

On the optimal supply of urban bus services: new elements and extensions

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SYDNEY



1. Introduction
2. Background: Microeconomic models of urban public transport
3. New elements and extensions
 - Investment in dedicated infrastructure
 - Choice of fare payment technology and boarding policy
 - Optimal number of seats inside a bus (crowding and standing disutilities)
 - Bus queuing delays: congestion
4. Results
5. Conclusions and policy implications

- Decisions on urban public transport provision
 - Network design
 - Technological choice (bus, tram, light rail, metro, etc.)
 - Investment in infrastructure
 - Number of services per hour and day
 - Fare collection method
 - Location of stations or bus stops
- › Choices have a profound impact on the cost of the system and the level of service provided (accessibility, waiting time, in-vehicle time, comfort, etc.)
- › Microeconomic literature on public transport operations: Several papers that attempt to find optimal values of:
 - Frequency (veh/h)
 - Vehicle size (pax/veh)
 - Network density (lines/km²)
 - Stop spacing (stops/km)



The basic model (Mohring, 1972)

Operator cost

$$C_{op} = c \cdot f \cdot T$$

c : bus operating cost
 f : frequency, T : cycle time

Users cost (waiting time)

$$C_u = P_w \frac{Y}{2f}$$

P_w : Value of waiting time savings
 Y : demand



The basic model (Mohring, 1972)

Operator cost

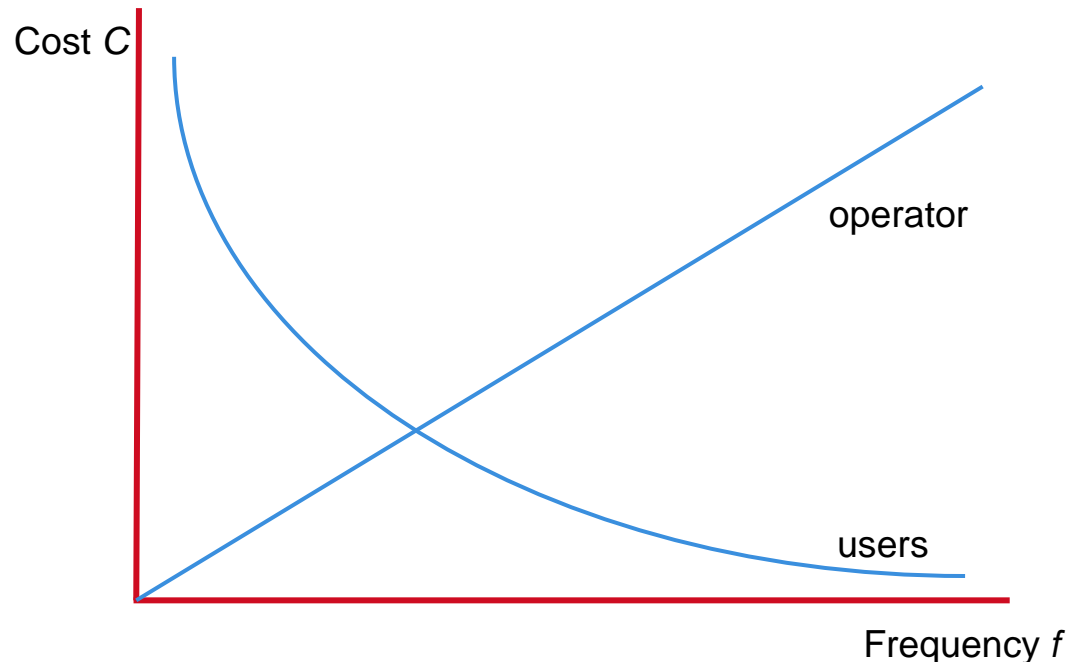
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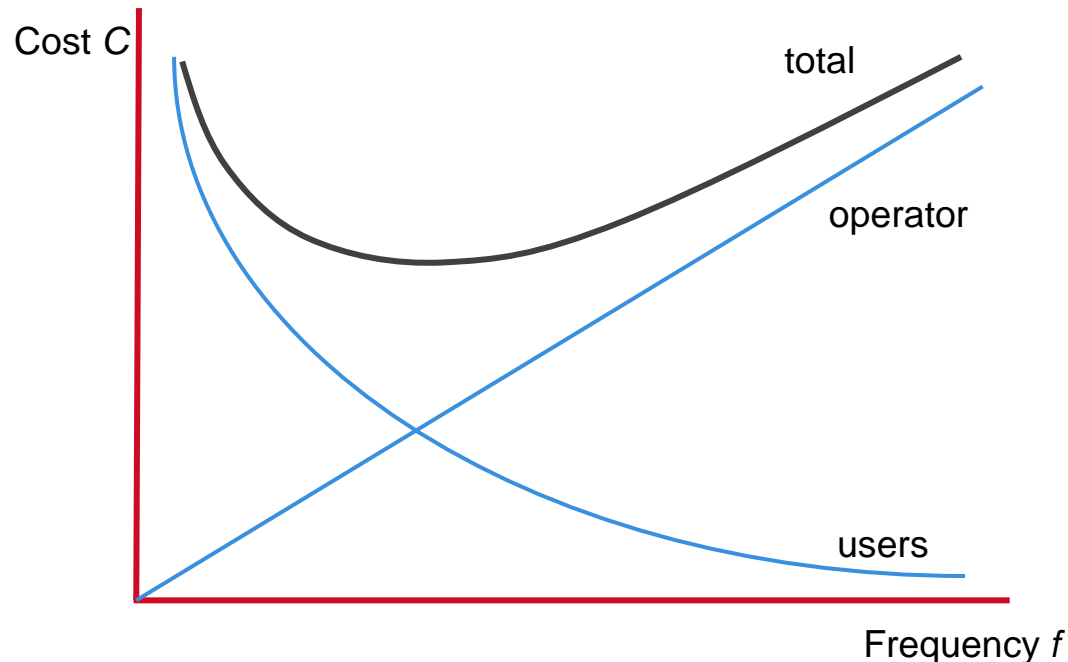
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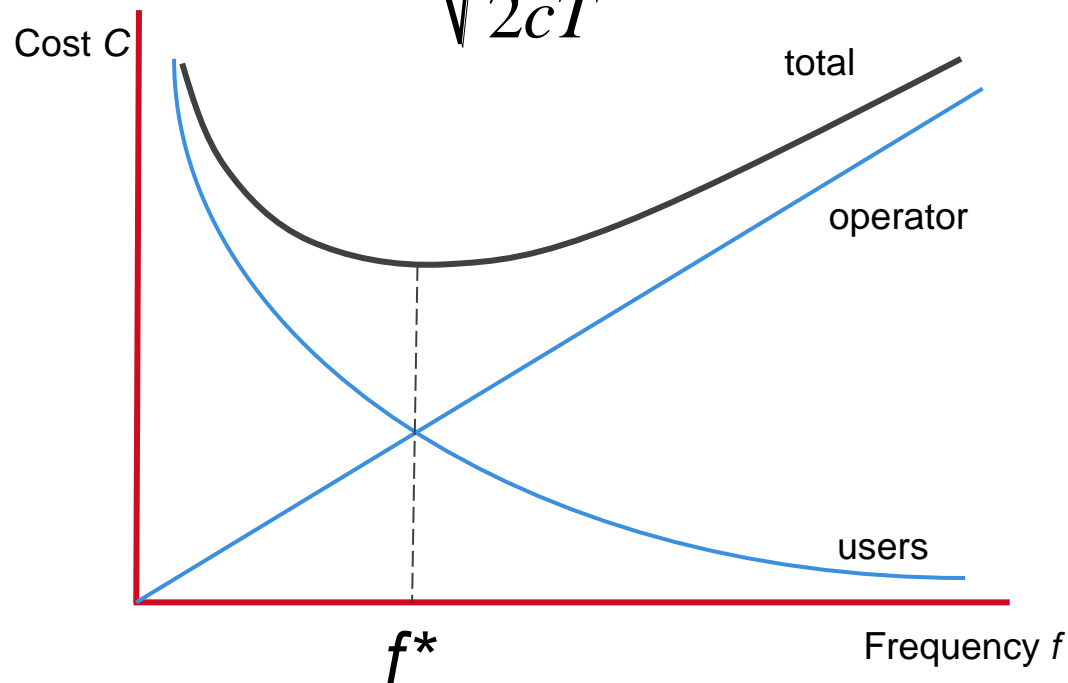




