Route choice decision under uncertainty

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Abstract

We study route choice behavior when travel time is uncertain. In this case, users do not minimize expected travel time but also take into account travel time variability. We collected survey data and analyze them with a method based on the ordered probit model. This allows us to have an ordinal as well as cardinal measures of risk aversion. Such an approach is therefore consistent with expected as well as with non-expected utility theory. Survey data suggest a series of key factors which explain the degree of risk aversion.

Keywords : Risk aversion, ordered probit, ordinal, cardinal, structural estimation, expected utility, non expected utility, stated preferences, survey, Paris.

1 Introduction

The route choice plays a central role in Transportation Economics, engineering and Operations Research. We consider here the simple case of two routes in parallel. The equilibrium concept is due to Wardrop, or equivalently, it is a non-cooperative Cournot (quantity) equilibrium with a continuum of players. The Wardrop principle states that each driver selects the shortest travel time route and as a consequence, if the two routes are used at equilibrium, the travel time is necessarily the same on both routes. Many authors have questioned this deterministic route choice behavior, i.e. the fact that if the travel times are different on both routes, all users select the shortest one. If the users have different values of time and minimize the travel cost c_i^k of alternative i, with $c_i^k = \alpha_k tt_i$ (α_k value of time for individual k and tt_i travel time), again all users out of equilibrium have the inclination to select the route with the shortest travel time.[1] Note that when the travel time is the same on the two routes, the model is again questionable since it says nothing about route choice. The assumption underlying deterministic route choice behavior has been challenged by researchers and several modifications have been proposed. We will discuss one by one three alternative complications, which remove the assumption that usually (i.e. when travel times differ) all users select the same route. The third change is concerned with the impact of the variability of travel time on route choice, which studied in the paper. Of course a combination of those complication would lead to a more realistic model.

[1] Any increasing function of travel time will lead to the same discussion. We choose here linearity in travel time, for simplicity.

Model 1: variety of route choice attributes. Deterministic route choice has been criticized on the ground that a large variety of factors, other than travel time play a role. If those factors are observable, the above model can be easily extended, since it su±ces then to replace the travel time function by the generalized cost function. The cost function of individual k is $c_i^k = F(X_i; ¯_k)$, where X_i is a vector representing the number of tra±c lights, scenery, safety, travel time, etc. and $¯_k$ are individual-speci¯c parameters to be estimated. In this case, the travel cost depends on the user preferences (via $¯_k$), so that the choice of two di®erent users facing the same alternative routes may di®er.

Model 2: factors unobservable to the modeler but observable to the users. The situation is more complex if factors (other than the travel time) that a®ect route choice are not observable by the modeler. For example, a user may select a route because he may want to make a stop over. In this case, the travel cost for individual k of route i can be written as $c_i^k = ®_k tt_i + e_i^k$, where e_i^k is a factor known by user k but unobservable by the modeler. This additive speci¯cation is the most commonly used in the literature. The maximization principle discussed in the deterministic case remains the same from the user perspective. For two routes in parallel, individual k selects route 1 if and only if $c_1^k < c_2^k$. This is a deterministic model from the individual perspective. If the factors e_i^k are continuously distributed over R in a population, a positive fraction of users will select route 1 and route 2. For simplicity, we consider a population of statistically identical individuals. Since the idiosyncratic terms $fe_i^k g$ are not observable by the modeler, the best he could do is to describe the probability P_i^k that an individual randomly selected in the population chooses route i. This probability is given by $P_i = PrfC_i^k < C_j^k g$, with $C_i^k = ®_k tt_i + "_i^k$, where $"_i^k$ is a random variable. This is a probabilistic choice model, even if there is no uncertainty from the individual perspective.

Model 3: factors unobservable to the users. Attitudes towards risk play a major role in route choice decisions and the user choice behaviour under uncertainty is examined in the framework of Model 3. Preliminary work in the mean variance context but without explanatory variables has been done by Noland and Small (1995) and by Noland, Small, Koskenoja and Chu (1998). Contrarily to these authors, we do not consider the departure time dimension in our analysis. This last case, treated in this paper corresponds to the situation where some characteristic of the route (here the travel time) is not observable by the individuals. The fact that this factor is observable or not by the modeler is irrelevant in our analysis. We assume that the travel time is stochastic and wish to describe the choice behavior of a speci¯c individual. Note that the perceived travel time can be biased or not. Although the individuals ignore the current travel time, we assume that he knows the distribution of travel time. Given that the travel time is random, the expected travel time is unlikely to be a su±cient statistic, since the other moments of the distribution, and in particular the standard deviation of the travel time are likely to in°uence route choice. Some authors have proposed to reinterpret model 2 to address the current case. This would correspond to a situation where there is a distribution of travel time, but each individual has a idiosyncratic idea about the value of travel time and bases his decision on his subjective belief. Unfortunately, this model is based on very strong assumption that individual do not learn (otherwise, day after day, they would accumulate information and get better estimate of the travel time) and cannot communicate with each others. This interpretation can be acceptable in the case of non-recurrent congestion but it is certainly a poor approximation for recurrent congestion.

In the third model, the expected travel time should be replaced by a function which depends on the distribution of travel time. Some parameterization have been used in the ¯nance and to a lesser extent in the Transportation literature (such as the mean-variance model; see Noland, Small, Koskenoja, and Chu, 1998). It is well known that the mean-variance models su®ers from severe biases since in particular it generated irrational behaviors (as discussed in the paper) if the random variable (here the travel time) is not normally distributed.

The most widely \aggregation rule" used is the expected utility theory (see, Machina, 1982, and Machina and Pratt, 1997). It relies on two separate assumptions. First, it implies that preference can be described by a utility function (which is known by the modeler). Second, it is based on the assumption that the preferences can be rationalized by the corresponding expected utility function. This approach, initially introduced by Bernoulli in a rather trivial way (without the utility concept) was later reformulated by Von Neuman and Morgerstern. Despite its popularity, it has been questioned by empirical and theoretical scholars (see Khaneman and Tversy, 1979, and the contributions of the French School: La®ont, 1993 and Gollier, 2001). However, expected utility theory and discrete choice models, can lead to behavior speci¯c to non-expected utility theory (this topics will not be treated in this article).

The objective of this paper is to develop a model, which accommodates at the same time expected and non-expected utility functions (see, the brilliant discussion of Epstein, 2003 on this issue). This is achieved by studying both the ordinal and the cardinal components of preferences towards risk (see the discussion of Camerer, 1989, on the use of di®erent utility functions). In our belief, this use of these two approaches (cardinal and ordinal) is the major methodological contribution of the paper. This method allows us to evaluate the impact of socio-economic factors on the tolerance towards risky travel time.

We have recently discovered that some researchers have developed estimates of risk aversion comparable to ours either in the case of expected utility theory (see Hartog, Ferrer-i-Carbonell, and Jonkerand, 2000) or in the case of non expected utility theory (see Donkers, Melenberg, and van Soest, 2000). This last paper also introduces a methodology to estimate risk aversion, which is based on the idea of ranking of choices (ordinal description of choices). However, contrarily to these authors, we did not asked certainty equivalent questions, but asked the respondents to rank lotteries. It is our experience (see Ben-Akiva, de Palma, and Bolduc) that respondant, even when they have the possibility to ask for some explanations are unable to respond to certainty-equivalent questions.

An extended road map of the paper is presented below. In Section 2, we introduce the survey experiment, which was conducted in Ile-de-France (Fontan, 2002). We collected a large sample (about 2,300 respondents) who were asked lottery-type questions as well as several questions concerning their socio-economic factors. In Section 2.1, we brie°y present the survey questionnaire. The risk tolerance is measured using lottery questions discussed in Section 2.2. In Section 2.3, we present the main aggregate results. In Section 3, we discuss the empirical model. The methodology based on the ordered Probit is introduced in Sections 3.1 and 3.2. In the remaining sections, we envisaged the four standard utility functions (mean-standard deviation, mean-variance, CRRA and CARA). In section 4, we present the empirical results. We ¯rst present estimates with no explanatory variables and then estimates with explanatory variables. In the analysis with explanatory variables, we explicitly recognize that attitude towards risk does vary across individuals. We discuss the impacts of several key socio-economic variables (employment status, purpose of the trip, gender, primary transportation mode) on the level of risk aversion and compute the distribution of risk tolerance. Finally, Section 5 briefly concludes the paper.

