very competitive, and prices fluctuate with the current market situation. For these reasons a random-utility model was used to model shipclass choice.

For practical reasons a user equilibrium is assumed between level of service obtained from assignment of the ship movement matrix and traffic generation (including shipclass choice)

It was decided to use a within-day dynamic traffic assignment model in the model system based on micro-simulation

In this paper however, we will not use the simulation model but present a VI formulation for a model system for inland shipping with combined shipclass choice and route choice, taking account of consignment size, variable water levels, shipclass shortage penalties, empty trip penalties in a within-day dynamic traffic assignment.

3. Modelling the object system

The model we propose needs the following input: the inland shipping network, the inland navigation fleet, and O/D demand for inland shipping (assumed fixed and given), cost functions, and interference from recreational navigation in the form of a pre-load onto the network. The model is designed to reflect the following effects:

- interaction between shipclass and the level of service available to that shipclass due to navigability constraints
- the effect of level of service on shipclass-specific cost
- the effect of shipclass-specific cost on the joint choice of shipclass choice and route
- the effects of scarcity of a particular ship class on shipclass choice.

The components and the effects are discussed in sections 3.1 - 3.4; a diagram of the interactions in the model is shown section 3.5 in Figure 7.

3.1 Model components considered

The model component we consider deals with simultaneous choice of route and shipclass, for which we will seek a user equilibrium.

Route choice and shipclass choice are modelled as a joint decision because different parts of the inland waterways network impose different limits on maximum ship size. The Rhine for example can accommodate the largest ships, whereas canals and locks can often accommodate only medium-size ships. Depending on the origin and destination, this requires a shipper to choose among available ship sizes and routes.

The impact of logistical considerations of individual companies on the choice of consignment size, frequency of supply, and mode choice was recognised, but on balance it was decided that they should not be included in an inland navigation model system and they will therefore not be discussed here. Several reasons for excluding these considerations from the model exist. First of all, the processes that lead to a consignment size depend on the management decisions of many individual companies, and all of the required data is proprietary, company-specific and highly sensitive for the companies involved. Secondly, modelling the logistics of individual companies would lead to unacceptable complications in the model system. Thirdly, at the level of individual companies so many factors determine the logistic decision that such individual decisions have a high degree of stochasticity. In summary, the amount of effort required to include this level of detail cannot be justified by the potential increase in model quality.

In addition, the water level in the rivers undergoes significant seasonal changes which limit the maximum allowable draught of ships during low water periods. For this reason the largest ships can only be loaded to about 50% of their maximum capacity when they have to navigate rivers during low-water periods, which is reflected in their transport cost per tonne.

3.2 Route choice

On the one hand routes impose restrictions on the width and length of ships, and on the other on their draught and height. It is important to distinguish between the two because width and length restrictions cannot be compensated for, whilst height and draught can be influenced by the loading factor of a ship (0: unloaded, 1: maximum load in tonnes, i.e. maximum draught).

As the inland shipping network is sparse, choice set generation can be carried out by route enumeration, i.e. all routes between r and s that are feasible for shipclass y. The result is a set of combinations of routes p shipclasses y that can navigate the routes. Each combination of route and shipclass carries a generalised cost C_{yp}^{rs} (consisting of travel time, fuel use, wages, etc.), and a choice will be made amongst these alternatives on basis of their generalised cost; a random utility maximisation model will be used. A stochastic choice is needed to ensure sufficient route dispersion.

As is often the case in route choice modelling, alternative routes between origin and destination in the inland waterways have significant overlap, which means that e.g. a multinomial logit (MNL) model is not appropriate. For practical reasons it was not feasible to use a probit model, and the applicability of alternative models such as paired combinatorial logit (PCL) and cross-nested logit (CNL) is not completely understood from a theoretical point of view. For these reasons it was decided to use the C-logit model to account for route overlap.