The basic model (Mohring, 1972)

$$C_{tot} = c \cdot f \cdot T + P_w \frac{Y}{2f} \quad \frac{dC_{tot}}{df} = c \cdot T - P_w \frac{Y}{2f^2}$$

$$f^* = \sqrt{\frac{P_w}{2cT} Y}$$



“The right approach is to escape the implicit notion that the only costs which are relevant to optimisation are those of the bus operator. **The time-costs of the passengers must be included too, and fares must be equated with marginal *social* costs.**”

(Turvey and Mohring, 1975, p. 280)

“(...) in the wide field of scheduled transport it has only recently been realised that **the principle of marginal cost pricing is practically impossible to apply correctly unless all users sacrifices and efforts are, at least conceptually, treated as costs on a par with producers costs.**”

(Jansson, 1979, pp. 270-271)



Literature summary

Model	Freq	Bus size	Dist stops	Route density	Fare level	Run speed	Fare pay board policy	Num seats	Special feature/contribution
Mohring (1972)	*		*		*				Square root formula
Jansson (1980)	*	*							Vehicle size
Kocur and Hendrickson (1982)	*	*		*	*				Elastic demand, number of lines
Oldfield and Bly (1988)	*	*			*				Waiting time not constant
Kuah and Perl (1988)	*		*	*					Stop spacing in feeder system
Chang and Schonfeld (1991)	*			*	*				Multiperiod analysis, elastic demand
Chien and Schonfeld (1998)	*		*	*					Rail line length optimization
Jara-Diaz and Gschwender (2003)	*	*							Crowding penalty effect
This work	*	*	*		*	*	*	*	Bus congestion and crowding



1. Decision on infrastructure for buses (choice of running speed)



(Sydney)



(Sydney)



(Delhi)

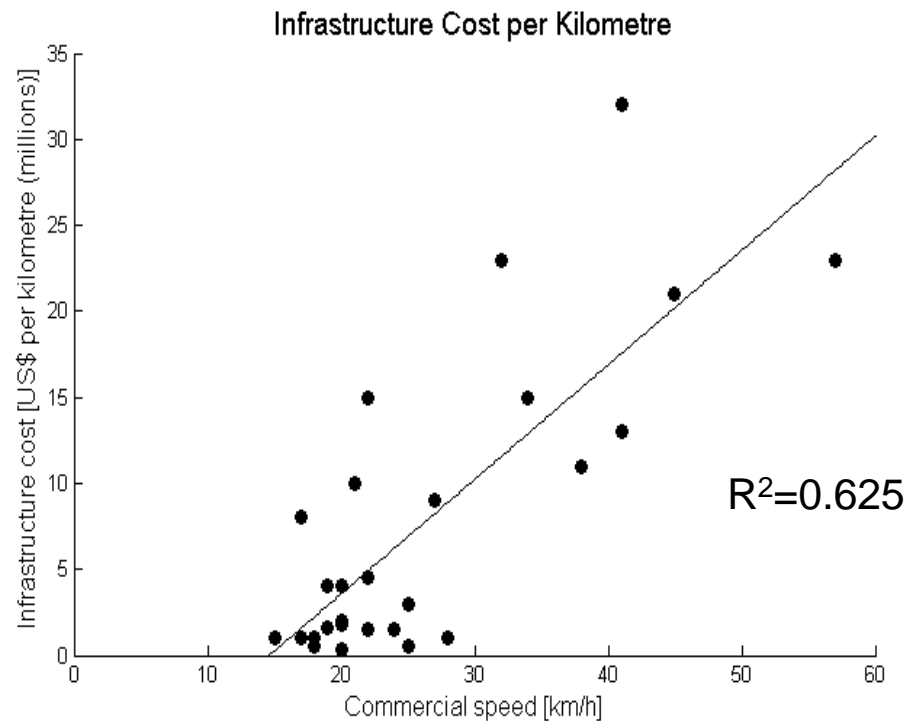


(Brisbane)



1. Decision on infrastructure for buses (choice of running speed)

‘The cost of *buying* speed’



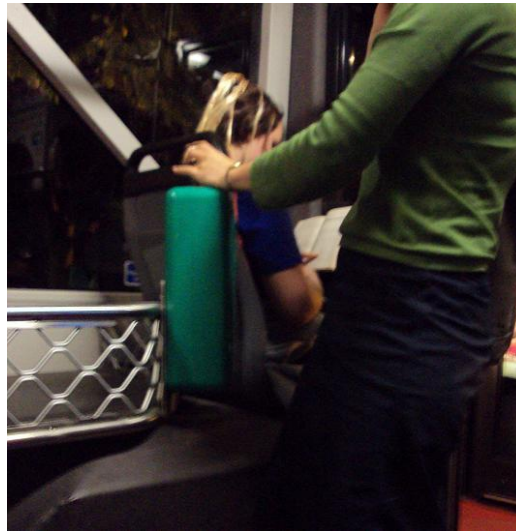
(Tirachini and Hensher, 2011)



2. Choice of fare collection system

› Several decisions to make

- Technological choice: cash, magnetic strip, contactless card, SMS message, etc.
- On-board or off-board payment
- Number of doors to board (1 or all)

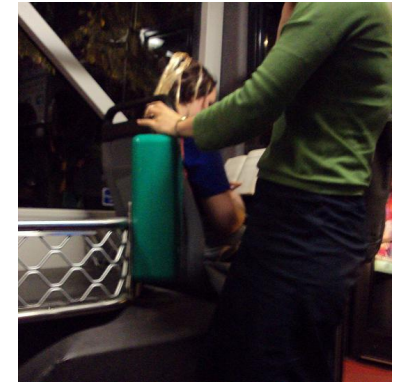


2. Choice of fare collection system

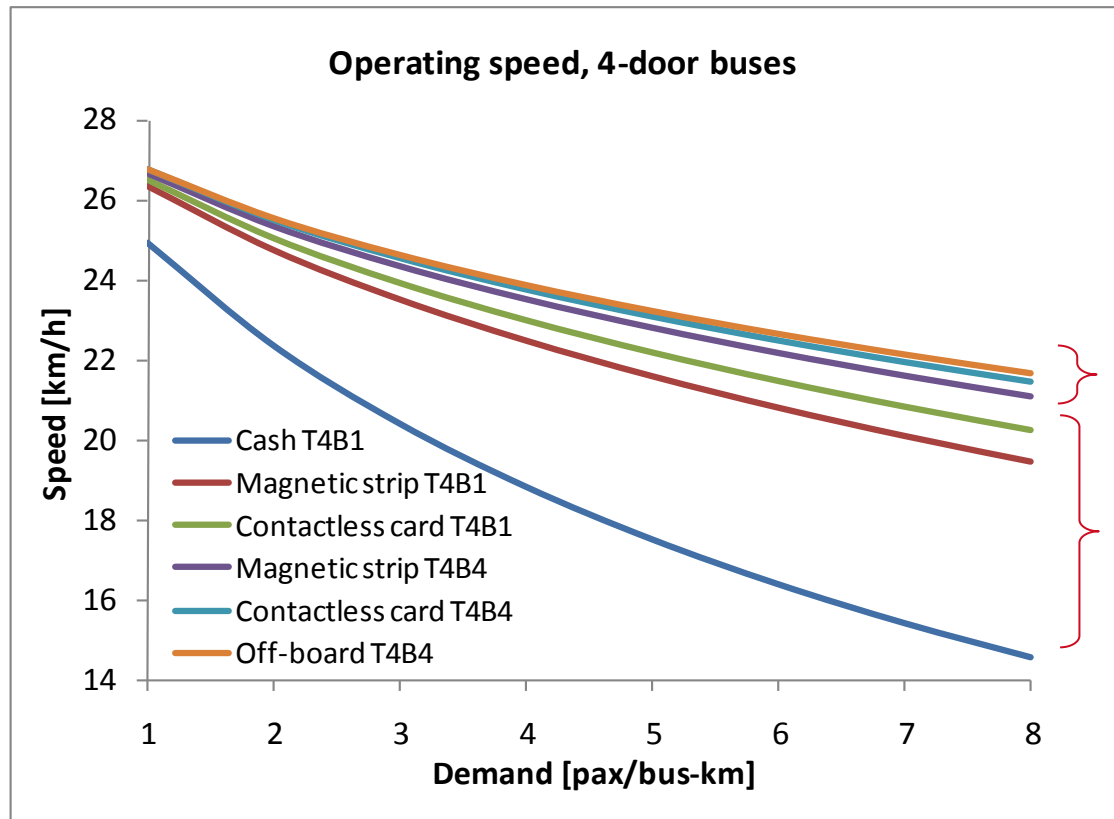
› Relevant for both users and operators. Differences on:

- Travel time:
 - Users' cost
 - Fleet size
 - Fuel and labour cost
- Capital cost
- Ability to integrate fares across routes and modes
- Transaction costs
- Evasion level
- Capacity to handle different fare structures
- Complexity of use

› Relevant... but under-researched



- › **Estimation bus operating speed:** Total speed including running time and stops of any sort



Boarding all doors

Boarding front door only

Technology Effect
Door Effect

Estimated with travel time model for Sydney buses, Tirachini (2010)



3. Bus congestion



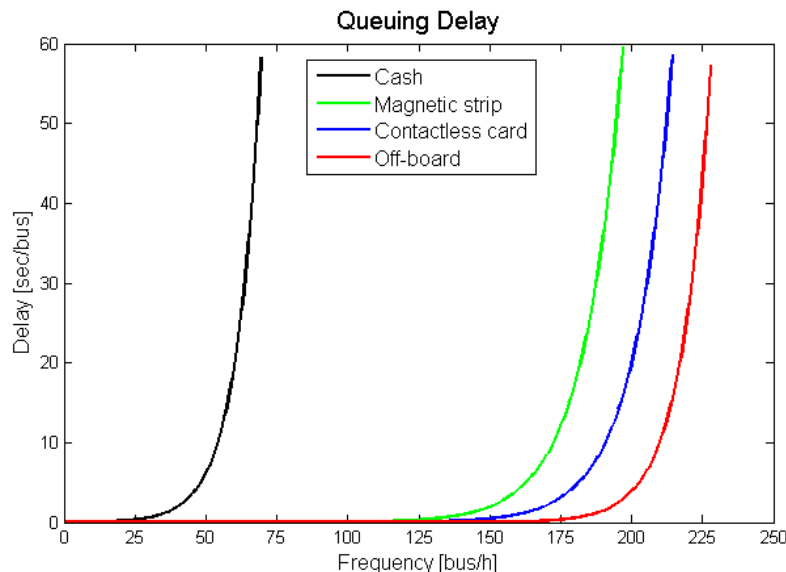
Oxford



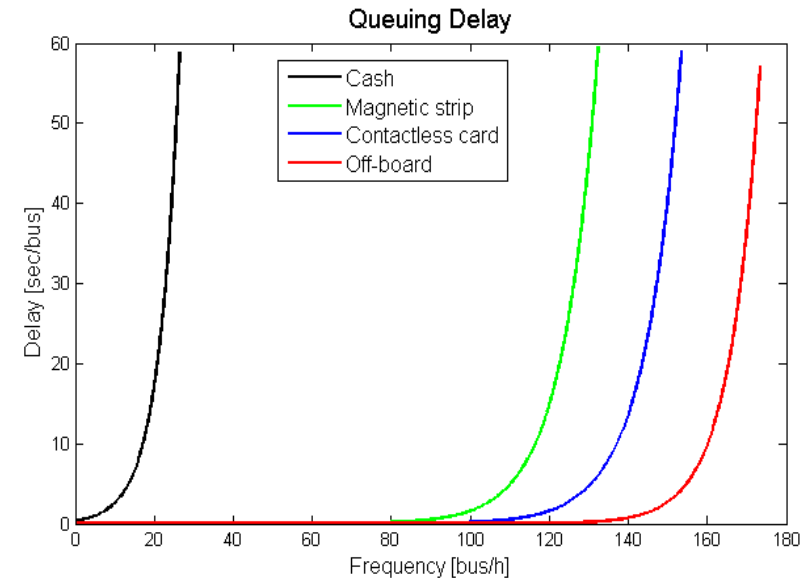
Santiago

3. Bus congestion

- › Bus congestion is an issue for **high frequency - high demand services**.
- › Usually disregarded in the economic analysis of pricing policies. If considered: Linear or BPR (*Bureau of Public Roads*) functions.
- › **More comprehensive approach: Bus queuing delay is function of frequency, demand and fare payment policy.**