2 Stated preferences survey

2.1 The general structure of the survey questionnaire

A computer-based phone survey (CATI) was administered during May-June 2000 in Paris area and suburbs. 4137 respondents were asked questions concerning their morning trips (the phone interviews took place between 5 and 7 PM the same day). The main purpose of this questionnaire was to study the departure time decision of commuters and non commuters using private and public transportation (see [8] for details).

The first part of the questionnaire collected information on the selected trip : schedule and constraints at the origin and the destination, purpose of the trip, characteristics of the mode(s) used (e.g. travel time and cost), knowledge of the route, etc...

In the second part of the questionnaire, the respondents were asked randomly selected questions involving several trade-offs between travel time and schedule delay cost.

The third part of the questionnaire, analayzed in this paper, collected data concerning the impact of travel time variability on route choice. Lottery-type questions were asked to a sub-sample of $2;387$ individuals (See details below). The purpose of this part was to better understand route choice behaviour when travel time is uncertain.

The last part of the questionnaire was concerned with individual and household characteristics such as gender, age, income, number of children, occupation, leisure, etc...

2.2 The evaluation of risk aversion

The risk part was based on the actual travel time for the first trip in the morning. In each question, the individual was faced with the same risk free alternative (fixed travel time corresponding to his revealed actual travel time) and a risky alternative (lottery) involving travel time variability. In the context envisaged here, the individual faced a route with a fixed travel time (risk free route) and another one with a variable travel time (risky route). The travel time for the risky route entails a low and a high value (both proportional to the revealed individual travel time) ; high and low values occur with the same probability.

Each respondent was faced with a series of three sequential similar questions, as shown on Figure 1.

For the first question L_4, each respondent had to choose between the risk free alternative L_s (a route with the guaranted actual travel time $\overline{tt}$) and a risky alternative (a route with a low travel time $tt_1 = 2/3\,\overline{tt}$ and a high travel time $tt_2 = 4/3\,\overline{tt}$). As a consequence both routes have the same expected travel time and only risk averse individual will select the risk free route,

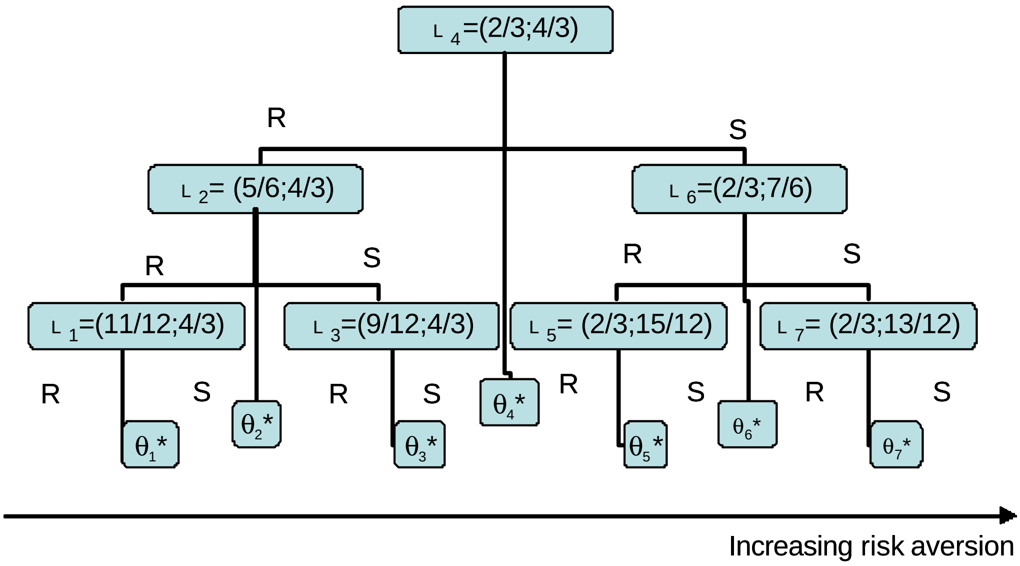

Figure 1: The hierarchy of lotteries

whereas risk lovers would choose the risky route and risk neutral agents (if any) would say they are indi®erent between the two routes.

For the second question, all individuals had to choose between the same risk free alternative L_S, and a new risky alternative. This risky alternative was more favourable for those who had previously selected the risk free alternative and less favourable for those who had previously selected the risky alternative. Lottery L_i is more favourable than lottery L_j if either the lower level tt_1 or the higher level tt_2 is lower for L_i than for L_j and the other level is the same for the two lotteries. For example, L_1 is less favorable than L_2 since $11/12 > 5/6$. The seven lotteries are ranked from the least favourable L_1 to the most favourable L_7.

Finally, the third lottery proposed depended on the answers to the previous lotteries, according to the same reasoning as above (see table below). For example, an individual who selected the lottery $L_2 = (5=6; 4=3)$ next faced the less favourable lottery $L_1 = (11=12; 4=3)$.

Notice that the relation \more favorable" can be given an interpretation in the context of expected utility theory. Clearly, the utility is a decreasing function of travel time. Consider two lotteries L_i and L_j where L_i is more favourable than L_j (in the sense de¯ned above). If $tt_{1i} < tt_{1j}$ and $tt_{2i} = tt_{2j}$, then

$$EU(L_i) = 0{:}5U(tt_{1i}) + 0{:}5U(tt_{2i}) > 0{:}5U(tt_{1j}) + 0{:}5U(tt_{2j}) = EU(L_j):$$

Therefore, the more favourable lottery L_i is preferred to less favourable lottery L_j for any decreasing utility function. Note that this result still holds in the context of the following non-expected utility theory. Let assume that individuals have a biased perception of probabilities. For example, a pessimistic individual may systematically underestimate the probability of the lower travel time ($\tilde{P}_1 < 0{:}5$ instead of $0{:}5$) and overestimate the probability of the larger travel time ($\bar{P}_2 > 0.5$ instead of 0.5) so that non-expected utility of lottery L_i is $NE(L_i) = \bar{P}_1 U(tt_{1i}) + \bar{P}_2 U(tt_{2i})$. One still has

$$NE(L_i) = \bar{P}_1 U(tt_{1i}) + \bar{P}_2 U(tt_{2i}) > \bar{P}_1 U(tt_{1j}) + \bar{P}_2 U(tt_{2j}) = NEU(L_j);$$

so L_i is still preferred to L_j by all individuals.

All individuals agree with this ranking $L_1 \succ L_2 \ldots \succ L_7$, but position the safe alternative L_s di®erently. The individuals who prefer the least favourable lottery L_1 to the safe alternative have the larger tendency to enjoy risk, that is they are the least risk averse.[2] Conversely, those who prefer the most favourable lottery L_7 are the most risk averse.

If an individual prefers the safe alternative to L_3, he necessarily prefers the safe alternative to L_1 and L_2. If he prefers L_4 to the safe alternative, he also necessarily prefers L_5, L_6 and L_7 to the safe alternative. This allows to de¯ne an ordinal notion of risk aversion and the corresponding standard cardinal representations. Consider a cardinal representation corresponding to a chosen utility function and let θ_j^* represent the risk aversion of an individual who is indi®erent between L_j and the safe alternative L_s. Consider the individual who prefers L_s to L_3 but prefers L_4 to L_s. Then his risk aversion lies in the interval $[\theta_3^*; \theta_4^*]$. For any chosen utility, the ordering of lotteries then gives a natural ordering of risk aversions $\theta_1^* < \theta_2^* \ldots < \theta_7^*$. De¯ning $\theta_0^* = -\infty$ and $\theta_8^* = +\infty$ will allow us to consider in the same way the extreme intervals. Risk aversion lies in the interval $]\theta_0^*; \theta_1^*]$ for the individuals who prefer the less favourable lottery L_1 to the safe one L_s; similarly, it lies in the interval $[\theta_7^*; \theta_8^*[$ for the individuals who prefer the safe lottery L_s to the most favourable one L_7.

This set of questions allows to rank the individuals in 8 categories. Since the lotteries are naturally ranked for any decreasing[3] utility function, the above reasoning is independent of the cardinalization of preferences. It is only based on a cardinal notion of risk aversion and as a consequence and importantly does not preclude the use of non-expected utility.

Note that the lotteries are proportional to the actual travel time $\overline{tt}$, so the scale e®ects if any will be introduced through our measure of risk aversion.

If an individual places the safe alternative L_s at the same position whatever the level of the reference travel time $\overline{tt}$, then one can say that our ordinal notion of risk aversion is free of scale; alternatively, we say that preferences are scalable. In the framework of expected utility theory, this condition corresponds to the case of constant relative risk aversion.