The perceived cost of path p from r to s when starting in time period k with shipclass y is denoted as $\tilde{C}_{yp}^{rs}(k)$, which has a systematic component and an error term:

$$\tilde{C}_{yp}^{rs}(k) = C_{yp}^{rs}(k) + \varepsilon_{yp}^{rs}(k)$$

with:

$C_{yp}^{rs}(k)$ the systematic component, and

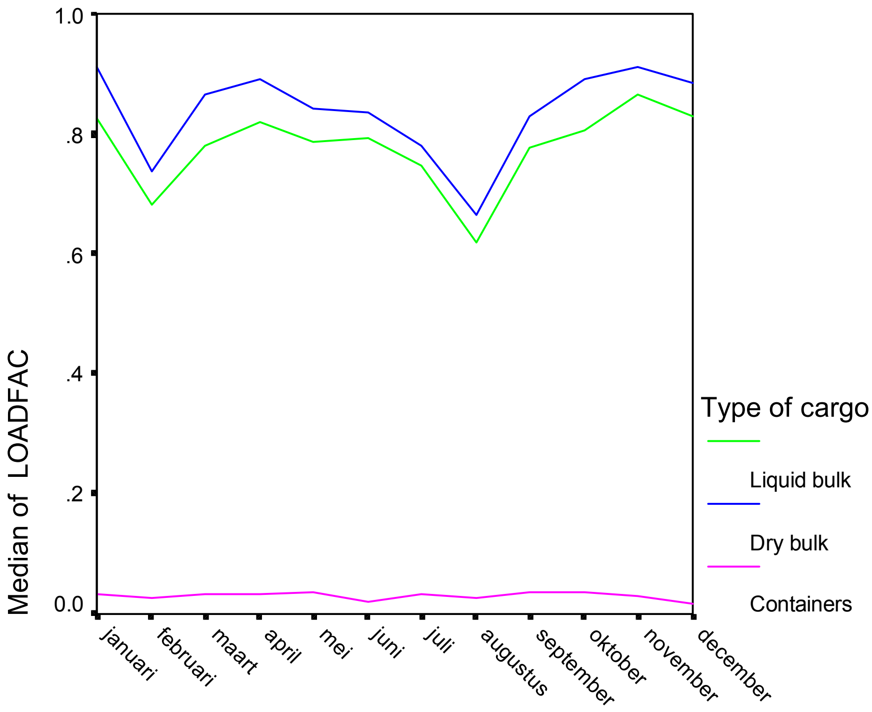
$\varepsilon_{yp}^{rs}(k)$ the error term

We will assume i.i.d. Gumbel distributions for the error term, so that we obtain an MNL logit model.

3.3 Loading factors and cost

Although the water levels in canals are controlled through judicious use of locks, the bulk of the inland navigation goods flows uses natural rivers such as the Rhine. During summer the water level in these rivers drops to the extent that the maximum allowable draught is significantly limited or (in extreme cases) that navigation becomes impossible for certain shipclasses. In this case the larger ships can only navigate the river when partially loaded.

Figure 2: Monthly load factors for Rhine shipping by goods type



Source: TNO-INRO (2003)

This is illustrated in Figure 2, which shows the median loading factors for Rhine shipments by type of cargo (liquid bulk, dry bulk, and containers). Bulk cargo usually has a high loading factor (i.e. loaded weight / carrying capacity), which varies from 90-80% during March - May to about 70% in February and August.

We interpret this data as follows: ships carrying bulk goods are usually filled up, and because of the density of the cargo a full ship has a high loading factor. With general cargo the main constraint is seems to be size, not weight; note the low loading factor for container shipments. During august and February water levels are low and ships carrying bulk goods are partially loaded. This affects the transport cost per tonne (somewhat) since the fixed costs are almost the same. Therefore the relative attractiveness of shipclasses changes, and with it the shipclass choice probabilities.

3.3.1 Link performance functions

Within the inland shipping network, four separate types of links are distinguished:

- Waterways
- Bridges
- Locks
- Quays

Each link has its own special link performance function.

Waterways usually don't constitute bottlenecks, and therefore do not cause any travel time increase at current shipping levels so that the link-performance function of waterways can be considered to be flat.