5 pax/bus



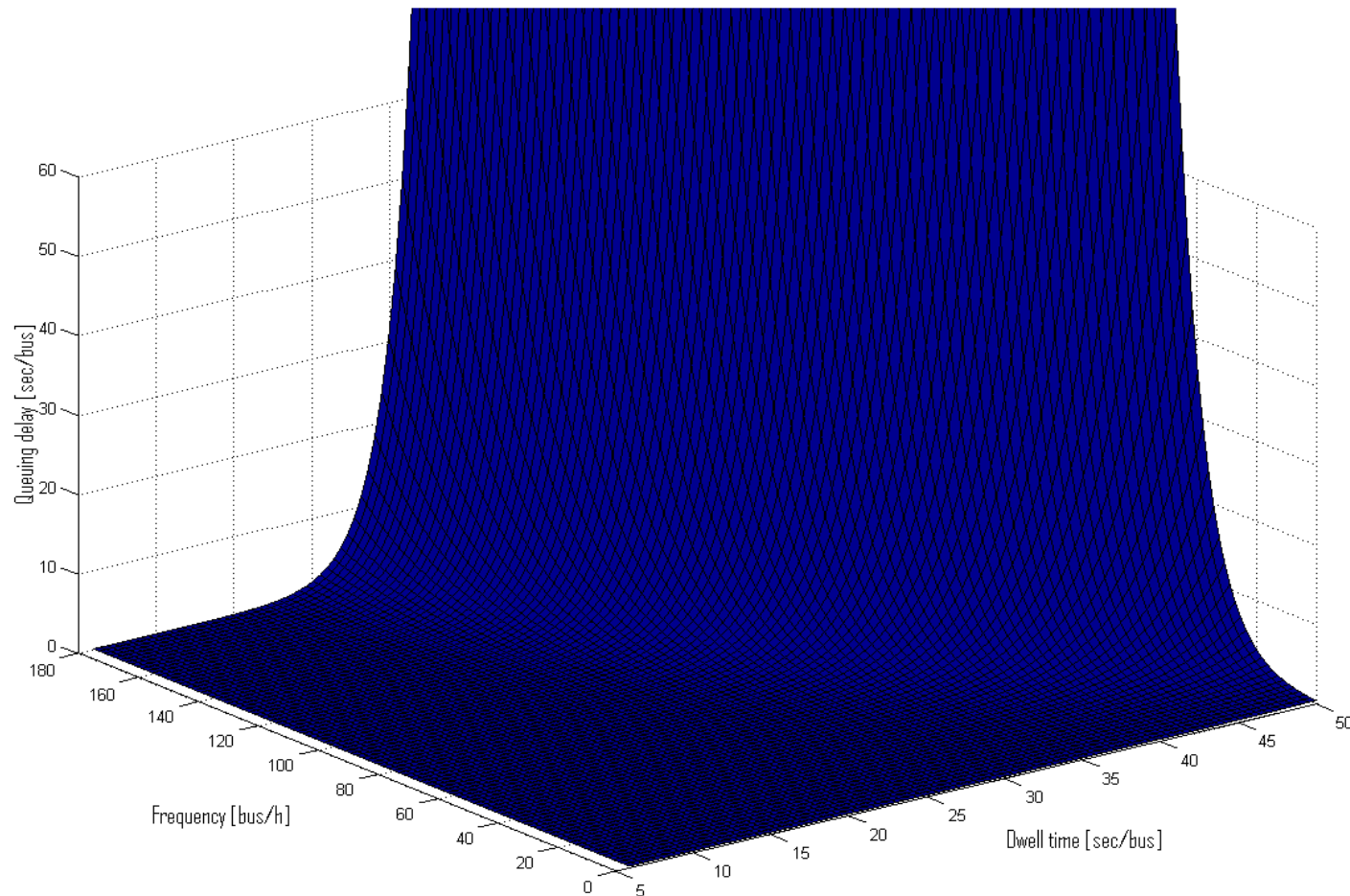
15 pax/bus

$$t_q = (b_0 + b_1 P_b) e^{(b_2 + b_3 P_b) f} \quad (\text{Fernández et al., 2000; Tirachini and Hensher, 2011})$$



3. Bus congestion

Queuing delay at bus stops





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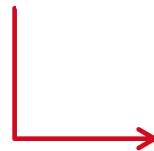
crowding



4. Crowding



- Source of disutility for users
 - It increases the value of in-vehicle time savings (e.g. Whelan and Crockett, 2009; Hensher *et al.*, 2011)
 - It increases the boarding and alighting time itself
 - For trains (Lin and Wilson, 1992)
 - For buses (Tirachini, 2011)
- Behavioural effect*
- Performance effect*



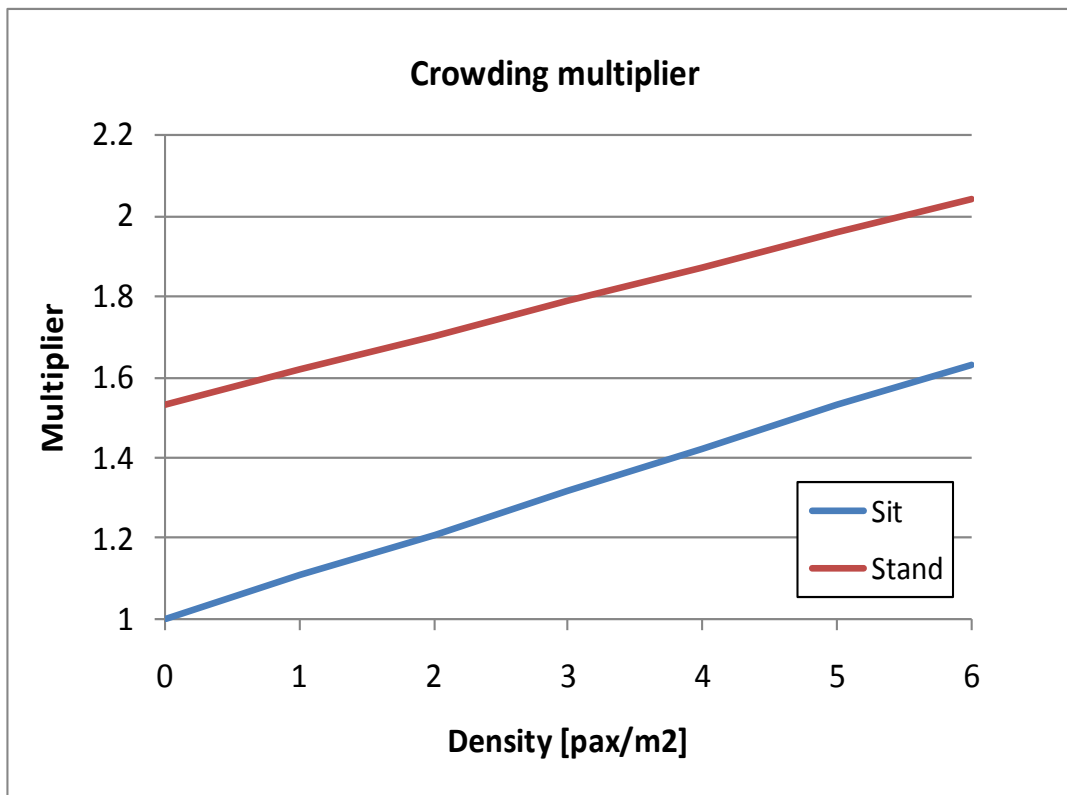
**Impact of these effects on
the optimal design of bus systems**

5. Optimal number of seats inside a bus

- People dislike crowding
- People dislike standing



Increase in valuation of in-vehicle time



UK rail services
(Whelan and Crockett, 2009)

Impact on:

- Frequency and bus size
(e.g., Jara-Diaz and Gschwender, 2003)
- **Number of seats**
 - Pax sitting: 0.50 m²
 - Standee: 0.15-0.20 m²

More seats: Comfort and the expense of capacity



Number of seats as a decision variable: not a crazy idea!

Different allocation of space for seating and standing



Model 1: Total cost minimisation

Total cost = Operator cost + User cost

$$\text{Min } C_{tot}(f, K, S, v_0, \beta) = \underbrace{C_o(f, K, S, v_0, \beta)}_{\text{Operator cost}} + \underbrace{C_a(S)}_{\text{Access time cost}} + \underbrace{C_w(f)}_{\text{Waiting time cost}} + \underbrace{C_v(f, K, S, v_0, \beta)}_{\text{In-vehicle time cost}}$$

- › f : Frequency (veh/h)
- › K : Vehicle size (pax/veh)
- › S : Number of bus stops
- › V_0 : Running speed (km/h)
- › β : Payment method

Operator cost

$$C_o(f, K, p, v_0, \beta) = \underbrace{c_1(v_0)L}_{\text{Land+busway}} + \underbrace{c_2(\beta)S}_{\text{Stations}} + \underbrace{c_3(K, \beta)fT_c(f, K, S, v_0, \beta)}_{\text{Rolling Stock+personnel}} + \underbrace{2c_4(K)Lf}_{\text{Running cost}}$$