In the other case, the position of the safe alternative L_s depends on $\overline{tt}$. In expected utility theory terms, the relative risk aversion is variable. A particular example corresponds to case of constant absolute risk aversion, for which relative risk aversion is inversely proportional to $\overline{tt}$.

For any parametric speci¯cation of preferences, it is straightforward to compute a numerical value for the thresholds θ_j^* of relative risk aversion. This is what we do in sections 3.3 to 3.6.

[2] Here, the concept of risk aversion is not based on the (cardinal) measure of the Arrow-Pratt index. For the time being, it is only linked to an ordinal notion supported by the preference relationship between lotteries.

[3] Travel time is obviously a bad rather than a good : people are better o® with lower travel times. Most utility functions considered here will take negative values, which is atypic but not inconsistent.

2.3 Descriptive statistics

The total sample is composed by 56% women and the average age is 38:25% of households have two individuals, 18% one individual, 20% three individuals, 22% four individuals and 15% ¯ve or more individuals. 55% of households count at least one child under 18:47% of households do not have a car and the income of 60% of households is less than 18;000FF per month.[4]

The trip to work represent 67:2%, trip for shopping 9:2% and the trip to school represent 8% of total trips. Note that men more frequently trip to work and women go more frequently shopping or visiting people.

About 73% of individuals use only one mode. Cars are used by 49% of individuals and transit by 45% (4% of car passengers and 2% of motorcycle users). Note that individuals between age 15 and age 30 use frequently public transportation. The average travel time is 39 minutes by private transportation and 38 minutes by public transportation.

The aggregate result for the risk question are brie°y presented below. The distribution of the choices of the 2;387 individuals we could retain for the estimates is displayed in the following table. 785 individuals representing 32:9% of the sample are risk lovers; this seems quite a high percentage.

Choice	Frequency	%
SSS	393	16:46
SSR	379	15:88
SRS	358	15:00
SRR	315	13:20
I	157	6:58
RSS	198	8:29
RSR	143	5:99
RRS	150	6:28
RRR	294	12:32

Table 1 : Distribution of choices

The purpose of this article is to show how risk aversion depends on individual characteristics. To do that, we need to develop a methodology for the estimation procedure. Since several utility functions can be used in our estimates, and since the mere idea of expected utility theory is questionable, we prefer to start with an ordinal approach, and afterwards use a cardinal approach with relies on speci¯c utility function. For the cardinal analysis, we concentrate on the standard utility functions envisaged in the literature.

3 Methodology for the empirical study

3.1 The ordered probit model

We wish to develop a choice model to describe the probability that an individual selects a given lottery. Since the proposed alternatives are naturally ordered, we decided to introduce an

[4]One Euro is 6:55957 French francs.

estimation procedure based on the ordered probit model (see the seminal article of Small, 1997, on the ordered probit model).

The ordered probit model describes the probability that an individual selects a given alternative among a series of ordered alternatives (here the order is the one of the lotteries).

First, we consider a cardinal representation of risk aversion corresponding to a chosen utility. The utility of the choice involves a random variable θ^k with a strictly increasing cdf F_k, to be determined. θ^k is interpreted as the degree of risk aversion. The choice depends on thresholds, which determine the ranges of choices. For this chosen utility function, the numerical value of the corresponding threshold θ_j^* is given by the condition of indifference between the lottery L_j and the risk-free alternative. The individual who has a higher value θ^k ($\theta^k > \theta_j^*$) will stricly prefer the lottery L_j to the risk-free alternative. Moreover, the individual with a risk aversion $\theta^k \in \left]\theta_{j-1}^*; \theta_j^*\right]$ prefers L_j to L_s but prefers L_s to L_{j-1}. The probability P_j^k that individual k places L_s between L_{j-1} and L_j is:

$$P_j^k = \Pr\left(\theta_{j-1}^* < \theta^k \leq \theta_j^* | X_k\right) = F_k\left[\theta_j^*\right] - F_k\left[\theta_{j-1}^*\right]$$

Second, let ν denote any other cardinal representation of the same ordinal risk aversion, involving the random variable ν^k with pdf Φ_k and the corresponding thresholds ν_j^*[5], to be estimated. This second representation may be chosen according to an arbitrary rule and we are not interested in finding the corresponding utility function. For example, we can choose ν so that the random variables ν^k has a gaussian distribution because it leads to simple calculations.

Since θ^k and ν^k correspond to the same ordinal preferences, they lead to the same localisation of L_s in the lotteries scale and the probability P_j^k that individual k places L_s between L_{j-1} and L_j is:

$$P_j^k = \Phi_k\left[\nu_j^*\right] - \Phi_k\left[\nu_{j-1}^*\right].$$

In addition to the conditions $\nu_0^* = \theta_0^* = -\infty$ and $\nu_8^* = \theta_8^* = -\infty$, this implies that the threshold values θ_j^* and ν_j^* are linked by the conditions

$$F_k\left[\theta_j^*\right] = \Phi_k\left[\nu_j^*\right], \quad \theta_j^* = F_k^{-1}\left[\Phi_k\left[\nu_j^*\right]\right]. \tag{1}$$

Whenever θ^k is a continuous random variable with support $\mathbb{R}$, F_k is a strictly increasing function and can be inverted. The conditions (1) provides the correspondence between θ^k and ν^k in 7 points.

The individual likelihood for individual k is simply the probability of the observed outcome j_k, that is $P_{j_k}^k = \Phi_k\left(\nu_{j_k}^*\right) - \Phi_k\left(\nu_{j_k-1}^*\right)$ and the sample log-likehood is simply given by the sum of the individual log-likelihoods.

$$\ln L = \sum_{i=1} \ln\left(P_{j_k}^k\right);$$

[5] With $\nu_0^* = \theta_0^* = -\infty$ and $\nu_8^* = \theta_8^* = -\infty$

where n represents the sample size.

We will develop in sections 4.1 and 4.2 the expression of Φ_k both with and without explanatory variables, with a special focus on the observed individual travel time $\overline{tt}_k$.

On the one hand, the estimation procedure is only based on the ordinal representation of preferences reflected in ν; threshold values ν_j^* are estimated using a maximum likelihood technique and are not affected by the choice of any cardinal representation of preferences. On the other hand, the specification of a utility function allows to give a numerical (cardinal) value to the estimated (ordinal) risk aversion.

3.2 Mass point

Since a fairly large number of individuals (157 representing 6.5% of the sample) happened to be indifferent in the first lottery L_4, we decided to create an additional category, denoted by I, regrouping "indifferent" individuals. Since the expectation of lottery L_4 corresponds to the safe alternative, indifferent individuals are risk neutral. Therefore, their risk aversion is $\theta = \theta_4^* = 0$. This generates a mass point in the distribution of θ.

We therefore explicitly recognize that the function $F_k^{-1} \circ \Phi_k$ may be constant on some interval. We can deal with this by associating two different values $\nu_{4\inf}^*$ and $\nu_{4\sup}^*$ to the threshold $\theta_4^* = 0$. In this case, ν^k is still a continuous random variable, whereas θ^k has a mixed distribution with

$$\Pr\left(\theta^k = 0\right) = \Pr\left(\nu_{4\inf}^* < \nu^k \leq \nu_{4\sup}^*\right) = \Phi_k\left(\nu_{4\sup}^*\right) - \Phi_k(\nu_{4\inf}^*). \qquad (2)$$

The function $F_k^{-1} \circ \Phi_k$ shows a discontinuity in 0 and remains invertible elsewhere. The condition (1) $\theta_j^* = F_k^{-1}\left(\Phi_k\left(\nu_j^*\right)\right)$ still holds for $j \neq 4$.

In the two cases (with or without mass point), θ^k can be expressed as a function of ν^k. The only difference is that this function is strictly increasing in the case of no mass point but constant on the interval $\left[\nu_{4\inf}^*; \nu_{4\sup}^*\right]$ in the case of a mass point.

In order to get numerical values for the risk aversion, we consider 4 utility functions and compute the series of thresholds for each of them.