Bridges may cause delays, but mainly for the secondary waterways, as practically all of the bridges in the main waterways network can be passed without the need for opening them. Even when bridges do have to be opened, ships have precedence over land traffic and can request opening several minutes in advance by mariphone, thereby minimising their delay.

Locks do cause delays either because they have fixed operating schedules, or because their cycle-time is influenced by the amount of traffic that they have to process. Interestingly different shipclasses can cause different amounts of delay, so that the total amount of time lost at a lock depends on the composition of the traffic stream.

Quays are not usually incorporated as links into the transportation network, but behave as links that sometimes require ships to queue for them. As their availability is inversely related to the traffic flow, their expected waiting time for quays can be expected to be proportional to the traffic intensity, so that they can be treated as ordinary congestion-sensitive links.

3.4 Shipclass shortage penalties and empty trip penalties

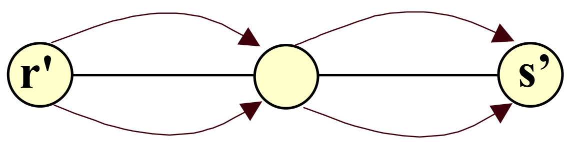
There is a finite supply of each shipclass, even when foreign flag ships are taken into consideration. During periods of low water levels many ships can only be operated with loading factors lower than 1, so that more ships than usual are needed to move the same amount of goods. This may lead to a shortage for certain shipclasses causing other, less cost-effective shipclasses, to be considered. This is can be modelled by adding a shortage penalty to shipclasses when demand exceeds supply.

In addition the freight matrix is asymmetric, causing many empty trips. The cost of an empty trip after delivering a cargo is the cost of moving the ship to a location where it can pick up a new cargo. This cost depends on the location, the shipclass, and the season.

Both effects can be modelled by extending the network as will be shown below.

The physical network with origin *r*', an intermediate node, and destination *s*' is shown in Figure 3.

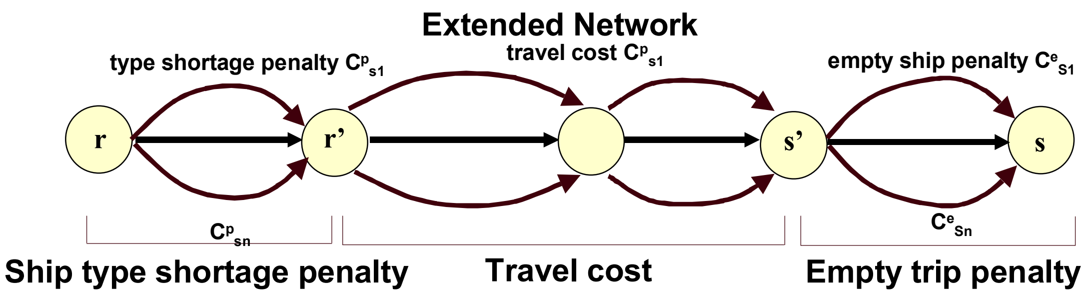
Figure 3: The physical network



The shipclass shortage penalty can be modelled as part of the path cost by introducing an artificial link between a fictitious origin *r*, and physical origin *r*', one link for each shipclass. The link cost is a function of the shipclass, the number of ships of that type that are available, and the number of that type that are needed.

Empty trip penalties can be modelled through an artificial link between the physical destination s and an extended node *s*'. The empty trip penalties do not depend on the traffic load, but only on shipclass y and destination *s*.