Access time cost

$$C_a(S) = P_a \frac{L}{2v_w S} N_b$$

Waiting time cost

$$C_w(f) = P_w \left(t_0 + \frac{t_1}{2f} \right) N_b$$

In-vehicle time cost

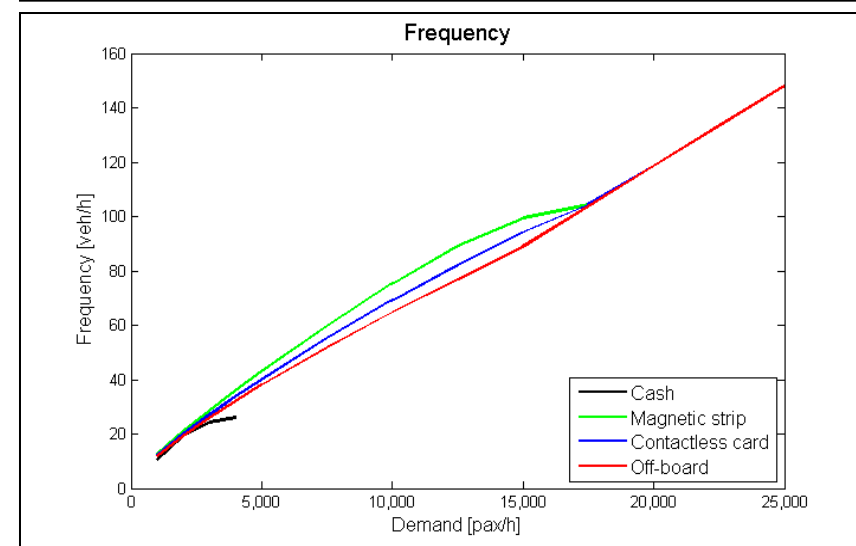
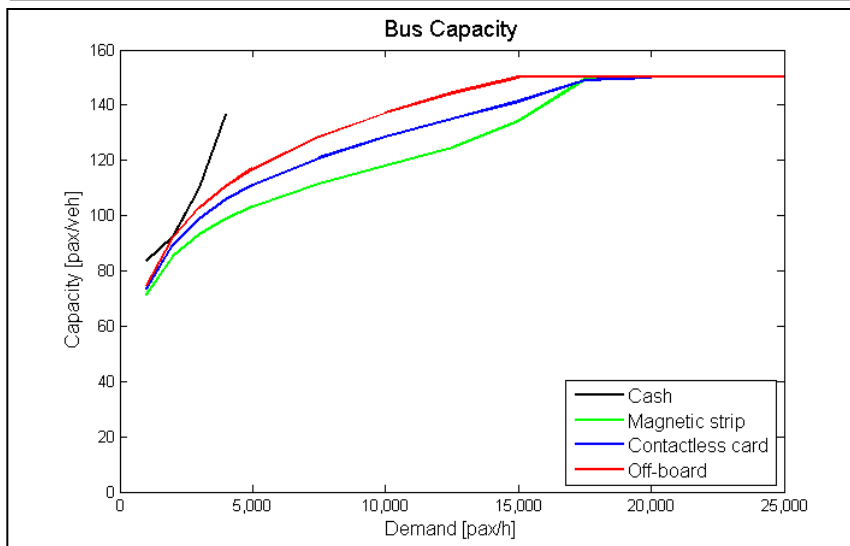
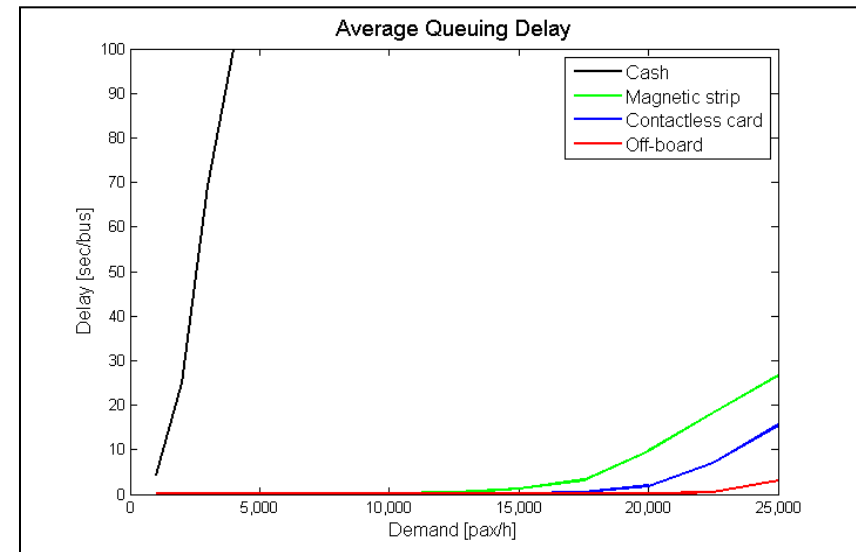
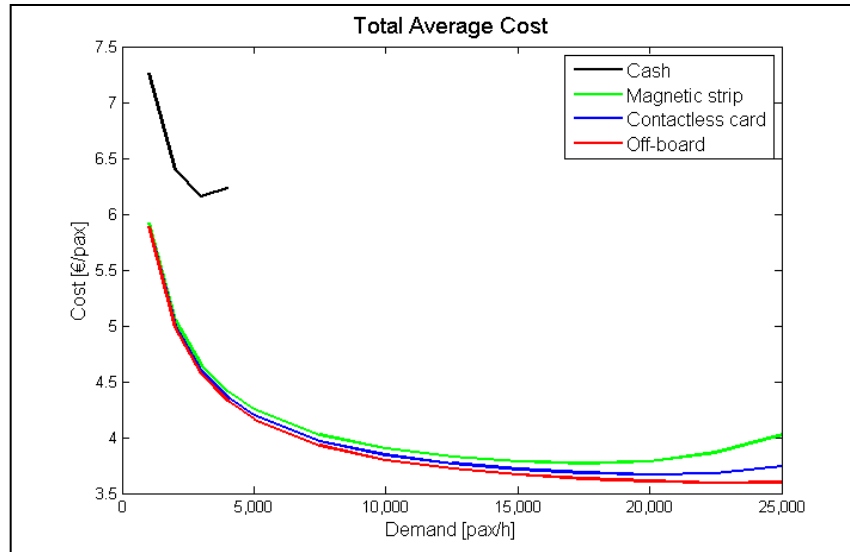
$$C_v = P_v \frac{l_1}{L} T_{t1} N_{b1} + P_v \frac{l_2}{L} T_{t2} N_{b2}$$

Travel time

$$T_{t1} = \underbrace{\frac{L}{v_0}}_{\text{Running time (cruising speed)}} + \underbrace{\left[\frac{0.5C_T(1-u)^2}{1-ux} + \frac{v_0}{2} \left(\frac{1}{a_0} + \frac{1}{a_1} \right) \right] \frac{1-u}{1-ux} I}_{\text{Time lost at intersections}} + \underbrace{\left[p_h t_{s1}^h + (1-p_h) t_{s1}^l \right] S}_{\text{Time lost at bus stations}}$$