3.3 Mean-standard deviation formulation

For example, consider the following expected utility function corresponding to the lottery $L_j = (\alpha_{j1}; \alpha_{j2})$ with travel times $\left(tt_1 = \alpha_{j1}\overline{tt}; tt_2 = \alpha_{j1}\overline{tt}\right)$, expected travel time $E(tt) = (tt_1 + tt_2)/2$ and standard deviation $\sigma = (tt_2 - tt_1)/2$:

$$EU(tt_1; tt_2) = -E(tt) - \theta^S\sigma = -\overline{tt}\left(\frac{\alpha_{j1} + \alpha_{j2}}{2} + \theta^S\frac{\alpha_{j2} - \alpha_{j1}}{2}\right).$$

For this specification, an individual indifferent between the lottery $(\alpha_1; \alpha_2)$ and the risk free choice $\overline{tt}$ has a risk aversion θ^{*S} such that $EU(tt_1; tt_2) = -\overline{tt}$ or

$$\theta^{*S} = \frac{2 - (\alpha_{j1} + \alpha_{j2})}{\alpha_{j2} - \alpha_{j1}}.$$

Note that $EU(tt)$ is proportional to $\overline{tt}$, just as the lotteries proposed, so if choices were driven by mean-standard deviation preferences, the choices would not be affected by the actual travel time $\overline{tt}$ if the estimates θ^S are independent of the actual travel time $\overline{tt}$. In this case the threshold values are also independent of $\overline{tt}$.

Straightforward calculations confirm that the thresholds θ_j^{*S} are correctly ranked in this case. This gives the thresholds displayed in the following figure.

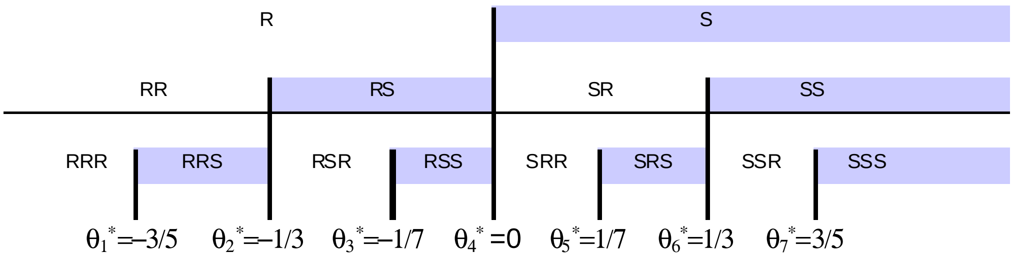

Figure 2: Ranking and threshold values for mean-standard deviation utility

3.4 Mean-variance formulation

In that specification, the standard deviation is replaced by the variance of travel time, so the utility becomes :

$$EU(tt_1;tt_2) = -E(tt) - \theta^V \sigma^2 = -\overline{tt}\left(\frac{\alpha_{j1}+\alpha_{j2}}{2}\right) - \theta^V \overline{tt}\left(\frac{\alpha_{j2}-\alpha_{j1}}{2}\right)^2 :$$

Note that for the loteries considered, $E(tt)$ is proportional to $\overline{tt}$, whereas σ^2 is proportional to $\overline{tt}^2$. According to the standard formulation of the mean-variance model, θ^V represents the absolute risk aversion. There are no scale effect, if the estimate show that θ^V in inverseley proportional to the actual travel time $\overline{tt}$.

The threshold values are given by the condition $EU(tt_1;tt_2) = -\overline{tt}$:

$$\theta^{*V} = 2\frac{2-(\alpha_{j1}+\alpha_{j2})}{(\alpha_{j2}-\alpha_{j1})^2\,\overline{tt}}:$$

Note that the threshold values for the relative risk aversion are inversely proportional to the actual travel time. Indeed, if the estimates show that the parameter θ^V is inversely proportional to the actual travel time $\overline{tt}$, then the utility function is scalable.

3.5 CRRA utility

$U(x) = -\frac{x^{1+\theta^R}}{1+\theta^R};\ \theta^R \neq -1$. Note that the case $\theta^R = -1$ corresponds to the logarithmic utility function $U(x) = -\ln(x)$. It can easily be checked that the individual who is indifferent between the safe and the risky lottery L_3 (4/3; 3/4) has a logarithmic utility function. Note also that that $\gamma = 0$ corresponds to risk-neutral agents (in this case: $U(x) = -x$).

We now compute the thresholds for $\theta^R \neq -1$. The expected utility of the alternative $(\alpha_{j1}\overline{tt}; \alpha_{j2}\overline{tt})$ in lottery L_j is:

$$EU(L_j) = -\frac{1}{2}\left(\frac{\left(\alpha_{j1}\overline{tt}\right)^{1+\theta^R}}{1+\theta^R} + \frac{\left(\alpha_{j2}\overline{tt}\right)^{1+\theta^R}}{1+\theta^R}\right) = -\frac{1}{2}\frac{\overline{tt}^{1+\theta^R}}{1+\theta^R}\left((\alpha_{j1})^{1+\theta^R} + (\alpha_{j2})^{1+\theta^R}\right);$$

whereas the utility of the riskless choice is $\frac{\overline{tt}^{1+\theta^R}}{1+\theta^R}$. Therefore the preference are scalable if the estimate of θ^{RA} are constant (with respect to $\overline{tt}$). The indifference condition between the lottery L_j and the safe choice L_S is

$$(\alpha_{j1})^{1+\theta^R} + (\alpha_{j2})^{1+\theta^R} = 2.$$

We verify that this condition is independent of the value of the actual travel time. Therefore, finding a significant effect of actual travel time on decisions leads to the rejection of the CRRA specification (with constant parameters) against a more general specification such that standard relative risk aversion θ^R is not constant.

Straightforward calulation shows that $\Phi\left(\theta^R\right) = (\alpha_{j1})^{1+\theta^R} + (\alpha_{j2})^{1+\theta^R}$ is a convex function of γ and has a unique minimum for

$$\underline{\theta}^R = \frac{\ln\left(\frac{-\ln\alpha_1}{\ln\alpha_2}\right)}{\ln\frac{\alpha_2}{\alpha_1}} - 1$$

and that $\Phi\left(\theta^R\right) \underset{\gamma\to\pm\infty}{\longrightarrow} +\infty$. Since in addition $(\alpha_{j1})^{1+\theta^R} + (\alpha_{j2})^{1+\theta^R} = 2$ for $\theta^R = -1$, there is a unique solution $\vartheta \neq -1$ such that $\Phi(\gamma) = 2$. Therefore the thesholds are uniquely defined (although they are not explicit).

3.6 CARA utility

$U(x) = \frac{1-e^{\theta^{AR}x}}{a}; a \neq 0$. The limit case $\theta^{AR} \to 0$ correspond to risk neutral individual (then $U(x) = x$). The expected utility of the lottery is :

$$EU(L_j) = \frac{1}{\theta^R} - \frac{1}{2\theta^R}\left(e^{\theta^R\alpha_{j1}\overline{tt}} + e^{\theta^R\alpha_{j2}\overline{tt}}\right);$$

whereas the utility of the risk free choice is $\left(1 - e^{\theta^R\overline{tt}}\right)/\theta^R$.

So the lottery is preferred to the safe choice iff $e^{a\overline{tt}(\alpha_{j1}-1)} + e^{a\overline{tt}(\alpha_{j2}-1)} > 2$. Clearly, the solution are independent of the actual travel time $\overline{tt}$ iff the parameter θ^R is inversely proportional to $\overline{tt}$ (and in this case the preferences are scalable).

The table below sums up the threshold values for the risk measure θ for the different utility functions (we chave chosen $\overline{tt} = 1$ hr, when the threshold value are not constant). We can check that they are increasing with the lottery rank.

	α_{j1}	α_{j2}	θ_j^{*S}	θ_j^{*V}	θ_j^{*R}	θ_j^{*A}
L_1	$11/12$	$4/3$	$-3/5$	$-72/25$	-8.223	-7.875
L_2	$5/6$	$4/3$	$-1/3$	$-4/3$	-2.975	-2.887
L_3	$9/12$	$4/3$	$-1/7$	$-24/49$	$-1(\ln)$	-0.993
L_4	$2/3$	$4/3$	0	0	0	0
L_5	$2/3$	$15/12$	$1/7$	$24/49$	0.958	0.993
L_6	$2/3$	$7/6$	$1/3$	$4/3$	2.744	2.887
L_7	$2/3$	$13/12$	$3/5$	$72/25$	7.455	7.875

Table 2 : Thresholds for the different utility functions

4 Application and results

4.1 Estimates with no explanatory variables

In the absence of any explanatory variable, the distributions of θ^k and ε^k are the same for all individuals and we can drop the indexes k. So we get a function $\psi = F^{-1} \circ \Phi$ identical for all individuals. In that case, it is not restrictive to choose for Φ the standard normal, so that the threshold values $\hat{\theta}_j^*$ can be estimated using the ordered probit formulation. The threshold values θ_j^* are given by the lotteries and the parametrization of preferences introduced in section 3.