Figure 4: The extended network with route choice, and penalties for type shortage and empty trips

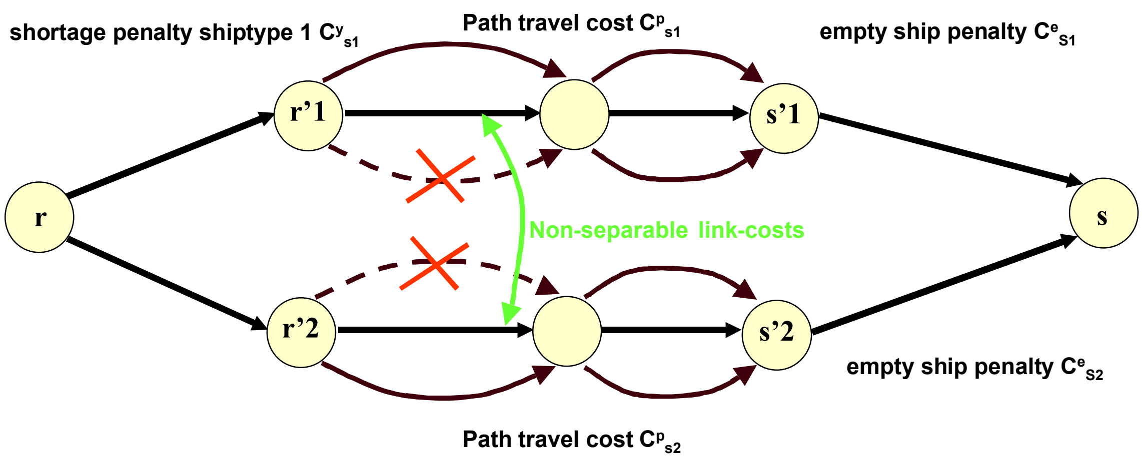


A complication is that not all routes are equally accessible to all shipclasses, so that each shipclass "sees" a different network. Conceptually this aspect can be incorporated by splitting the physical network into virtual copies (one per ship-type), through which each shipclass finds its shortest route: see Figure 5.

The advantage is that this modelling approach can deal with shipclass choice, route choice, and empty-ship penalties simultaneously by casting the decisions as the search for a least-cost path in an extended network.

The disadvantage is that the virtual copies of the network now have non-separable link costs. As the time needed per ship to pass a lock depends on the shipclass and is shared by all ships passing the lock, we have asymmetric link performance functions at locks (see §3.3). This presents a problem in that traffic assignment with asymmetric link cost functions cannot be formulated as the usual optimisation problem (see e.g. [Patriksson, M. (1994)])

Figure 5: Shipclass choice as route choice, leading to non-separable path costs



3.4.1 Path-based approach

The disadvantage of non-separable link-costs can be remedied by using a path-based approach, in which for each combination of O/D pair, cargo type, and shipment size feasible paths are defined that include all relevant costs (shipclass cost, travel cost dependent on shipclass, empty-ship penalty).

In addition the relative weight of cost and travel time may differ by cargo type.

3.4.2 Shipclass choice model

Given origin r, destination s, and a set of shipclasses $y \in Y$ that can transport goods of goods category *g*, a set of feasible routes and corresponding loading factors $\{p_y, \varphi^{\max}_{y,p}\}_{y \in Y}$ can be determined for each shipclass *y*.

An MNL model will be used as the base specification for the ship choice model.

It is expected that the attractiveness of available routes will contribute to the attractiveness of shipclasses, so that a utility logsum from the route choice model may be considered for inclusion into the shipclass choice model, leading to a nested logit model.

In this way the problem of solving for the equilibrium is decomposed into three sub-problems: determining the loading factors $\varphi_{p,y}$, determining the path costs, and solving a multi-userclass path-based equilibrium assignment on the supernetwork with non-separable link costs.