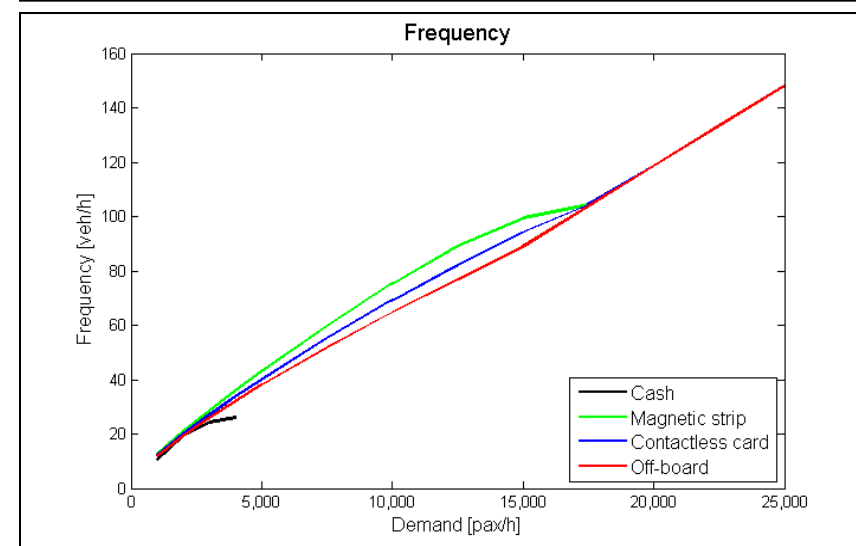
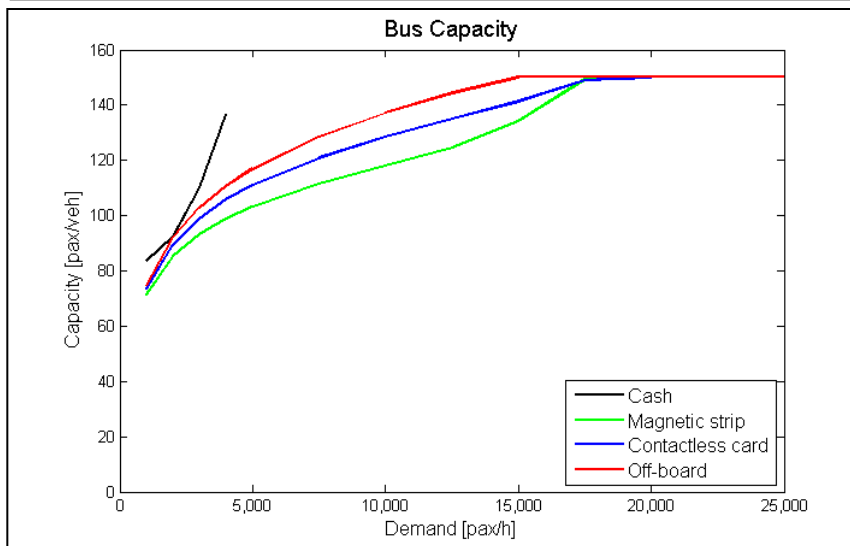
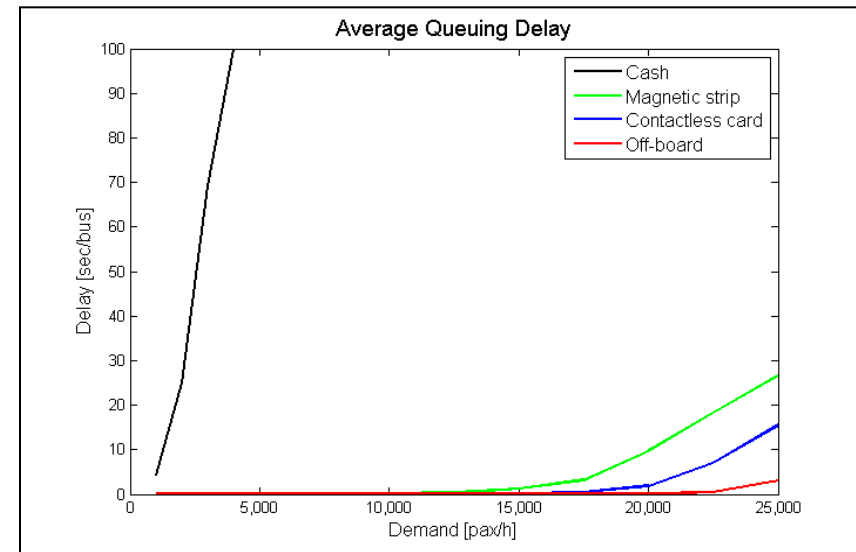
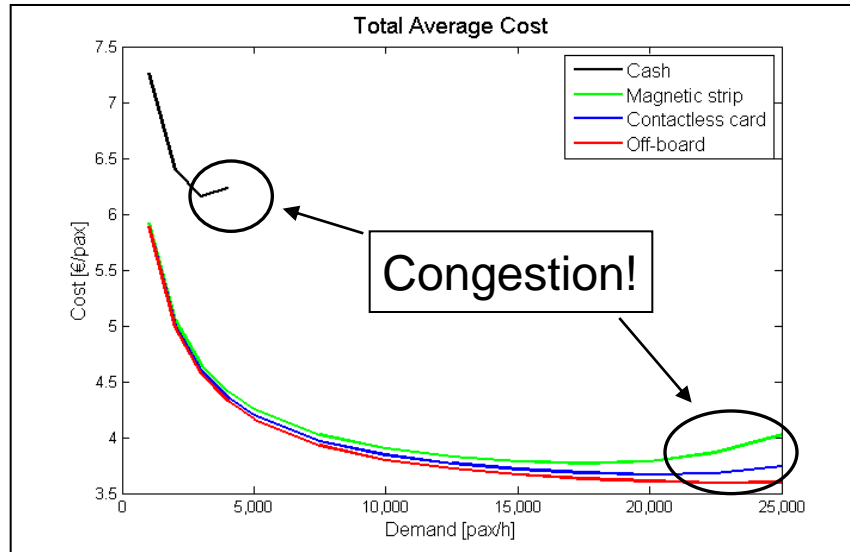


Results: Total average cost and variables

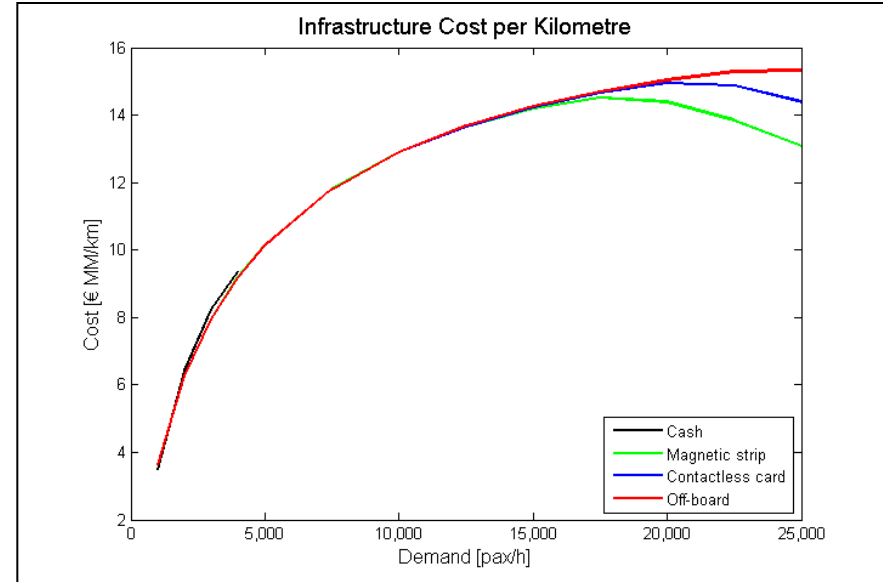
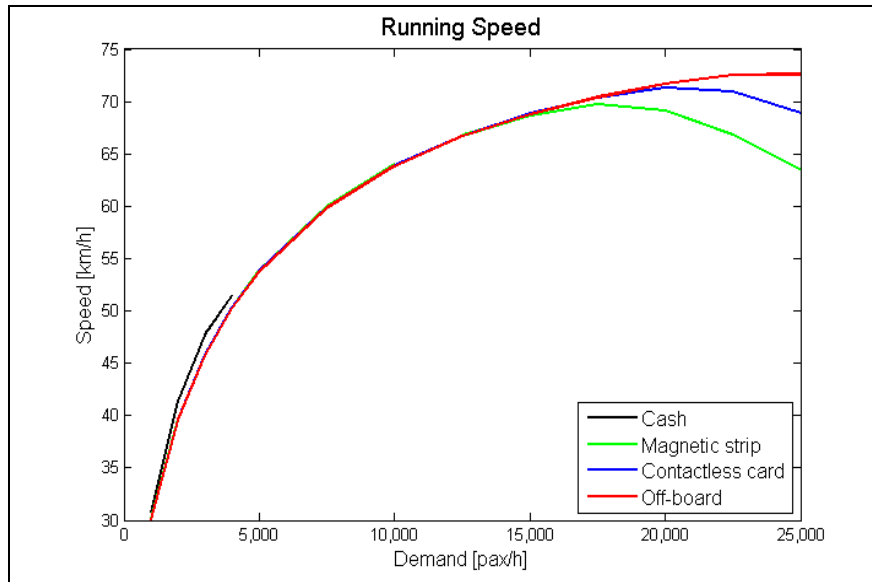