Since the model is just identified, the estimated values $\hat{\theta}_j^*$ are such that the estimated probability of each interval $\Phi\left(\hat{\theta}_j^*\right) - \Phi\left(\hat{\theta}_{j-1}^*\right)$ is exactly the observed percentage $\hat{P}_j$ in the corresponding interval of individuals who choose L_j.

With no explanatory variables, no restriction is imposed to the model. However, the model is not very useful since the probabilities it generates are simply the corresponding proportions observed in the sample. However, it can be easily generalized by introducing explanatory variables, as we do in the following sub-section. Before that, we present the properties of the model wihout explanatory variables, since it is simpler to understand in that case and it can be easily extended to the case of explanatory variables.

The main interest of the model in the case of no explanatory variables is to estimate the distribution of risk aversion for any chosen utility function.

Figure 4 represents the 7 thresholds for the different utility functions. Since Table 2 shows that thresholds are very low for the mean-standard deviation and mean-variance formulations as compared to the CARA and CRRA functions, we used different scales : the right one (from -2 to 2) is used for the mean-standard deviation and mean-variance formulations; the central one (from -10 to 10) is used for the CARA and CRRA functions. Except for this scale effect, the thresholds obtained with the different utility functions are comparable.

Since θ has a mixed distribution, we will consider separately the two aspects of its distribution.

On the one hand, $\Pr(\theta) = 0$ corresponds to the observed percentage of risk neutral agents in the sample, whatever the utility function.

On the other hand, the conditional density function for the continuus part $f_{\theta|\theta\neq 0}$ can be simply deduced from the (gaussian) density of the latent variable ε (see Figure 3) and from the ψ transformation depicted in Figure 5 in the case of the mean-standard deviation model. As discussed in section (2), risk neutral individuals corresponds to the flat portion of the ψ

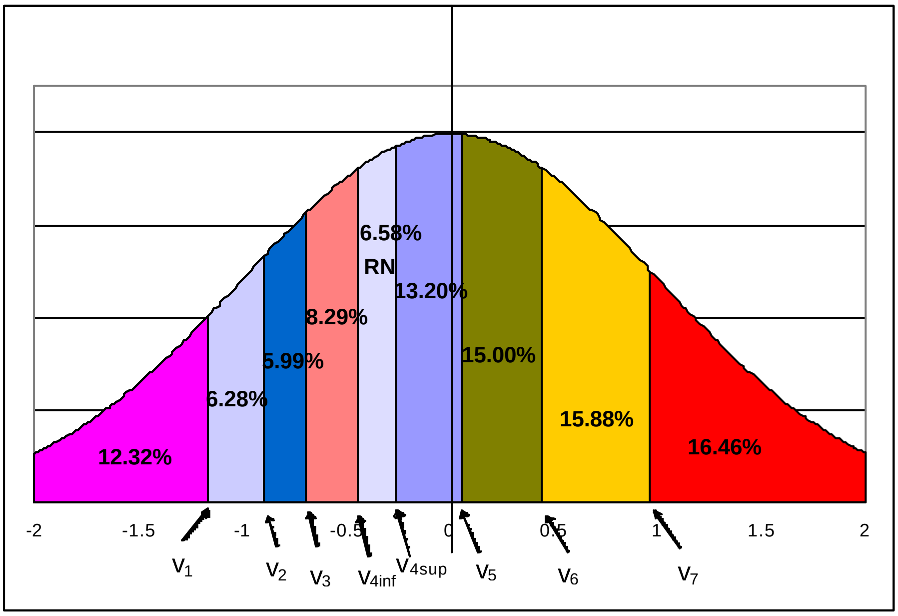

Figure 3: Distribution of the latent variable ν

curve. Since both ν_j^* and θ_j^* are ranked, we can select an increasing smooth function $\hat{\psi}$ such that $\theta_j^* = \hat{\psi}\left(\nu_j^*\right)$; $j = 0...8;\ j \neq 4$. Alternatively, we can select a polynomial approximation which is increasing in the relevant interval. Figure 5 shows that the ψ function is well described by an order 4 polynomial function ψ_{P4}, at least in the relevant interval $\theta \in [-0.6; +0.6]$, $\nu \in [-1.159; +0.976]$.[6]

The polynomial approximation ψ_{P4} is defined on $\mathbb{R}$. However, although ψ_{P4} is increasing on the relevant interval $[-1.159; +0.976]$, it happens to be decreasing for $\nu > 1.5$. Since Figure 5 shows that ψ is approximately linear for $\nu > 0$, we have replaced the polynomial approximation ψ_{P4} by a linear function for $\nu > 1.5$. This concerns less than 16.5% of the sample (those individuals who have the larger risk aversion $\theta > \theta_7^*$ and have always chosen the safe lottery L_S). The resulting approximation will be denoted $\hat{\psi}$.

The cdf for θ is given exactly in 7 points by the ψ transformation and by equations (1) : $\theta_j^* = F^{-1}\left(\Phi\left(\nu_j^*\right)\right) = \psi\left(\nu_j^*\right)$. The extension of ψ on $\mathbb{R}$ allows to recover the cdf of θ on $\mathbb{R}$: $F^{-1} \circ \Phi = \psi$, $F = \Phi \circ \psi^{-1}$ and the density

$$f = F' = \Phi' \circ \psi^{-1} \times \left[\psi^{-1}\right]' = \frac{\varphi \circ \psi^{-1}}{\psi' \circ \psi^{-1}};$$

where φ denotes the standard normal density. It can be approximated on $\mathbb{R}$ by $\frac{\Phi' \circ \hat{\psi}^{-1}}{\hat{\psi}' \circ \hat{\psi}^{-1}}$. We end

[6] For the conditional distribution of θ conditional on $\theta \neq 0$, risk neutrality ($\theta = 0$) corresponds to a single value for ν in the interval $[\nu_{4inf} = -0.443; \nu_{4sup} = -0.267]$.

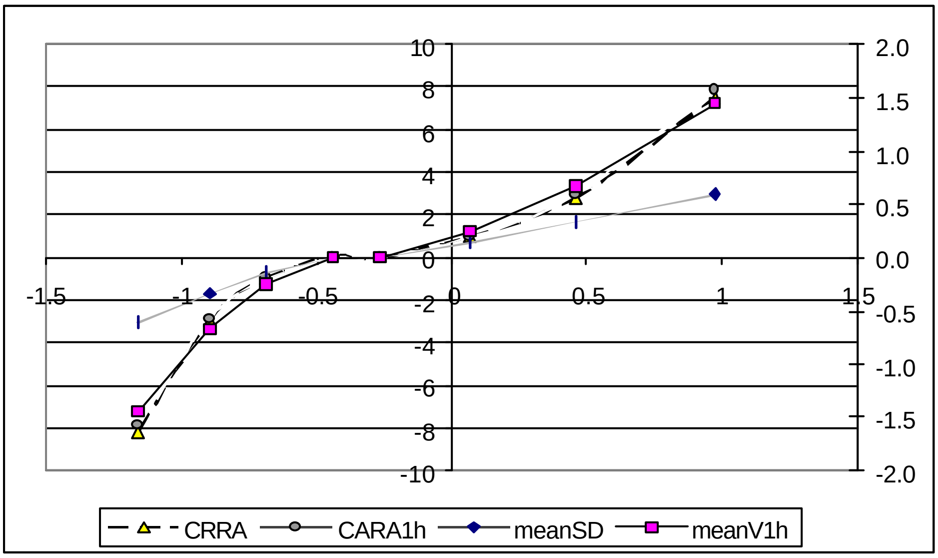

Figure 4: Threshold values for the di®erent utility functions

up with

$$f(µ) = \frac{Á \pm \hat{Ã}^{i\,1}(µ)}{Ã^{0} \pm \hat{Ã}^{i\,1}(µ)} = \frac{Á(º)}{\hat{Ã}^{0}(º)}:$$

Since Ã is a polynomial function (and a linear function for º > 1:5), its derivative is trivial to obtain. The estimated density for µjµ > 0 is depicted in Figure (6), where the vertical lines represent the thresholds. Note that the values of risk aversion outside the relevant interval $[µ_1^{¤}; µ_7^{¤}]$ are not estimated very precisely. The fact that the tail is larger on the left side than on the right side is a consequence of the linearization of $\hat{Ã}$ for large values of º, implying that µ takes only moderate values. On the other side, the polynomial approximation $Ã_{P4}$ took very negative values.