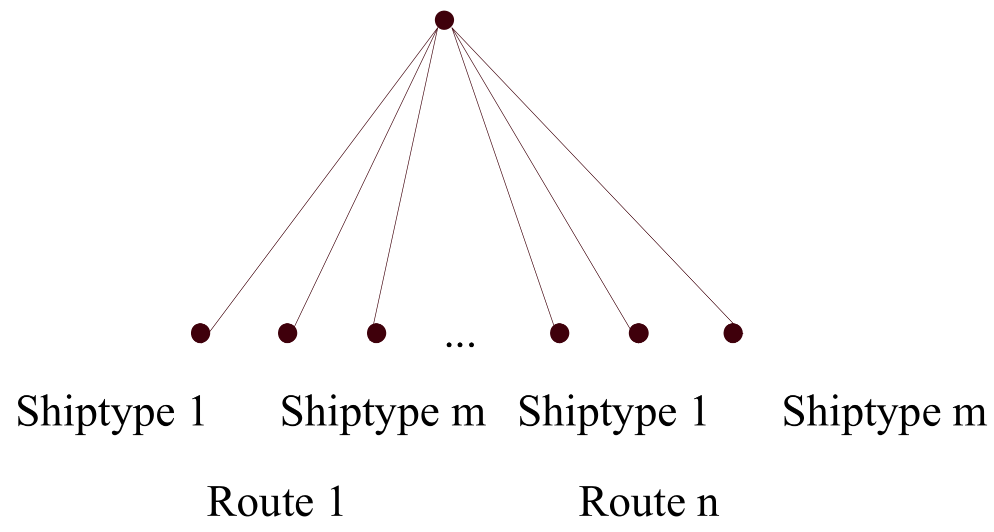
Determining the loading factors $\varphi_{p,y}$ is a separate problem, which could be solved using the heuristic proposed by De la Barra.

3.4.3 The simultaneous shipclass and route choice model

Assume that a shipment of n tonnes needs to be transported from i to j; the question is what is the best shipclass and what is the best route.

The structure of the simultaneous shipclass and route choice model is shown in Figure 6.

Figure 6: Simultaneous shipclass and route choice



The systematic part of the utility specification for shipclass y is:

$$V_{sn} = \beta_s + \beta_y C_p^y + \beta_c C_p^p + \beta_p C_{sn}^e$$

The loading factor is:

$$\varphi_{sn} = \frac{n}{n_s^{\max}}$$

The systematic part of the utility function for route r is:

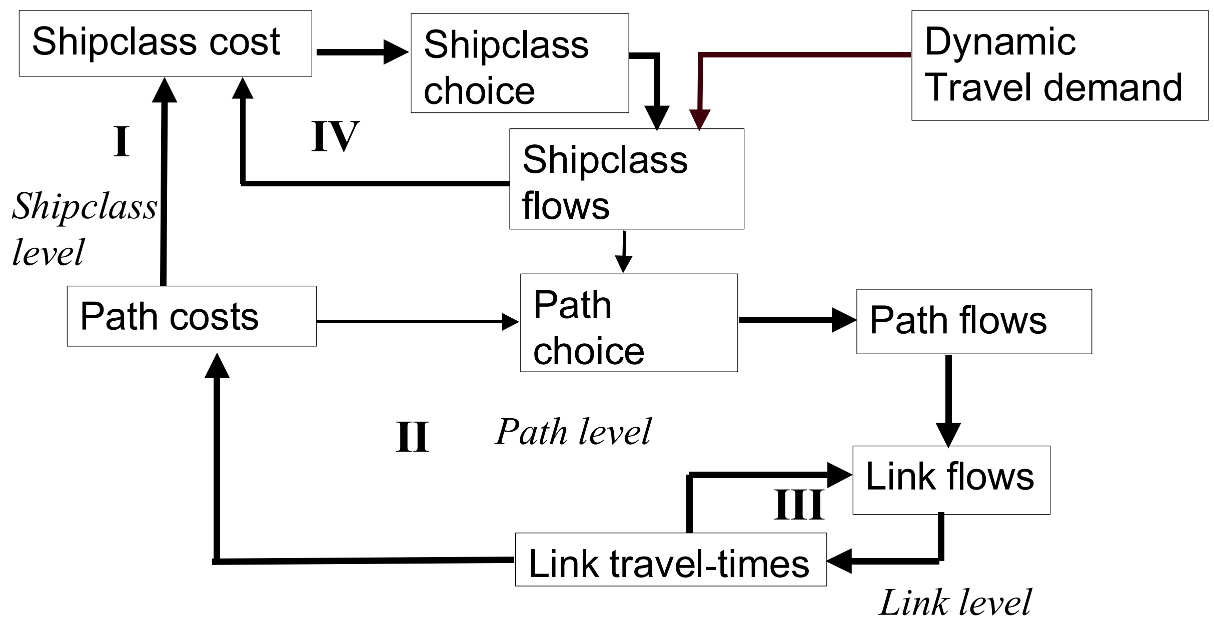
$$V_r(n) = \begin{cases} & \varphi_{sn} \leq \varphi_{rn}^{\min} \\ -\infty & \varphi_{sn} > \varphi_{rn}^{\min} \end{cases}$$

3.5 Feedback loops affecting the assignment

The combined shipclass / route choice problem is shown in Figure 7. Three feedback loops are shown:

Loop 1 represents the shipclass choice and the assignment to shipclasses, the main explanatory variable is path cost per ship class. Loop II represents the interaction between path flows and path costs in the route assignment. Loop III represents the interaction between link flows and link travel times, in waterways and at locks, bridges and quays. Loop IV represents the of the demand for a particular ship class on its price.

Figure 7: Structure of the interactions within the model



The choice of shipclass can be interpreted as a type of mode choice, so that existing VI formulations for combined mode/route choice can be applied. All the interactions shown are represented through their path cost.

4. VI formulation for user equilibrium

Our VI formulation for the BVMS model follows the approach taken in [Chabini and He (1998)], which specifies a stochastic within-day dynamic user equilibrium assignment. Due to the fact that we recast the combined ship choice / route choice as pure route choice within a supernetwork, we can apply the model as specified.

The validity of the equilibrium hypothesis, and indeed of the route choice models remain to be determined by empirical work.

We have a multi-userclass flow: $h_{yp}^{rs*}(k)$, with r and s origin and destination, y the shipclass, and p the path between r and s, equilibrium path costs $\tilde{C}_{yp}^{rs*}(k)$, and ordinary path costs $\pi_{yp}^{rs}(k)$.

The conditions for user equilibrium are:

$$\tilde{C}_{yp}^{rs*}(k)\begin{cases} = \pi_{yp}^{rs}(k) & if \quad h_{yp}^{rs*}(k) > 0 \\ \geq \pi_{yp}^{rs}(k) & if \quad h_{yp}^{rs*}(k) = 0 \end{cases} \quad \forall r,s,y,p,k \tag{1}$$

with $\varphi_1(k)$ the preference for shipclass 1

Where C is the cost of a combined choice of route and shipclass, h is the path cost, and D is a ship choice model, and q is a flow of ships.

4.1 The stochastic route-choice model of the DTA

The route-choice model assumes a stochastic user-equilibrium: for each O-D pair (r,s) at any time t, the perceived experienced travel time of a path that is chosen equals the minimum perceived experienced travel time. In the implementation we use, the path-choice probabilities are derived from a C-logit model, but the model (and the software) can work with more sophisticated route-choice models.

Given the path costs and the demand, the user-equilibrium path flow f_{kp}^{rs} is related to the O-D demand T_k^{rs} and the route-choice probability P_{pk}^{rs} as:

$$f_{kp}^{rs} = T_k^{rs} P_{pk}^{rs} \tag{2}$$

This is the dynamic generalisation of the conventional static user-optimal (Wardrop) condition with path travel time defined as experienced (actual) travel time. The condition can be expressed as follows:

$$c_{pk}^{*rs} \geq \pi_{pk}^{*rs} \tag{3}$$

$$f_{pk}^{rs}\left[c_{pk}^{*rs} - \pi_{pk}^{rs}\right] = 0 \tag{4}$$

$$f_{pk}^{rs} \geq 0 \tag{5}$$

The users' route choice behaviour model can be formulated (see [Chabini and He (1998)]) as an equivalent Variational Inequality (VI) problem:

$$\sum_{k=0}^{K} \sum_{rs} \sum_{p} F_{pk}^{*rs} \left[f_{pk}^{rs} - f_{pk}^{*rs} \right] \geq 0 \tag{6}$$

with:

$$F_{pk}^{*rs} = \left[f_{pk}^{*rs} - f_{pk}^{rs} \right] P_{pk}^{*rs} \frac{\partial c_{pk}^{rs}}{\partial f_{pk}^{rs}} \tag{7}$$

where the route costs depend on the travel time, which in turn depends on the link travel times that result from the DNL. If we assume that $\frac{\partial c_{pk}^{rs}}{\partial f_{pk}^{rs}} > 0 \quad \forall_{r,s,p,k}$, then we have that $\left[f_{pk}^{*rs} - f_{pk}^{rs} \right] P_{pk}^{*rs} = 0$ when $F_{pk}^{*rs} = \left[f_{pk}^{*rs} - f_{pk}^{rs} \right] P_{pk}^{*rs} \frac{\partial c_{pk}^{rs}}{\partial f_{pk}^{rs}}$. Therefore the equilibrium path flow is:

$$f_{kp}^{*rs} = T_k^{rs} P_{pk}^{*rs} \tag{8}$$

The route choice probabilities P_{pk}^{*rs} are calculated through a C-logit model to account for route-overlap (see [Cascetta (2001)]):

$$P_{pk}^{*rs} \frac{e^{V_{pk}^{rs} - CF_{pk}}}{\sum_{q} e^{V_{qk}^{rs} - CF_{qk}}} \tag{9}$$

with CF_{pk} the commonality factor of all routes p and all other routes q between r and s, and V_{pk}^{rs} the systematic part of the path utility function. Although the framework supports general forms of V_{pk}^{rs}, we will use the simplest form possible:

$$V_{pk}^{rs} = \sum_{a \in p} \tau_{ak} \left(X_{ak} \right) \tag{10}$$

with X_{ak} the total amount of traffic on link a during period k, as defined below.

Subject to the constraints specified in 11:

$$\sum_{y \in Y} \alpha_{yg} \lambda_{yg} q_y^{rs}(k) = \overline{Q}_g^{rs}(k) \quad \forall r, s, k, \tag{11}$$

with:

$\overline{Q}_g^{rs}(k)$ the flow of goods type g between r and s, and

α_{yg} the capacity of shipclass y for goods type g

λ_{yg} the average loading factor for goods type g and shipclass y, which is related to the average ship capacity and the average shipment size as follows:

$$\lambda_{yg} = \frac{s_{yg}^{rs}}{\alpha_{yg}} \tag{12}$$

s_{yg} the average shipment size for goods type g and shipclass y.

Externally given are:

$\overline{Q}_g^{rs}(k)$, α_{yg} and s_{yg}.

4.1.1 Solution algorithm: analytical algorithms

In principle, the solution algorithm should simply solve (11), analytical methods for which have been presented in [Chen (1999)], [Chabini and He (1998)] and [Bliemer (2001)]. A complicating factor is fact that only certain goods can be shipped in certain shipclasses, and that certain routes can only be used by a limited number of shipclasses.

In [Houtman et al. (1987)] we find a study into static multi-class equilibrium assignments on waterways. Their findings can be summarised as follows:

- Static multi-class assignments converge if speed-density relationships on waterways are taken into account;
- static multi-class assignments do not converge if time loss at key points such as locks and bridges are taken into account
- use of standard ship units (analogous to the use of PCU's) that depend both on shipclasses and on link types does not give stable solutions

Unfortunately significant travel time losses can be observed in locks and bridges, which essentially constitute queuing elements whose behaviour cannot be modelled using static assignments.

5. Discussion and conclusions

The model presented in this paper addresses the issue of combined choice of shipclass and route by extending the physical network with an additional structure in which the shipclass choice can be formulated. This is known as a supernetwork approach, and seems have very wide areas of application.

Finding a user-equilibrium with respect to the combined network and non-network decisions then reduces to the problem of finding a user-equilibrium stochastic assignment on the super-network.

In the specific instance of the shipclass-route choice, the supernetwork approach gives rise to non-separable cost functions, which cannot be handled properly within the framework of ordinary traffic assignment models or standard assignment software. The apparatus of finite-dimensional variational inequalities is needed to correctly solve such problems.

Finally the role of congestion and queuing at locks, quays, and (in some cases) bridges in the model formulation, requires the use of dynamic assignments. One approach is to undertake large-scale simulations, the other is to solve the dynamic assignment problem directly.

We have formulated the supernetwork assignment as an assignment in an ordinary network with several user-classes, which we solve using an existing DTA code. Unfortunately we have not been able to complete the computations in time for this paper, but we hope to be able to present DTA results during the presentation of this paper.

We note that we will not attempt to deal with the full complexity inherent in full-featured model system for inland shipping, and that we have made no attempt to calibrate our model against actual data. Therefore we regard our software as an illustration of the application of a supernetwork and as proof-of-concept rather than an operational assignment module.

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6.1 Disclaimer

The opinions presented here are those of the author, and do not necessarily reflect the views of either TNO-INRO or the AVV. Any factual errors are the sole responsibility of the author.

7. References

AVV (1997) Goederenvervoer (in Dutch). Report published by the AVV, Dutch Ministry of Transport, Public Works, and Water Management.

Bliemer, M. (2001) Analytical dynamic traffic assignment with interacting user-classes. Ph.D. thesis Delft University of Technology.

Catalano, S., van der Zijpp, N. (2001) A forecasting model for Inland Navigation based on Route enumeration. Proceedings of the AET conference 2001, Cambridge, UK.

Chabini, I., Y. He (1990) "A Flow-Based Approach to the Dynamic Traffic Assignment Problem: Formulations, Algorithms and Computer Implementations", Proceedings of TRISTAN III Conference, San Juan, Puerto Rico, June 17-23, 1998, Vol. 2

Chen, H. (1999) Dynamic travel choice models. A variational inequality approach. Springer verlag.

De Wit, J., Van Gent, H. (1996) Economie en Transport (in Dutch). Lemma.

Houtman, J., Van der Vlist, M. (1987) Ontwikkeling van een evenwichtsmodel voor meerdere voertuigtypen. VK 3102.201 Delft University of Technology (In Dutch)

Florian, M., Wu, J., He, S. (2000) A multi-class multi-mode variable demand network equilibrium model with hierarchical logit structures. PTRC 2000; seminar F, pp. 211-224.

Friesz, T. (2000) Strategic freight network planning models. In: Hensher, D., Button, K. (2000) Handbook of transport modelling. Pergamon.

Khoshyaran-Lebaque, M. (2000) A freight traffic modelling approach for the waterway network in the Netherlands. TU. Delft, report nr. VK 2000-005.

Lindveld, Ch.D.R., Catalano, S. (2002) A Variational Inequality formulation for combined shiptype and route choice. Proceedings of the AET conference 2002, Cambridge, UK.

Nagurney, A., Dong, J. (2002) Supernetworks. Edward Elger.

Patriksson, M. (1994) The traffic assignment problem; models and methods. Topics in Transportation. VSP, The Netherlands.

Roberts, P.O., Ben-Akiva, M., Terziev, M.N., Chiang, Y. (1977) Development of a policy sensitive model for freight demand. Phase I report. MIT center for transportation studies. CTS report number 77-11. April, 1977.

Sheffi, Y., (1985) Urban transportation networks: Equilibrium analysis with mathematical programming methods. Prentice Hall. (currently out of print).

Statistics Netherlands (2002) Statline. http://statline.cbs.nl

Tavasszy, L., Van der Zijpp, N.J., Van der Vlist, M., van Binsbergen, A., van Katwijk, R., Bovy, P.H.L. (1998) Masterplan modelsysteem binnenvaart (In Dutch). TNO report INRO/LOG 1998-06.

Van der Zijpp, N., Bliemer, M. (1999) A new simulation-based traffic assignment model for inland navigation. Colloquium Verkeerskundig Speurwerk, 1999, The Netherlands.