Results: Total average cost and variables



Results: Optimal running speed and infrastructure investment



Numerical analysis: **Logarithmic function**

$$v_0 = 14.39 \ln(N) - 69.20$$

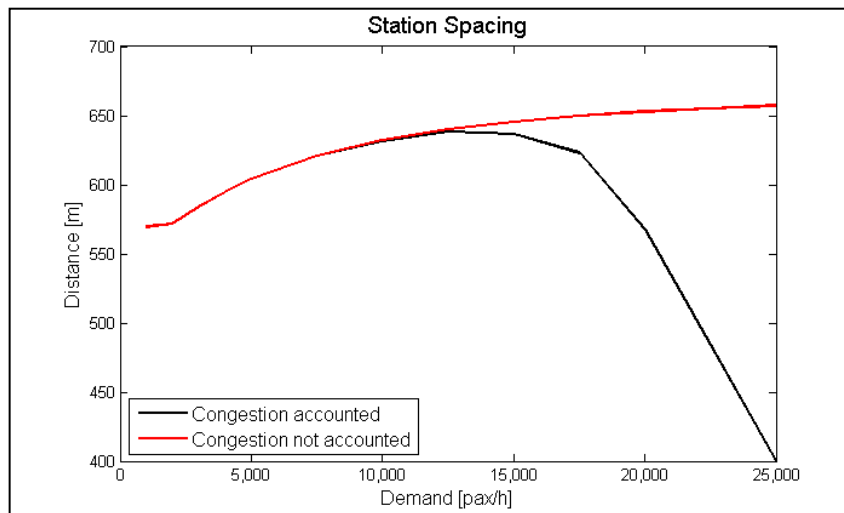
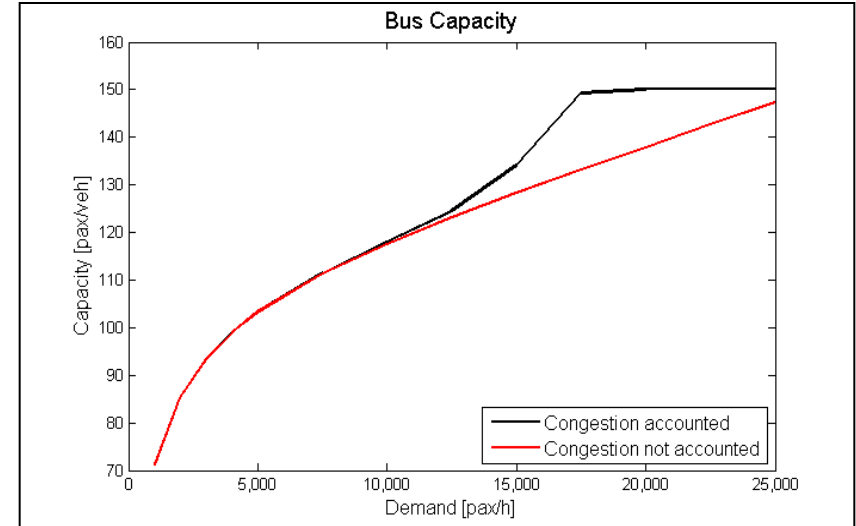
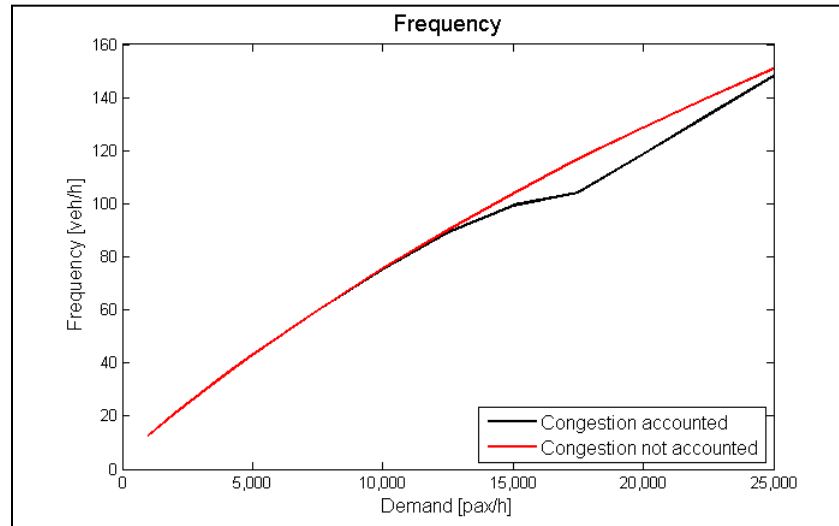
$$(R^2 = 0.998)$$

$$c_1 + c_2 \frac{S^*}{L} = 3.76 \ln(N) - 22.07$$

$$(R^2 = 0.994)$$



Results: Effect of (ignoring) bus congestion



Bus stop congestion should imply:

- Lower frequency
- Higher capacity
- Bus stops closer together

Model 2: Social welfare maximisation

- › Three modes: Bus, car, walk
- › OD matrix: Military Road, Sydney (12 zones, 3.5 km)
- › Congestion interaction bus-car (static) plus queuing delays for buses
- › MNL

$$SW = \underbrace{\sum_{ijp} \frac{y_{mp}^{ij}}{\lambda} \ln \sum_m e^{U_{mp}^{ij}}}_{\text{Users benefit}} + \underbrace{\sum_{ijp} y_{ap}^{ij} \pi_{ap}^{ij}}_{\text{Toll revenue}} + \underbrace{\sum_{ijp} y_{bp}^{ij} \pi_{bp}^{ij}}_{\text{Bus revenue}} - \underbrace{C_o}_{\text{Bus op cost}}$$

Variables:

Bus frequency

Congestion toll

Bus size

Bus fare

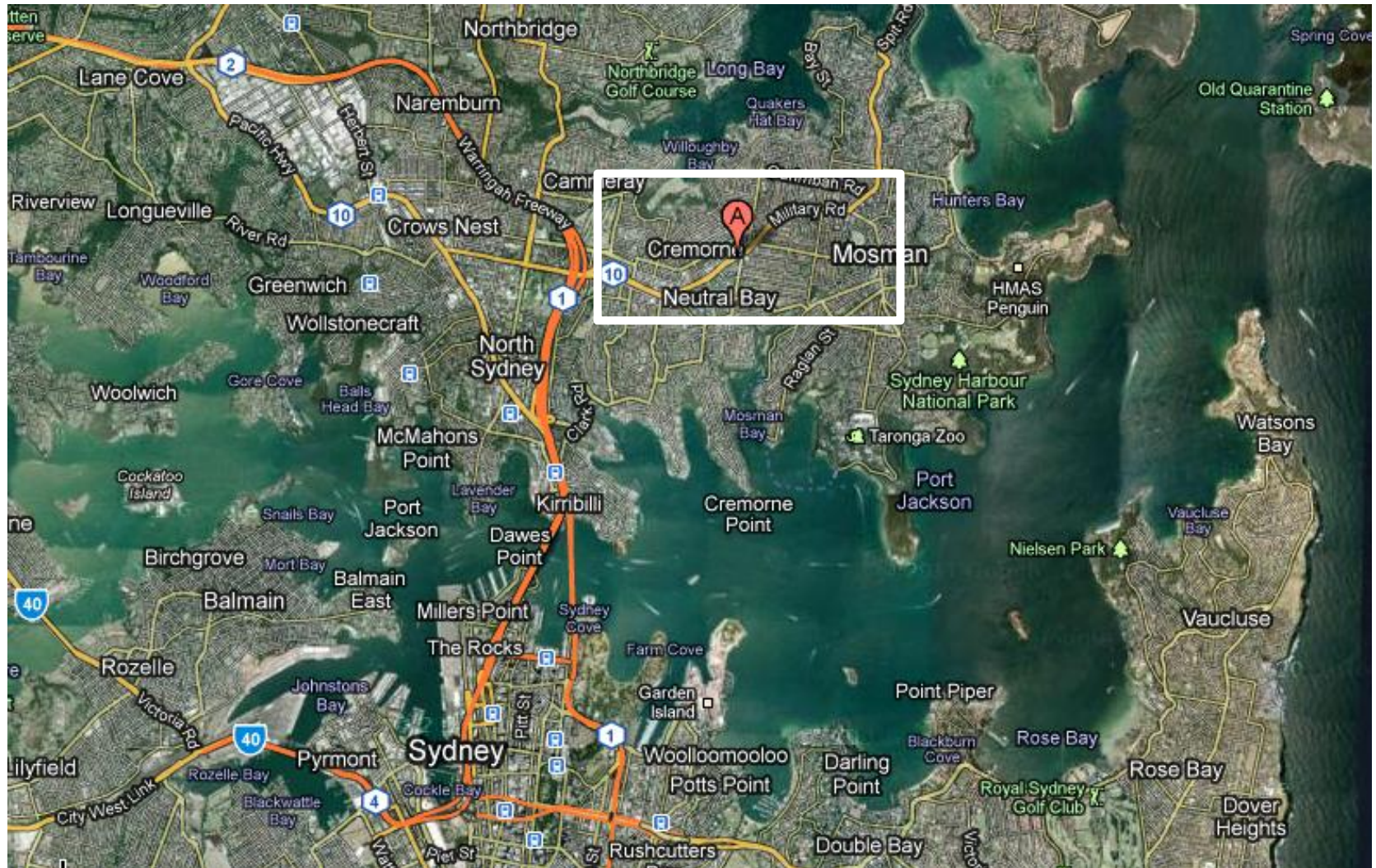
Fare collection technology

Boarding policy

Number of seats



Test corridor: Military Road, Sydney



› Estimated modal share (trips up to 5 km)

- Car: 62.5%
- Walk: 31.6%
- Bus: 5.9%

› Current bus service

- 12m long buses
- Two doors, boarding only at front door
- On-board magnetic strip payment
- Aprox 40 seats (65% of total bus area, 80% of area available for seating and standing)
- Morning peak frequency: 16 bus/h
- Fare: \$2



	Bus size (m)	Seats	Payment	Freq (bus/h)	Fare (\$)	Toll (\$)
Current situation	12	40	Magnetic strip	16.0	2.00	0.00
First best	8	24	Off-board	24.0	0.25	2.25
Best 12m bus	12	40	Off-board	17.2	0.15	2.25
Second best	8	24	Off-board	24.1	-0.95	0.00

Always maximum number of seats due to crowding and standing costs

	Pbus	Pcar	Pwalk	Welfare (\$)	Gain (\$)
Current situation	6.0%	62.5%	31.6%	43,633	0
First best	8.6%	56.9%	34.5%	45,130	1,498
Best 12m bus	8.7%	56.8%	34.5%	45,065	1,432
Second best	8.7%	60.7%	30.6%	44,419	786



8 metres



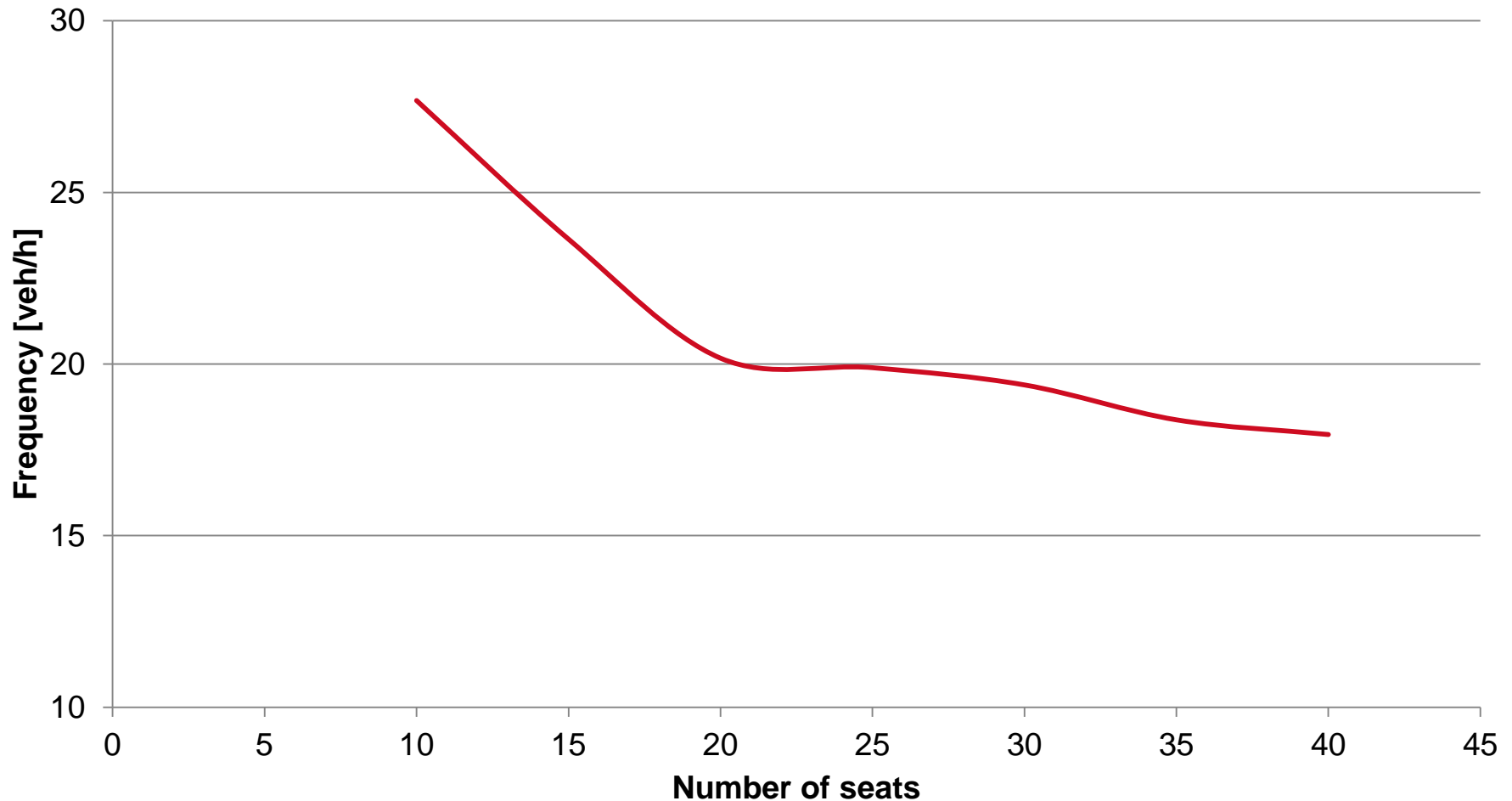
12 metres





Relation between frequency and number of seats

Optimal frequency



1. Running speed can be treated as a decision variable, linked to bus infrastructure investment
2. Comparison of different fare payment technologies and boarding policies
3. More proper treatment of congestion in the microeconomic modelling of bus operations (queuing delays)
4. Inclusion of crowding and standing disutilities in the optimisation of urban bus services
5. Selection of number of seats inside a bus

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Danke Zürich

- › Tirachini, A., Hensher, D. A. (2011) Bus congestion, optimal infrastructure investment and the choice of a fare collection system in dedicated bus corridors. *Transportation Research Part B: Methodological* 45(5), 828-844
- › Tirachini, A. (2011) Bus dwell time: The effect of different fare collection systems, bus floor level and age of passengers. Forthcoming in *Transportmetrica*
- › Jara-Díaz, S.R. and Tirachini, A. (2011) Urban bus transport: open all doors for boarding. To be presented in *Thredbo 12* Conference, Durban, South Africa, September 2011
- › Tirachini, A. (2010) Travel time, operating speed and the benefits of upgrading the fare payment technology in urban bus services. Submitted to *Transportation Research Part C: Emerging Technologies*



Minimisation Problem

>

$$\text{Min } C_{tot}(f, K, S, v_0, \beta) = C_o(f, K, S, v_0, \beta) + C_a(S) + C_w(f) + C_v(f, K, S, v_0, \beta)$$

Subject to:

$$\frac{\alpha N_b}{\kappa f} \leq K \leq K_{max}$$

$$f_{min} \leq f \leq f_{max}$$

$$v_{min} \leq v_0 \leq v_{max}$$

$$\beta \in \{\beta_1, \beta_2, \beta_3, \beta_4\}$$

Boarding and alighting times

Tirachini (2011) estimated the average boarding and alighting time per passenger for buses in Sydney, for the cases of **cash payment (10.02 sec/pax)**, **magnetic strip (4.61 sec/pax)** and **free service** (proxy for payment and fare verification outside bus, **1.46 sec/pax**), whilst a boarding time of **2.05 sec/pax** is used for the case with a **contactless card**, obtained by Fernández *et al.* (2009) for trunk services in Santiago de Chile.