4.2 Introduction of explanatory variables

As usual in the ordered probit approach, we make the (restrictive) assumption that the e®ect of explanatory variables is to shift the distribution of $º^k$ through a linear combination $X_k¯$ so that $©_k$ is the cdf of a normal with expectation $X_k¯$ and variance 1. The \reference individual" is de¯ned by the condition $X_r¯ = 0$ and the corresponding cdf of $µ^r$ is denoted F_r.

This assumption is restrictive because it implies that :

1. All the explanatory variables are perfect substitutes in risk aversion determination.[7]

[7] Let $¯_1$, $¯_2$ and $¯_3$ denote the coe± cients of the explanatory variables X^1, X^2 and X^3. Let consider any number ¢, an individual with characteristics X_1^1, X_1^2 and X_1^3 and another individual with characteristics $X_2^1 = X_1^1 + ¯_2 ¢$, $X_2^2 = X_1^2 ¡ ¯_1 ¢$ and $X_2^3 = X_1^3$. Then the model implies that the two individuals have the same distribution of risk aversion because $X_2¯ = \left(X_1^1 + ¯_2 ¢\right)¯_1 + \left(X_1^2 ¡ ¯_1 ¢\right)¯_2 + X_1^3¯_3 = X_1^1¯_1 + X_1^2¯_2 + X_1^3¯_3 = X_1¯$.

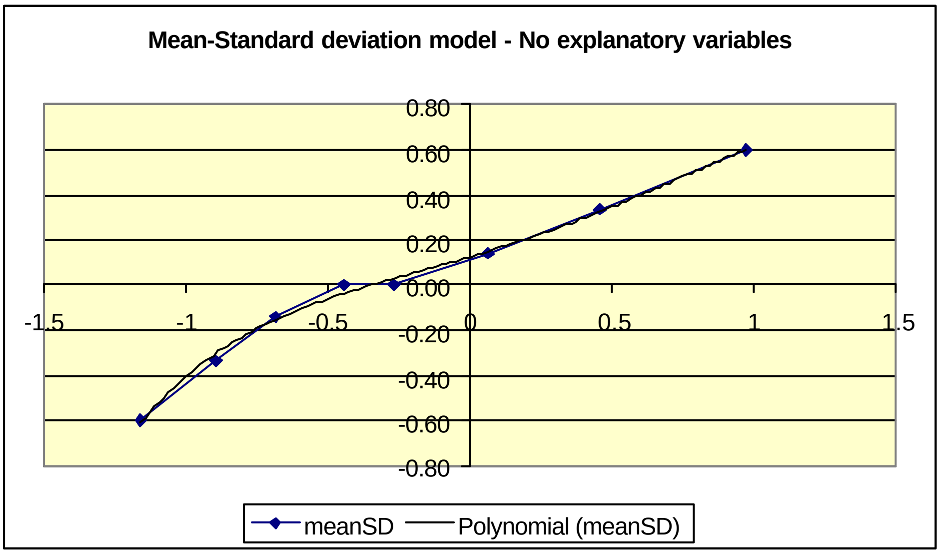

Figure 5: The Ã transformation and its order 4 polynomial approximation.

2. The variations of the probabilities P_j^k for a variation of the quantity X_k¯ are not totally free as shown on ¯gure 3. For example, if X_k¯ > 0 for individual k, the probabilities of riskiest choices are increased and the probabilities of the sa¯est choices are decreased (compared to the reference de¯ned by X_r¯ = 0).

It is more convenient to write º^k = X_k¯ + !^{k} where !^{k} standard normal with cdf ©. Then :

$$©_k\left(º_j^¤\right) = \Pr\left(X_k¯ + !^k \quad º_j^¤\right) = \Pr\left(!^k \quad º_j^¤ ¡ X_k¯\right) = ©\left(º_j^¤ ¡ X_k¯\right):$$

Let Ã denote $F_r^{¡1}$ ± © as in the case of no explanatory variable. Then the model implies that the transformation between the two cardinal representations of individual risk aversion is the same for all individuals, that is :

$$Ã = F_r^{¡1} ± © = F_k^{¡1} ± ©_k \qquad 8k$$

This assumption is restrictive as explained above (1 and 2). However, with a larger dataset, it could easily be relaxed in the following ways :

² split the sample so that $Ã_k$ is the same within each subsample but may vary from one subsample to the other. This is only limited by the condition that each subsample is large enough for the parameters to be estimated with enough precision.

² introduce heteroskedasticity in the distribution of º^k. This corresponds to a heteroskedastic ordered probit model. The variance of º^k is then modelled as $V\left(º^k\right)$ = exp (Y_k°), where Y_k is another set of explanatory variables that may intersect (but may not include) X_k. This model is known to be especially di±cult to estimate when there are many variables common to X_k and Y_k.

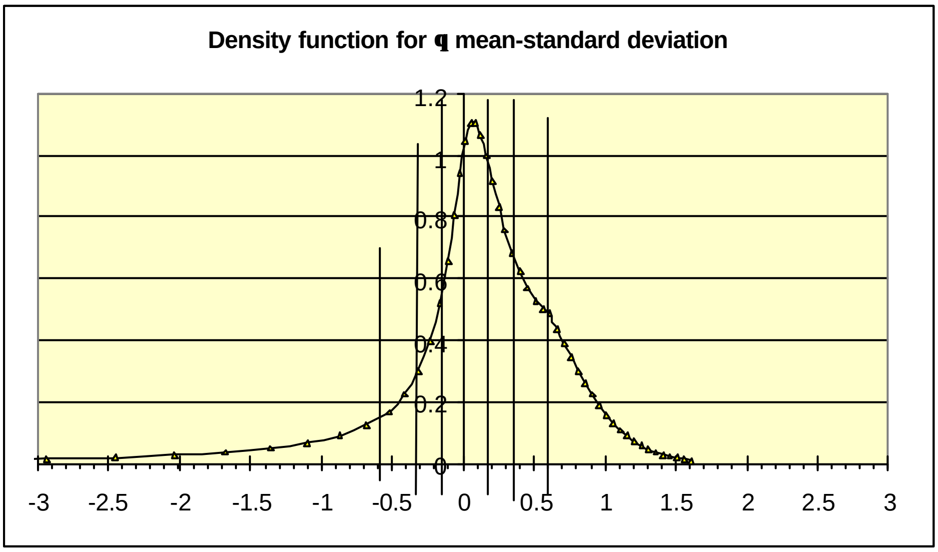

Figure 6: Distribution of risk aversion for the mean-standard deviation formulation

We first develop an analysis based only on the ranking of the choices. Let μ^k denote the risk aversion of individual k. As shown on Table 1, individual k selects lottery L_j if and only if his risk aversion is in the interval $\left[\mu^*_{j-1}; \mu^*_j\right]$. The threshold values depend on the choice of the utility function. Our ultimate objective is to predict the probability that a given individual with given characteristics selects one among several alternatives. Therefore, we are only interested in the ranking of the alternatives and on an ordinal notion of risk aversion rather than on a cardinal one. We will rely on an ordered estimation technique and will build bridges between the ordinal and cardinal representations of preferences and risk aversion. Since only the ranking of the μ matters, we can work on any convenient monotonic transformation of μ, which therefore preserves the ranking.

For a given individual k, the value of μ^k is assumed to be unobservable by the econometrician (although it is known by the individual). The best the modeller can do is to describe μ^k as a random variable with pdf F_k. However, the modeller observes individual characteristics X_k that were collected in the survey questionnaire. We model below the dependency of μ^k on X_k.

Let consider another cardinal representation of risk aversion involving the random variable o^k and the thresholds values o^*_j.

The distribution function F_k depends a priori on the observable characteristics X_k Let P^k_j denote the probability that individual k chooses L_j, given his socioeconomic characteristics X_k

$$\begin{aligned} P^k_j &= \Pr\left(\mu^*_{j-1} < \mu^k \quad \mu^*_j | X_k\right) = F_k\left[\mu^*_j\right] - F_k\left[\mu^*_{j-1}\right] \\ &= \Phi_k\left[o^*_j\right] - \Phi_k\left[o^*_{j-1}\right]: \end{aligned}$$

Since we work here on an ordinal notion of risk aversion, there exists an increasing function

ψ such that

$$\theta^k = \psi\left(X_k\beta + \varepsilon^k\right);$$

where ε^k has a standard normal distribution $N(0,1)$.

Let Φ denote the standard normal cumulative distribution function. Notice that Φ and ψ are not indexed by k since they are assumed to be the same for all individuals in the population, whatever their characteristics.

$$\begin{aligned} P_j^k &= P\left(\theta^*_{j-1} < \psi\left(X_k\beta + \varepsilon^k\right) \leq \theta^*_j\right) \\ &= P\left(\psi^{-1}\left(\theta^*_{j-1}\right) - X_k\beta < \varepsilon^k \leq \psi^{-1}\left(\theta^*_j\right) - X_k\beta\right) \end{aligned}$$

or

$$P_j^k = \Phi\left(\varepsilon^*_j - X_k\beta\right) - \Phi\left(\varepsilon^*_{j-1} - X_k\beta\right); \qquad (3)$$

where $\varepsilon^*_j = \psi^{-1}\left(\theta^*_j\right)$ denotes the thresholds for ε^k, $j = 1,\ldots,7$. Equation (3) corresponds exactly to the ordered probit specification (with $\varepsilon^*_0 = -\infty$ and $\varepsilon^*_8 = +\infty$). Note that the thresholds ε^*_j are specific to the population considered and are independent of the individual characteristics. According to the above formula, individual characteristics are assumed to shift the distribution of ε. This is equivalent to a shift of all the thresholds of the same quantity $X_k\beta$, as shown on the following figure. A shift on the right increase all the probabilities on the right part of the figure (less risky choices) and decreases all the probabilities on the left size (more risky choices) .

The increasing function ψ is known in 7 points $\theta^*_j = \psi\left(\varepsilon^*_j\right)$. Recall that the θ^*_j can be computed given lotteries and a choice of the utility function. The ε^*_j are estimated using the usual ordered probit technique (see (3)).

This allows to compute the conditional distribution F_{X_k} :

$$F_{X_k}(\theta) \equiv F\left(\psi^{-1}(\theta) - X_k\beta\right) = \Phi\left(\varepsilon - X_k\beta\right):$$

4.3 Estimation results

4.3.1 Whole sample

As expected, the thresholds ε^*_{jr} obtained with explanatory variables are a simple linear transformation of the initial thresholds ε^*_j, obtained without explanatory variables : $\varepsilon^*_{jr} = a + b\varepsilon^*_j$. The new transformation ψ_r is then given by the condition : $\theta^*_j = \psi_r\left(\varepsilon^*_{jr}\right) = \psi_r\left(a + b\varepsilon^*_j\right) = \psi\left(\varepsilon^*_j\right)$. So ψ_r is obtained by combining ψ with a linear transformation and it is not necessary to give a the graph of $\theta_j = \psi_r\left(\varepsilon^*_{jr}\right)$.

The results for the whole sample estimation are presented in Annex Table 1. The explanatory power is very low (pseudo-R^2 = 2.23%) and most of the explanatory variables have no significant effect on risk aversion. Concerning socio-economic characteristics, we can only say that risk aversion is significantly larger when the purpose of the trip is a business appointment and lower

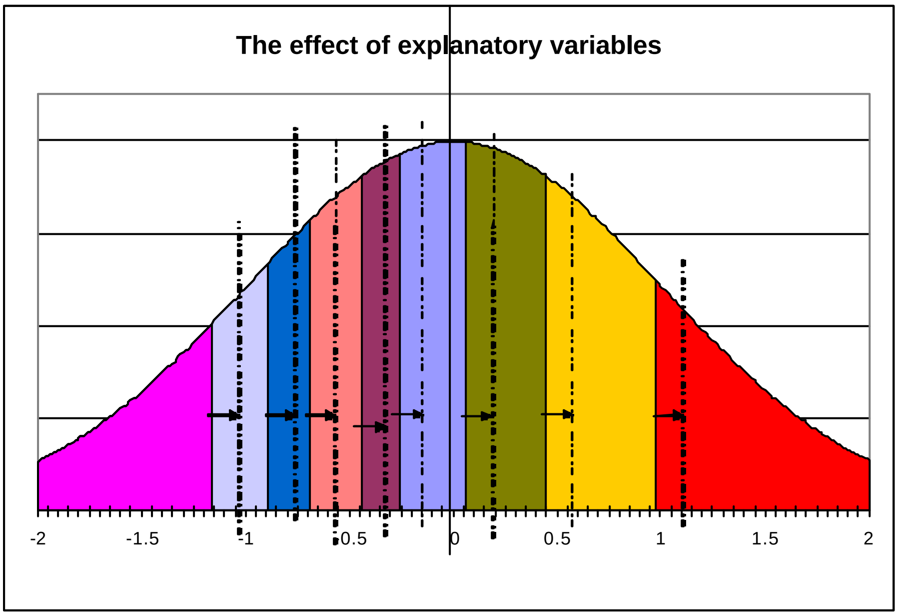

Figure 7: The e®ect of explanatory variables on º thresholds

for middle income earners. One possible explanation for the low explanatory power of individual characteristics is that risk averse individuals and risk lovers respond to di®erent models. So we will restrict the next section to risk averse individuals.

On the opposite, actual travel time $\overline{tt}$ has a very signi¯cant (positive) e®ect on risk aversion. In order to measure this e®ect in a non restrictive way, we simply considered a polynomial approximation and estimated an order 3 polynomial for the e®ect of $\overline{tt}$ on the latent variable º. The 3 corresponding parameters are highly signi¯cant and the shape of the e®ect of $\overline{tt}$ looks like a 1=$\overline{tt}$ function (see Figure), which is consistent with the mean-standard deviation and with the CARA formulations.

4.3.2 Risk averse agents only

The results for risk averse agents presented in annex table 2 show signi¯cant e®ects of many explanatory variables. The explanatory power (pseudo-R^2) has increased up to 11:87% and most explanatory variables have a signi¯cant e®ect on risk aversion.

Risk aversion is very signi¯cantly larger when the purpose of the trip is a business appointment and for public transportation users. Intermediary professions, employees and especially blue collars have a larger risk aversion than other workers or non workers. Women are slightly more risk averse than men, but the di®erence is not really signi¯cant. Older individuals (more than 60 years old) are far less risk averse. Risk aversion is a U-shaped function of income, but the e®ets is not very signi¯cant.

In the case of the probit model, we take advantage of the fact that the derivative of the density

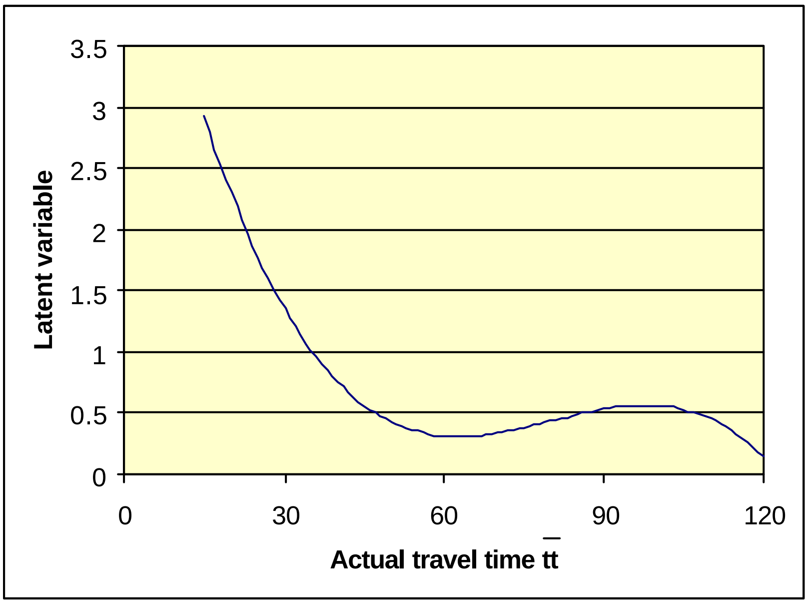

Figure 8: Estimated effect of $\overline{tt}$ on the latent variable.

function ϕ is simply the product of the identity and ϕ itself ($\phi'(x) = -x\phi(x)$) to calculate the conditional expectation of ν in any given interval. The conditional density of ν knowing that $\nu^*_{j-1} < \nu < \nu^*_j$ is $\frac{\phi(\nu)}{\Phi(\nu^*_j) - \Phi(\nu^*_{j-1})}$ on the interval $\left[\nu^*_{j-1}; \nu^*_j\right]$. So the conditional expectation of ν on this interval is

$$E\left[\nu | \nu \in \left[\nu^*_{j-1}; \nu^*_j\right]\right] = \int_{\nu^*_{j-1}}^{\nu^*_j} \nu \frac{\phi(\nu)}{\Phi\left(\nu^*_j\right) - \Phi\left(\nu^*_{j-1}\right)} d\nu = \frac{\phi\left(\nu^*_j\right) - \phi\left(\nu^*_{j-1}\right)}{\Phi\left(\nu^*_j\right) - \Phi\left(\nu^*_{j-1}\right)}.$$

The formula holds both on closed intervals and on open intervals with limits $-\infty$ and $+\infty$.

5 Concluding comments

This paper present a new methodology to measure risk for a population of travellers. We concentrate our analysis on route choice and there is no reason to believe that the quantitative results could be extended to other decisions under uncertainty. Our main focus was to analyses which factors influence the user's attitude towards risk. We collected our data via a telephone interview administered to a large sample of private and public transportation users. We introduce an index which measures the preference towards risk, either from an ordinal or from a cardinal point of view. The ordinal approach does not rely on expected utility theory. Indeed, one could argue that the ordinal neither rely of expected utility theory since it is based on discrete choice theory which does not satisfy, for example, the axiom of transitivity: if an individual prefers lottery L_2 to lottery L_1, and lottery L_3 to lottery L_2 it is not true that he prefers lottery L_3 to

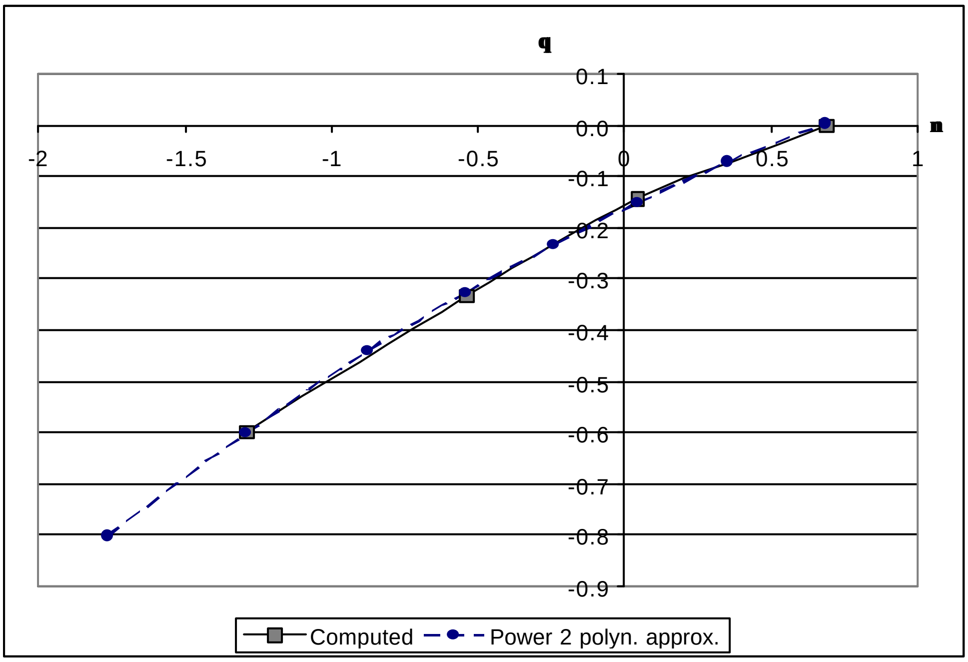

Figure 9: Conditional expectations for mean-standard deviation (Risk averse individuals)

lottery L_1. This is because, for each decision stage, some unobservable factors do in°uence the decisions processes (see Anderson; de Palma, and Thisse, 1992). This suggests that there is no need to reject expected utility theory, for the deterministic component of the model (without non-obversable error term variables), since the econometric model that we have developed can possibly explain example of choice behavior with seem to contradict expected utility theory.

We have headlight the impact of key socio-economics factors (gender, education, employment status, purpose of the trip) which explain the level of risk aversion. However, the statistical signi¯cance remains rather low (low adjusted R^2 of about 12%) and more data are needed (a few thousands are still not enough) to get better results and in particular to construct panel data. Embedded in a discrete choice model (ordered probit), the expected utility is not strongly rejected. However, we did not estimate the perceived probabilities, in order to check is the individual under (or over) estimates the probabilities of occurrence of additional travel time (for example). This would justify deviation from expected utility theory. Several questions raised in this paper remain open and much more work has to be performed in the theoretical and empirical study of risk aversion.

Finally, we believe that our methodology can potentially be applied for the analysis of risk aversion in other contexts: see, Ben-Akiva, de Palma, and Bolduc (2002) for the investment context, Goeree, Holt, and Palfrey (2002) for the game theory context and Holt and Susan (2002) for the use of experimental economics in the study of risk aversion. We expect that the web site that we are developing to collect data will be able to bridge this lacuna.

Acknowledgments

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Table A1 : Regression results (whole sample)

Ordered probit estimates
Number of obs : 2385
Log likelihood : -4960.4775
LR chi2(14) : 225.78
Prob> chi2 : $<10^{-5}$
Pseudo R2 : 0.0223

Variable	**Coef.**	**Std. Err.**	**z-stat**	**P>|z|**
Travel time	0.0730831	0.0095221	7.68	0
Travel time ²	-0.0008866	0.0001523	-5.82	0
Travel time ^{3}	3.24E-06	0.000000728	4.45	0
Purpose of the trip				
Business appointment	0.2365417	0.1207646	1.96	0.05
Other reason	Reference category			
Age				
Age>60	0.0007697	0.0928495	0.01	0.993
Younger	Reference category			
Profession				
Intermediary professions	0.1005022	0.055519	1.81	0.07
Employees	0.0838141	0.0570618	1.47	0.142
Blue collar	0.0179638	0.0952012	0.19	0.85
Other profession, non worker	Reference category			
Gender				
Woman	-0.0104343	0.0441392	-0.24	0.813
Man	Reference category			
Transportation mode				
Public transportation	0.2063947	0.0452522	4.56	0
Private transportation	Reference category			
Household income				
Income <1000€/month	-0.1318078	0.1072436	-1.23	0.219
1000-1500€/month	-0.3675041	0.0797591	-4.61	0
1500-2000€/month	-0.0850272	0.0658713	-1.29	0.197
2000-2500€/month	-0.055934	0.053451	-1.05	0.295
Income >2500€/month and missing	Reference category			
Threshold values				
threshold1	0.5218828	0.1896114		
threshold2	0.7835648	0.1885422		
threshold3	0.9874257	0.1880498		
threshold4	1.23507	0.1878847		
threshold5	1.420189	0.1882091		
threshold6	1.783224	0.1895806		
threshold7	2.213197	0.1916457		
threshold8	2.774855	0.1945354		

Figure 10:

Table A2 : Regression results (risk averse and risk neutral)

Ordered probit estimates
Number of obs : 1600
Log likelihood : -2211.2167
LR chi2(14) : 595.72
Prob> chi2 : $<10^{-5}$
Pseudo R2 : 0.1187

Variable	Coef.	Std. Err.	z-stat	P>|z|
Travel time	0.2102987	0.0154637	13.6	0
Travel time ²	-0.0027497	0.0002568	-10.71	0
Travel time ³	0.0000114	1.30E-06	8.72	0
Purpose of the trip				
Business appointment	0.3860796	0.1483898	2.6	0.009
Other reason	Reference category			
Age				
Age>60	-0.3522765	0.1256943	-2.8	0.005
Younger	Reference category			
Profession				
Intermediary professions	0.1734804	0.0694839	2.5	0.013
Employees	0.2115525	0.0722139	2.93	0.003
Blue collar	0.3368098	0.1251842	2.69	0.007
Other profession, non worker	Reference category			
Gender				
Woman	0.1030768	0.0559359	1.84	0.065
Man	Reference category			
Transportation mode				
Public transportation	0.158499	0.0565664	2.8	0.005
Private transportation	Reference category			
Household income				
Income <1000€/month	0.04663	0.1362516	0.34	0.732
1000-1500€/month	-0.1864096	0.1086728	-1.72	0.086
1500-2000€/month	-0.0940956	0.0833161	-1.13	0.259
2000-2500€/month	0.0006692	0.0674461	0.01	0.992
Income >2500€/month and missing	Reference category			
Threshold values				
threshold1	3.383028	0.2899622		
threshold2	4.35633	0.2953728		
threshold3	5.088665	0.2999036		
threshold4	5.858514	0.3040333		

Figure 11: