The Reliable Hub-and-Spoke Network Design Problem: Models and Algorithms

ETH Zurich
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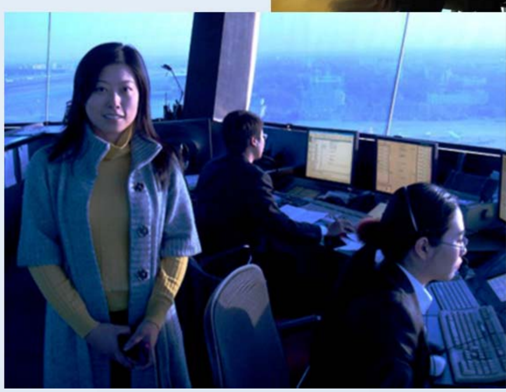
Yu Zhang
University of South Florida

Research Group



Research Areas:

- Air transportation
  - Airport surface movement operations and management
  - Airport system planning and management
  - Performance measurement and system analysis
- Traffic operations and management
- Non-motorized transportation
  - Performance evaluation
  - Safety

Sponsored Research Projects

Air Transportation

- **ACRP 02-50, Deriving Benefits from Alternative Aircraft-Taxi Systems (Institutional PI), sponsored by Transportation Research Board Airport Cooperative Research Program, 6/2014-12/2015**
- **The Role of Air Cargo in Tampa Bay Regional Goods Movement (PI), sponsored by Florida Department of Transportation, 1/2014-12/2014**
- **Users' Perception of Remote and Virtual Tower at Small Airport, sponsored by Transportation Research Board Airport Cooperative Research Program Graduate Research Award Program, 2014**
- **ACRP 02-38, Guidebook for Energy Facilities Compatibility with Airports and Airspace (Institutional PI), sponsored by Transportation Research Board Airport Cooperative Research Program, 10/2012-4/2014**
- **Impact of Single Airport Delay to National Airspace System, sponsored by Transportation Resertch Board Airport Cooperative Research Program Graduate Research Award Program, 2010**
- **Research on FAA Performance Indicators (PI), sponsored by Federal Aviation Administration (FAA) Air Traffic Organization (ATO), 2009-2010**
- **Performance Metrics Development and Analysis Support (PI), sponsored by Federal Aviation Administration (FAA) Air Traffic Organization (ATO), 2009-2010**

Sponsored Research Projects

Non-motorized Transportation

- **Bull Bikes - A Smart Bike Sharing Program for USF Bulls (PI), sponsored by Student Green Energy Fund of USF, 8/2013-8/2016**
- **Bulls Walk and Bike Week Campaign for Improving Pedestrain and Bicyclist Safety (Co-PI), sponsored by Florida Department of Transportation, 2012, 2013, 2014**

Traffic Operations and Management

- **Tampa Bay, FL In-Vehicle Driving Behavior Field Study (Investigator), sponsored by Strategic Highway Research Program (SHRP2) , 2010-2013**
- **Design of Advanced Traffic Responsive Signal System (PI), sponsored by Florida High Tech Corridor and Albeck Gerken Inc., 1/2012-12/2012**

Multidisciplinary Research

- **Graduate Scholarships to Achieve Sustainable Infrastructure at the Water-Energy-Global Nexus (Co-PI), sponsored by National Science Foundation (NSF), 2010-2014**

Outline

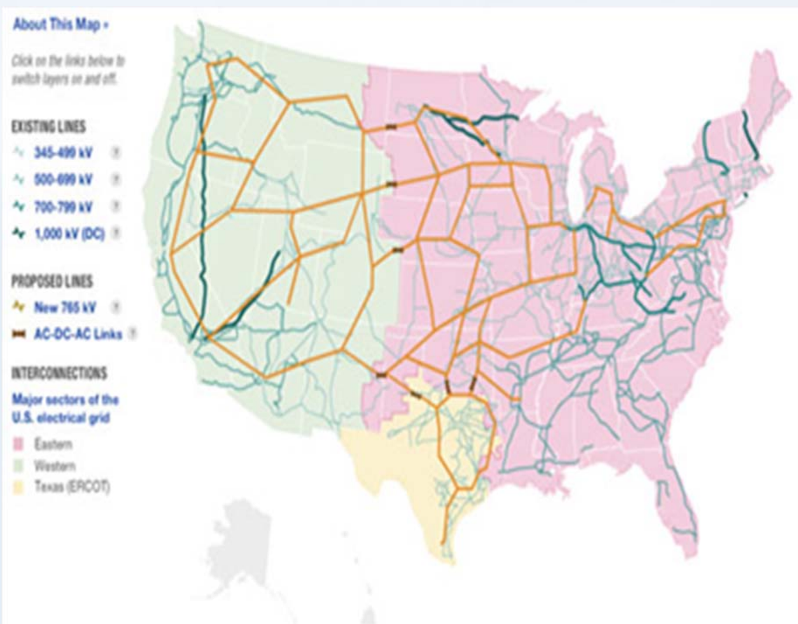
- Part I: Background and Motivation
- Part II: Literature Review
- Part III: Formulation
- Part IV : Solution Method
- Part V: Case Study and Conclusions

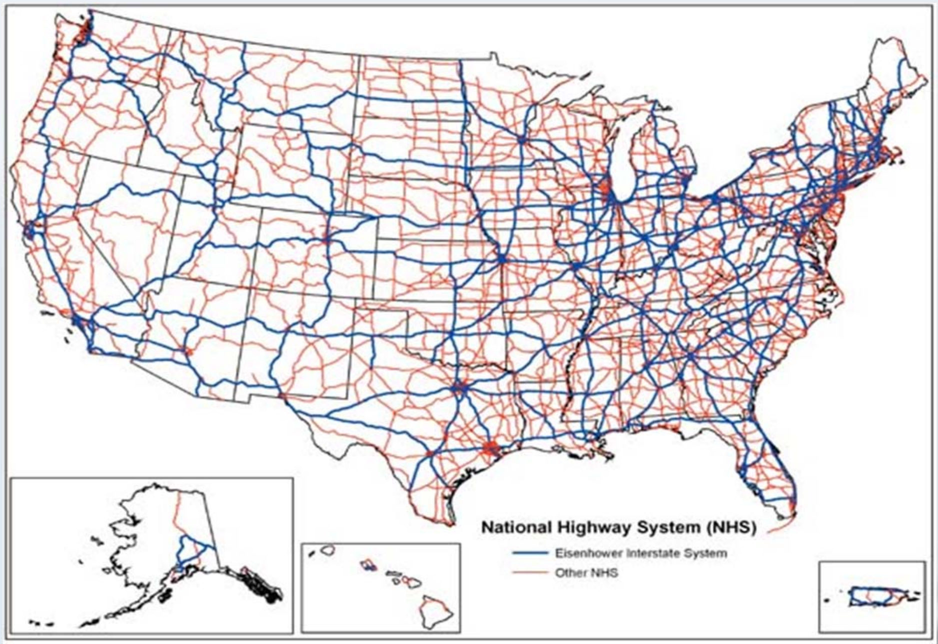
Collaborative work with Yu An and Dr. Bo Zeng at USF.

Part I
Background and Motivation

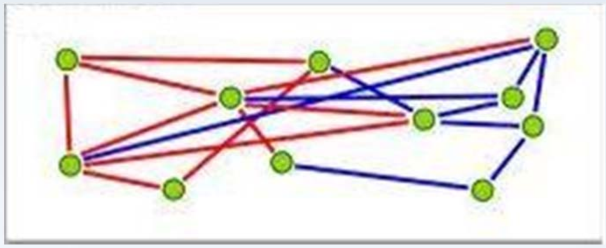
Application of Network Systems

- Electrical Engineering: power grid
- Transportation Science: roadway system

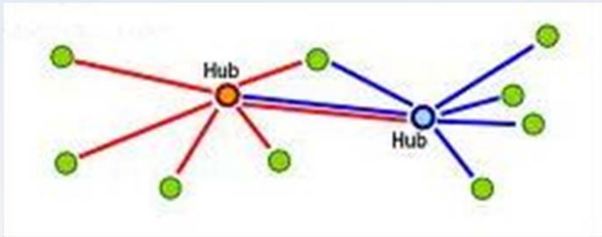




Typical Formats of Network Systems



Point-to-Point system

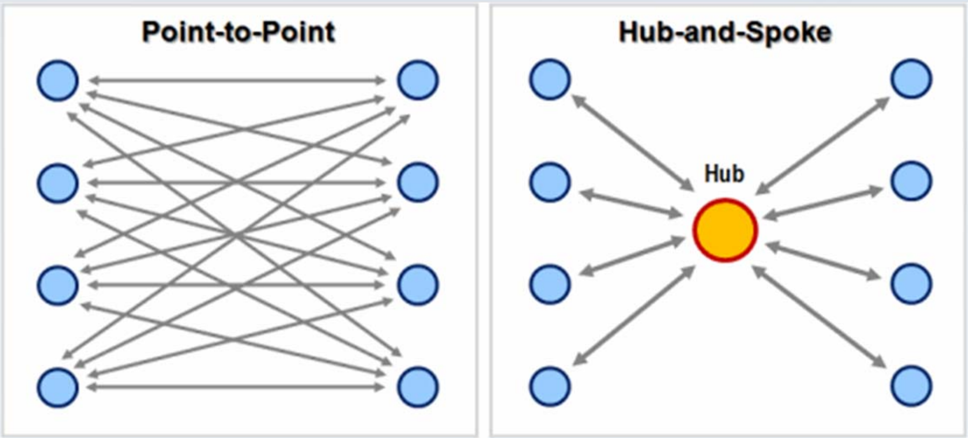


Hub-and-Spoke system

Hub-and-spoke structure is an important class of networks that is widely used in a variety of industries:

- *Air transportation (United Airlines, Delta Air Lines, American Airlines…)*
- *Postal delivery (Fedex, UPS…)*
- *Telecommunications*
- *Others*

Advantages of Hub-and-spoke System



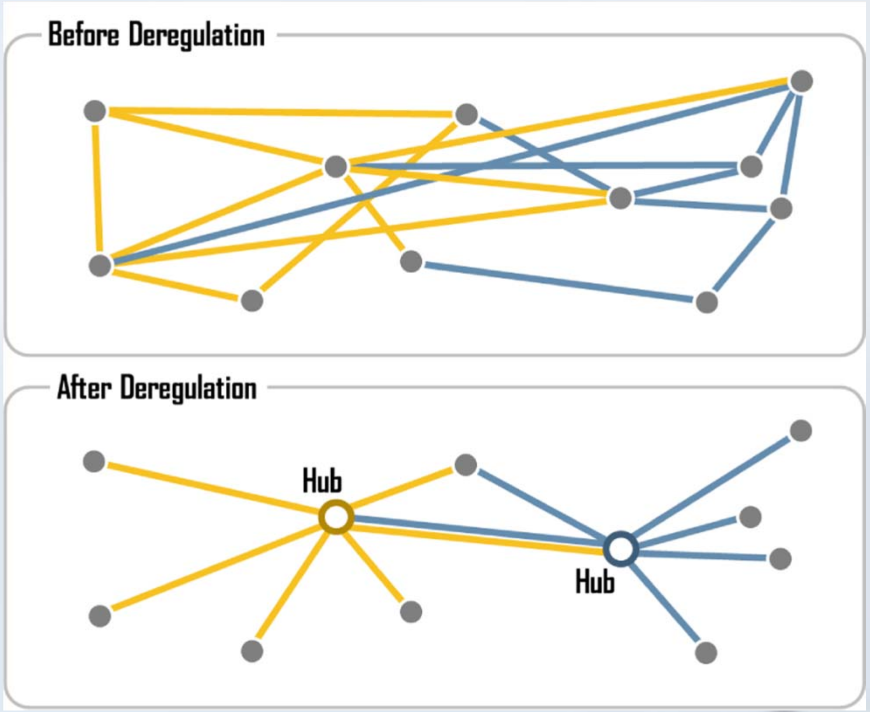
• **Simplify the structure of a network:** A small number of links are sufficient to keep the connection of the whole network.

•**Reduce the operating cost of the whole system:**

- •**Economies of scale on connections** by offering a high frequency of services.
- •**Economies of scale at the hubs,** enabling the potential development of an efficient distribution system since the hubs handle larger quantities of traffic.
- •**Economies of scope in the use of shared transshipment facilities.**

Disadvantages of Hub-and-spoke System

- Longer transportation time, transferring at hub airport
- More traffic pressure on hubs: congestion, delay
- Reliability issue: if one hub malfunctions, all connections to it will be discontinued



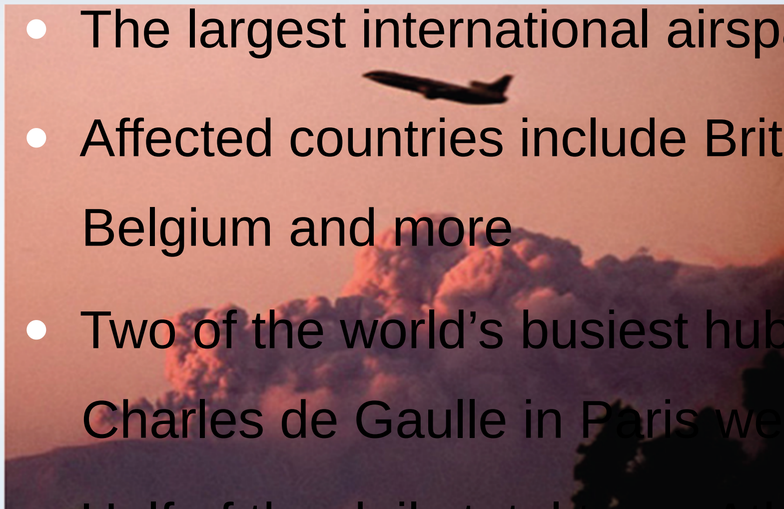
Statistics of Disruption Cost in Airline Industry

- In 2000, about 30% of the jet-operated flight legs of one major US airline were delayed, and about 3.5% of these flight legs were canceled(Ball 2007).

More than $440 million per year (Clarke and Smith (2000)) for major US domestic carrier.

Various delays cost consumers and airlines about $6.5 billion in 2000 (Air Transport Association).

Recent Story on Icelandic Volcano Eruption



- The largest international airspace shutdown in years
- Affected countries include Britain, France, Germany, Ireland, Belgium and more
- Two of the world's busiest hubs, Heathrow in London and Charles de Gaulle in Paris were closed
- Half of the daily total trans-Atlantic flights were canceled by U.S. and European carriers
- Overall 10 million passengers were affected, accumulated losses should be more than $1.7 billions

Extensive demand on alternative routes

Challenges to Airline Industry

- A volatile environment with emerging airline consolidation for avoiding financial straits
- Airspace capacity uncertainty caused by increasing traffic demand and more frequent extreme weather conditions
- Augmented international competition

We propose a reliable Hub-spoke network model that will:

- select hub nodes and explicitly include hub unavailability
- design back up plans in addition to primary routes

Part II
Literature Review

Early Studies on Hub-and-spoke Network

The hub-and-spoke system existed way back in the 1950's
Fedex? American Airline? Delta?

First Quantitative Model: O'Kelly(1986,87)
1.Proposition of Hub location problem(HLP).

- *Mathematical formulation to design the hub-and-spoke network*

2.Introduction of a data set.

- *Civil Aeronautics Board (CAB) data set (25 nodes):*
  - *Passenger flow between each node pair i and j(wij)*
  - *Unit transportation cost between each node pair i and j (cij)*

Hub Location Problem(HLP)

Minimization problem

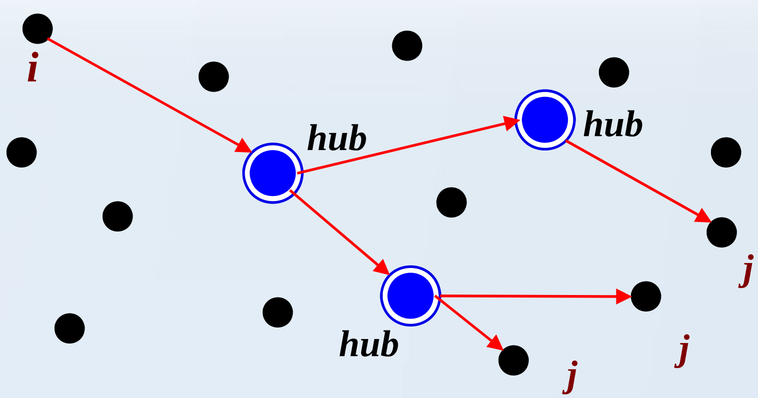
Objective function:
Total transportation cost

Decision variables:
1.Hub location variables
2.Spoke allocation variables

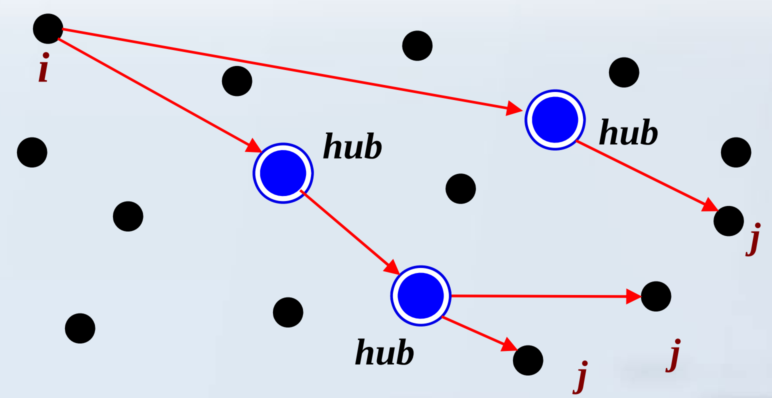
Constraints:
The traffic flow between each node pair has to be assigned to one or two hubs

Classification of HLPs

- SA (single allocation): all the flows from a single spoke go to the same hub in their routes;
- MA (multiple allocation): flows from a single spoke go to different hubs in their routes.



SA network



MA network

Several Existing Work on Reliable Network Designs

- Snyder and Daskin (2005): “Reliability models for facility location: The expected failure cost case”, uncapacitated facility location models with identical failure probability

- Cui, Ouyang and Shen (2010): “Reliable Facility Location Design under the Risk of Disruptions”, an uncapacitated facility location model with site-dependent failure probabilities

- Kim and O’Kelly (2009) : “Reliable p-Hub Location Problems in Telecommunication Networks”, only arc failure probabilities are considered, max expected flow, no backup routes, no algorithm development

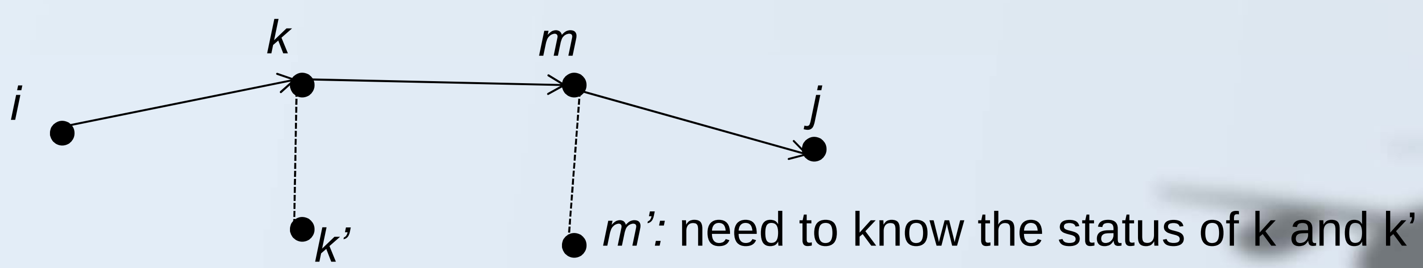
- Devari et al. (2010): a fuzzy variant of the model in Kim and O’Kelly (2009)

Part III
Formulation

Assumptions

Assume that:

- Each route has at most two hubs and hubs are uncapacitated.
- The failure of hub airports are independent of each other.



Notation and Decision Variables

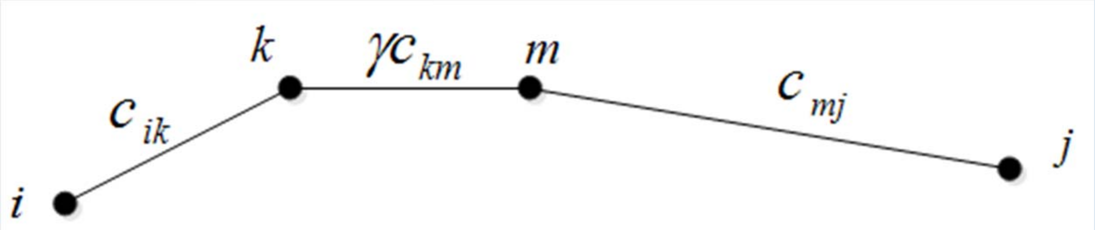
Let N: set of nodes

H: set of potential hubs, H=N

w_{ij}: the amount of traffic flow between i and j

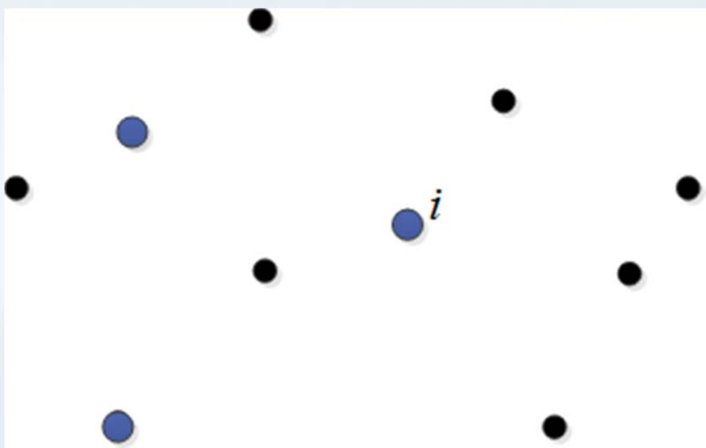
q_k: the failure probability of hub k.

c_{ij}: the cost of transportation per unit of traffic flow between i and j

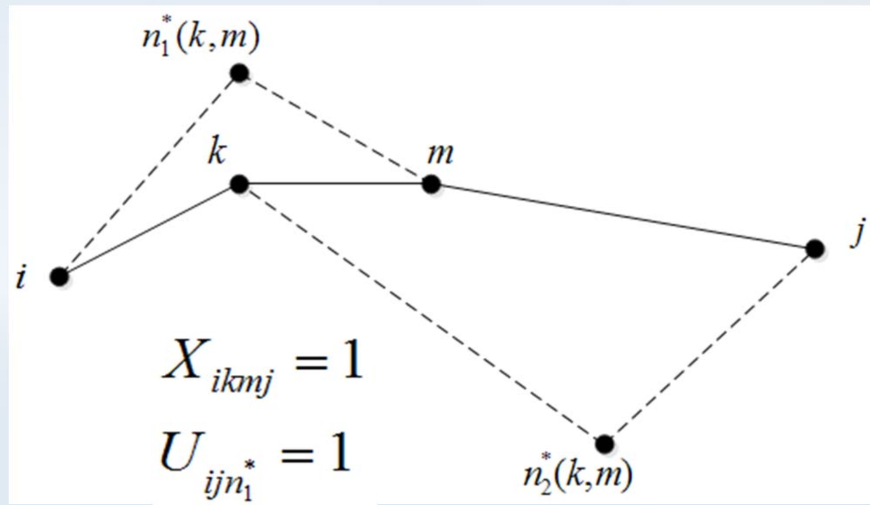


for the route i-k-m-j: $F_{ikmj} = c_{ik} + \gamma\, c_{km} + c_{mj}$

Notation and Decision Variables



$$Y_i = 1$$
$$Y_i \in \{0,1\}$$



$$X_{ikmj} = 1$$
$$U_{ijn_1^*} = 1$$
$$V_{ijn_2^*} = 1$$

- Hub location Variables

- Spoke allocation Variables
- **Variables indicating backup hubs of each route**

Formulation

Objective:

Total transportation cost

Variables:

1. Hub location Variables
2. Spoke allocation Variables
3. **Variables indicating backup hubs of each route**

Constraints:

1. The flow between any node pair i and j has to be routed through one or two hubs
2. Number of hubs is *p*
3. **Regular hub and the backup hub for each route have to be different**

Normal operation cost(no hub is down)

$$\min \sum_{i}\sum_{k\neq i}\sum_{m}\sum_{\substack{j>i\\ j\neq m}} F_{ikmj}w_{ij}(1-q_k-q_m^k)X_{ikmj}$$

Expected Failure cost

$$+\sum_{i}\sum_{j>i}\Big(\sum_{m\neq j} F_{iimj}w_{ij}(1-q_m^i)X_{iimj}+\sum_{k\neq i} F_{ikjj}w_{ij}(1-q_k^j)X_{ikjj}+F_{iijj}w_{ij}X_{iijj}\Big)$$
$$+\rho\Big(\sum_{i}\sum_{k}\sum_{m\neq k}\sum_{j>i}\sum_{n}(F_{inmj}w_{ij}q_kX_{ikmj}U_{ijn}+F_{iknj}w_{ij}q_mX_{ikmj}V_{ijn})$$
$$+\sum_{i}\sum_{k}\sum_{j>i}\sum_{n} F_{innj}w_{ij}q_kX_{ikkj}U_{ijn}\Big) \qquad (1)$$

subject to

$$\sum_{m} X_{ikmj}=Y_{ik} \qquad \forall i, j>i, k \qquad (2)$$
$$\sum_{k} X_{ikmj}=Y_{jm} \qquad \forall i, j>i, m \qquad (3)$$
$$\sum_{k} Y_{ik}=1 \qquad \forall i \qquad (4)$$
$$\sum_{k} Y_{kk}=p \qquad (5)$$
$$U_{ijk}+\sum_{m} X_{ikmj}\leq Y_{kk} \qquad \forall i, j>i, k \qquad (6)$$
$$\sum_{k} U_{ijk}=1-\sum_{m} X_{iimj}-\sum_{m} X_{ijmj} \qquad \forall i, j>i \qquad (7)$$
$$V_{ijm}+\sum_{k} X_{ikmj}\leq Y_{mm} \qquad \forall i, j>i, m \qquad (8)$$
$$\sum_{m} V_{ijm}=1-\sum_{k} X_{ikjj}-\sum_{k} X_{ikij} \qquad \forall i, j>i \qquad (9)$$
$$X_{ikmj}\in\{0,1\}\ \forall i, j>i, k, m;\ Y_{ik}\in\{0,1\}\ \forall i, k;\ U_{ijk}, V_{ijk}\in\{0,1\}\ \forall i, j>i, k \qquad (10)$$

Part IV
Solution Method

Cplex Performance

(Standard Linearization)

Table 2: Solver performance for *R-SAHMP* and *R-MAHMP*

|N|	p	α	*R-SAHMP* *StdLinear* Time(s)	*R-SAHMP* *StdLinear* Gap(%)	*R-SAHMP* *CptLinear* Time(s)	*R-SAHMP* *CptLinear* Gap(%)	*R-MAHMP* *StdLinear* Time(s)	*R-MAHMP* *StdLinear* Gap(%)	*R-MAHMP* *CptLinear* Time(s)	*R-MAHMP* *CptLinear* Gap(%)
10	3	0.3	33.7	0.032	T	0.514	641.1	0.100	1456.7	0.100
	5	0.3	24.5	0.047	T	1.827	3516.3	0.100	T	1.896
	7	0.3	5.2	0.000	2.2	0.000	138.4	0.100	4.4	0.100
	3	0.5	40.4	0.000	T	2.069	343.5	0.099	T	0.321
	5	0.5	35.3	0.000	T	3.414	T	0.164	2041.3	0.000
	7	0.5	7.1	0.006	4.5	0.094	520.9	0.100	76.5	0.100
	3	0.7	50.1	0.000	T	2.007	407.6	0.100	1335.4	0.100
	5	0.7	39.2	0.010	T	1.660	M	0.760	T	1.951
	7	0.7	7.6	0.000	19.7	0.099	M	0.330	M	0.740
15	3	0.3	M	NA	T	4.030	M	16.360	T	4.440
	5	0.3	M	NA	M	5.070	M	14.480	T	5.441
	7	0.3	M	NA	T	4.789	M	18.660	T	3.669
	3	0.5	M	NA	T	3.729	M	11.650	T	4.531
	5	0.5	M	NA	M	5.340	M	14.960	T	4.620
	7	0.5	M	NA	T	4.020	M	13.560	T	3.117
	3	0.7	M	NA	T	4.907	M	10.110	T	3.949
	5	0.7	M	NA	M	4.560	M	9.770	T	3.723
	7	0.7	M	NA	T	3.480	M	9.600	T	2.662

- Nonlinear mixed integer programming problem

- Commercial solver (Cplex) is unable to solve the model in most cases.

- Solution strategy:

  Lagrangian relaxation + variable fixing + branch-and-bound

Solution Method (Lagrangian Relaxation)

$$
\begin{aligned}
&f(\delta_1, \delta_2, \beta, \gamma_1, \gamma_2) = \\
&\min \sum_i \sum_k \bar{C}_{ik} Y_{ik} - \sum_i \beta_i \\
&+ \sum_i \sum_{k \neq i} \sum_m \sum_{\substack{j>i \\ j \neq m}} (F_{ikmj} w_{ij} (1 - q_k - q_m^k) + \delta_{ijk,1} + \gamma_{ijk,1} + \delta_{ijm,2} + \gamma_{ijm,2}) X_{ikmj} \\
&+ \sum_i \sum_{j>i} \sum_{m \neq j} (F_{iimj} w_{ij} (1 - q_m^i) + \delta_{iji,1} + \gamma_{iji,1} + \delta_{ijm,2} + \gamma_{ijm,2}) X_{iimj} \\
&+ \sum_i \sum_{j>i} \sum_{k \neq i} (F_{ikjj} w_{ij} (1 - q_k^j) + \delta_{ijk,1} + \gamma_{ijk,1} + \delta_{ijj,2} + \gamma_{ijj,2}) X_{ikjj} \\
&+ \sum_i \sum_{j>i} (F_{iijj} w_{ij} + \delta_{iji,1} + \gamma_{iji,1} + \delta_{ijj,2} + \gamma_{ijj,2}) X_{iijj} \\
&+ \sum_i \sum_k \sum_{m \neq k} \sum_{j>i} \sum_n \rho F_{inmj} w_{ij} q_k X_{ikmj} U_{ijn} + \sum_i \sum_{j>i} \sum_k \gamma_{ijk,1} U_{ijk} \\
&+ \sum_i \sum_k \sum_{m \neq k} \sum_{j>i} \sum_n \rho F_{iknj} w_{ij} q_m X_{ikmj} V_{ijn} + \sum_i \sum_{j>i} \sum_m \gamma_{ijm,2} V_{ijm} \\
&+ \sum_i \sum_k \sum_{j>i} \sum_n \rho F_{innj} w_{ij} q_k X_{ikkj} U_{ijn} \qquad (11)
\end{aligned}
$$

Relax the difficult constraints so that the original problems can be divided into two sub-problems Sub-1 and Sub-2 that are easier to solve.

Sub-1

$$\min \left\{\sum_i \sum_k \bar{C}_{ik} Y_{ik} - \sum_i \beta_i : \sum_k Y_{kk} = p,\ \ Y_{ik} \le Y_{kk}\ \forall i,k,\ \ Y_{ik} \in \{0,1\}\ \forall i,k.\right\}$$

$$\bar{C}_{ik} = \begin{cases} \beta_i - \sum_{j>i} \delta_{ijk,1} - \sum_{j>i} \delta_{jik,2}, & \text{if } i \neq k; \\ \beta_k - \sum_{i>k} \delta_{kik,1} - \sum_{i<k} \delta_{ikk,2} - \sum_i \sum_{j>i} (\gamma_{ijk,1} + \gamma_{ijk,2}), & \text{otherwise.} \end{cases}$$

Step 1: For each k, set $Y_{kk} = 1$. For i, k $(i \neq k)$, set $Y_{ik} = 1$ if $\bar{C}_{ik} < 0$ and $Y_{ik} = 0$ otherwise. Compute $S_k = \sum_i \bar{C}_{ik} Y_{ik}$, for each k.

Step 2: Sort S_k's in ascending order, choose p of the nodes with smaller S_k, and set the corresponding $Y_{kk} = 1$ and set the remaining Y_{kk}'s to 0. Calculate the optimal value of **SAsub-1** by $\sum_k S_k Y_{kk} - \sum_i \beta_i$.

Step 3: For i, k $(i \neq k)$, set Y_{ik} to 0 if $Y_{kk} = 0$.

Sub-2

$$
\begin{aligned}
\min \sum_{i}\sum_{k\neq i}\sum_{m}\sum_{\substack{j>i\\ j\neq m}} & (F_{ikmj}w_{ij}(1-q_k-q_m^k)+\delta_{ijk,1}+\gamma_{ijk,1}+\delta_{ijm,2}+\gamma_{ijm,2})X_{ikmj} \\
& +\sum_{i}\sum_{j>i}\sum_{m\neq j}(F_{iimj}w_{ij}(1-q_m^i)+\delta_{iji,1}+\gamma_{iji,1}+\delta_{ijm,2}+\gamma_{ijm,2})X_{iimj} \\
& +\sum_{i}\sum_{j>i}\sum_{k\neq i}(F_{ikjj}w_{ij}(1-q_k^j)+\delta_{ijk,1}+\gamma_{ijk,1}+\delta_{ijj,2}+\gamma_{ijj,2})X_{ikjj} \\
& +\sum_{i}\sum_{j>i}(F_{iijj}w_{ij}+\delta_{iji,1}+\gamma_{iji,1}+\delta_{ijj,2}+\gamma_{ijj,2})X_{iijj} \\
& +\sum_{i}\sum_{k}\sum_{m\neq k}\sum_{j>i}\sum_{n}\rho F_{inmj}w_{ij}q_k X_{ikmj}U_{ijn}+\sum_{i}\sum_{j>i}\sum_{k}\gamma_{ijk,1}U_{ijk} \\
& +\sum_{i}\sum_{k}\sum_{m\neq k}\sum_{j>i}\sum_{n}\rho F_{iknj}w_{ij}q_m X_{ikmj}V_{ijn}+\sum_{i}\sum_{j>i}\sum_{m}\gamma_{ijm,2}V_{ijm} \\
& +\sum_{i}\sum_{k}\sum_{j>i}\sum_{n}\rho F_{innj}w_{ij}q_k X_{ikkj}U_{ijn} \qquad (13)
\end{aligned}
$$

subject to

$$(7), (9)$$

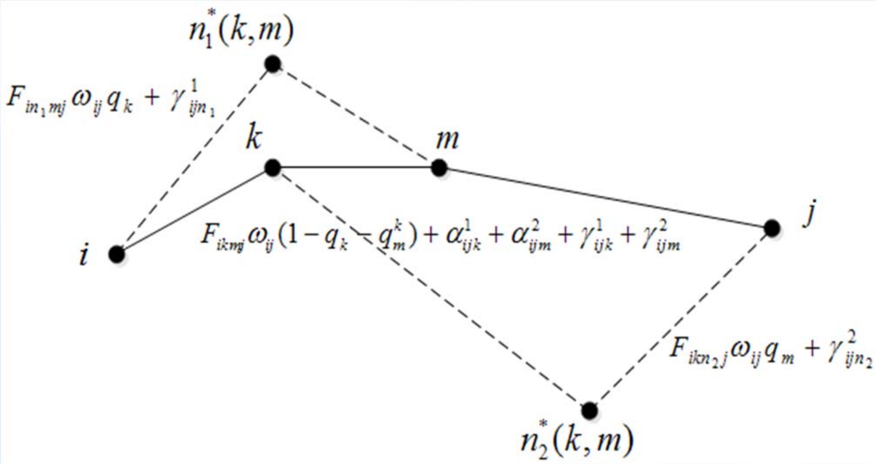
$$\sum_{k}\sum_{m} X_{ikmj} = 1 \qquad \forall i, j>i \qquad (14)$$

$$U_{ijk} + \sum_{m} X_{ikmj} \leq 1 \qquad \forall i, j>i, k \qquad (15)$$

$$V_{ijm} + \sum_{k} X_{ikmj} \leq 1 \qquad \forall i, j>i, m \qquad (16)$$

$$X_{ikmj} \in \{0,1\} \; \forall i, j>i, k, m; U_{ijk}, V_{ijk} \in \{0,1\} \; \forall i, j>i, k \qquad (17)$$

For each (i,j), enumerate (k,m)



Choose(k*,m*)

Variable Fixing

- Variable fixing is an approach that uses both primal information from a feasible solution and dual information from Lagrangian multipliers to fix some variables in Lagrangian solution procedure.

Proposition 1. *When UB is strictly greater than LB,*
(i) if $Y^*_{kk} = 1$ *and* $f(\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \boldsymbol{\beta}, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2 | Y_{kk} = 0) > UB$ *for some* k*, we have* $Y_{kk} = 1$ *in any optimal solution;*
(ii) if $Y^*_{kk} = 0$ *and* $f(\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \boldsymbol{\beta}, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2 | Y_{kk} = 1) > UB$ *for some* k*, we have* $Y_{kk} = 0$ *in any optimal solution.*

Proof. We provide the proof for (i). Results in (ii) can be proven using similar arguments.

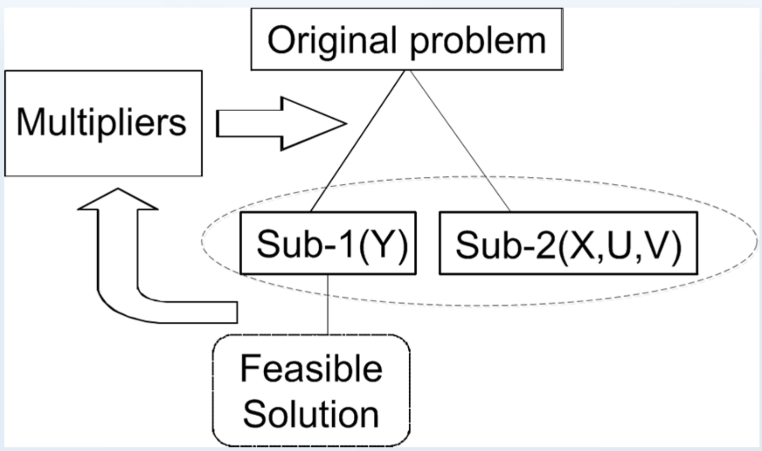
Note that $f(\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \boldsymbol{\beta}, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2 | Y_{kk} = 0)$ is a lower bound to *R-SAHMP* with a spoke node located in k for the given Lagrangian multipliers $(\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \boldsymbol{\beta}, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2)$. So, if

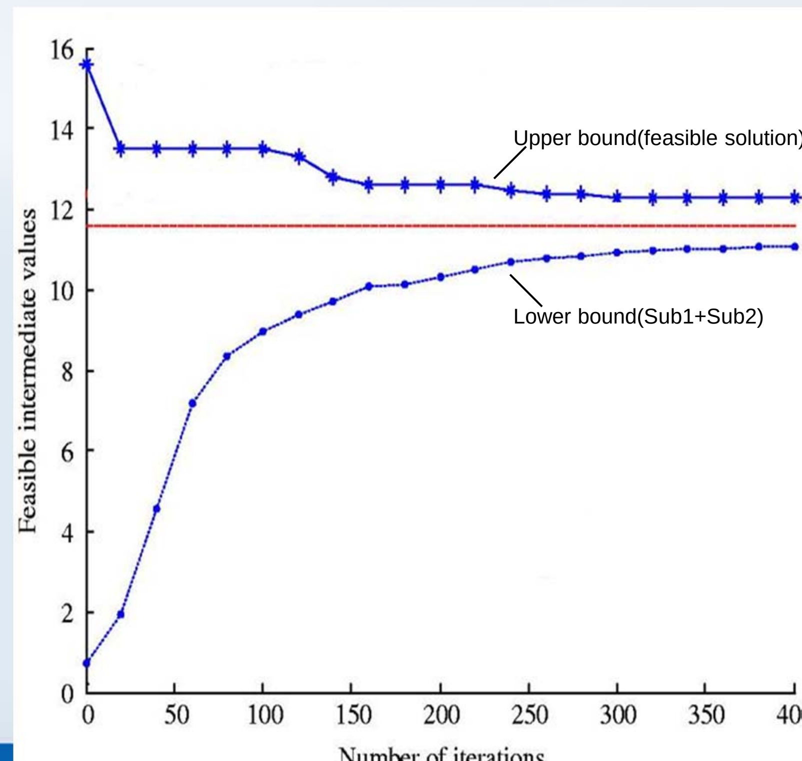
$$f(\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \boldsymbol{\beta}, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2 | Y_{kk} = 0) > UB,$$

any solution to *R-SAHMP* with a spoke node in k will generate more cost than the current best feasible solution. Therefore, we have $Y_{kk} = 1$ in any optimal solution to *R-SAHMP*. $\square$

Solution Method

- A feasible solution (that gives an upper bound of the optimal solution) can be derived from the solutions to Sub-1 and Sub-2.
- The solution information in turn can be used to update the multipliers such that the gap between upper and lower bounds decreases over iterations.



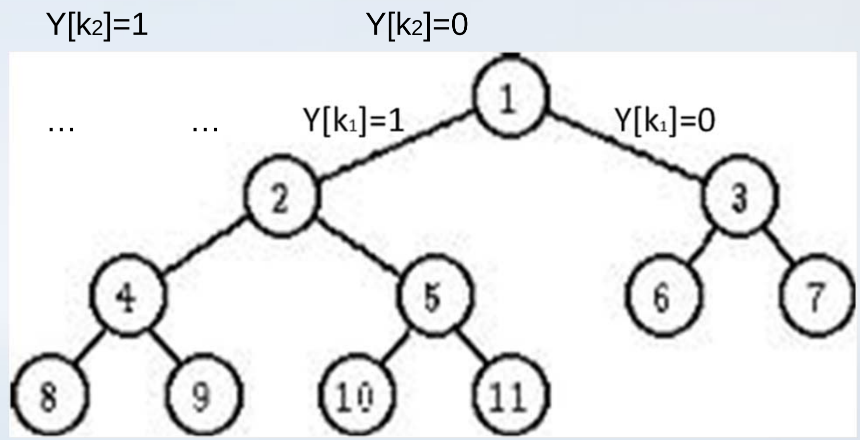


Branch-and-bound

- When the gap is small enough(<1%) we claim that the feasible solution now is the required optimal solution.

- If the gap is still large(>=1%) after 3000 iterations, branch and bound will be applied to close the gap.

- The breadth-first search algorithm can guarantee that the lower bounds will increase in the child nodes.



Part V
Case Study and Conclusions

Solution Algorithm Performance

Table 1: LR and Branch-and-Bound performance for *R-SAHMP* and *R-MAHMP*

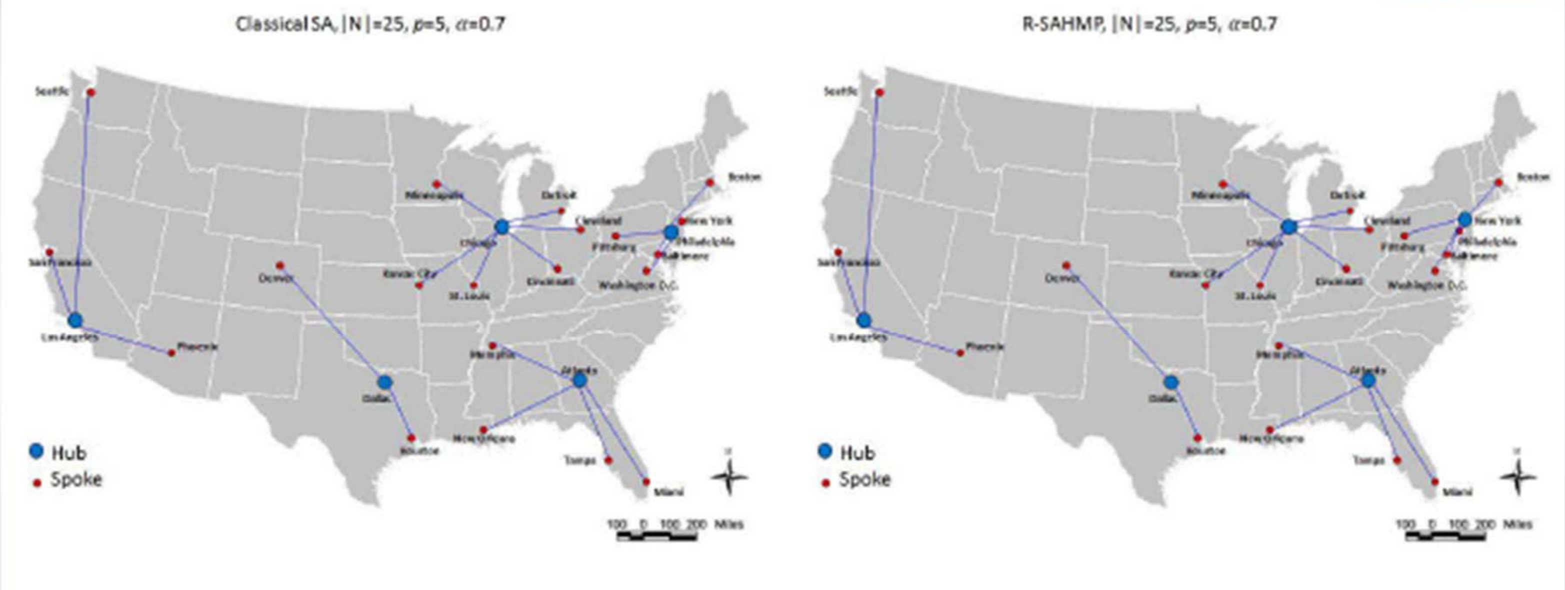
$\lvert\mathbf{N}\rvert$	p	α	*R-SAHMP* *Iter.*	*BB_Nodes*	*Gap(%)*	*Time(s)*	*R-MAHMP* *Iter.*	*BB_Nodes*	*Gap(%)*	*Time(s)*
10	3	0.3	250	0	0.099	1.3	878	2	0.100	2.6
	5	0.3	565	0	0.100	3.7	1057	0	0.090	3.5
	7	0.3	184	0	0.098	1.8	604	0	0.100	1.4
	3	0.5	257	0	0.098	4.6	830	2	0.000	2.6
	5	0.5	1902	6	0.095	17.6	866	0	0.097	2.3
	7	0.5	184	0	0.099	1.7	587	0	0.099	1.5
	3	0.7	182	2	0.099	1.5	607	2	0.070	2.7
	5	0.7	1515	4	0.096	5.8	731	0	0.098	1.8
	7	0.7	323	0	0.098	2.7	561	0	0.095	2.8
15	3	0.3	1015	2	0.016	15.0	1455	4	0.000	20.0
	5	0.3	1353	4	0.099	27.3	596	0	0.097	4.3
	7	0.3	1722	6	0.099	30.4	716	0	0.100	8.4
	3	0.5	1362	4	0.095	24.4	910	4	0.100	18.4
	5	0.5	1701	6	0.080	31.3	563	0	0.096	2.3
	7	0.5	1313	6	0.090	21.7	635	2	0.100	15.7
	3	0.7	980	2	0.099	21.6	1958	8	0.098	27.6
	5	0.7	1540	4	0.099	31.4	573	0	0.092	3.4
	7	0.7	512	0	0.099	13.7	564	2	0.099	12.7
20	3	0.3	482	0	0.098	32.3	1979	6	0.000	92.0
	5	0.3	553	0	0.099	37.6	608	2	0.000	36.1
	7	0.3	118	0	0.100	8.1	581	0	0.100	33.2
	3	0.5	1762	6	0.098	116.1	1441	6	0.000	68.1
	5	0.5	1584	4	0.099	107.8	605	0	0.100	26.6
	7	0.5	589	0	0.099	63.7	971	4	0.100	77.8
	3	0.7	3925	16	0.097	177.6	1660	8	0.044	82.8
	5	0.7	3871	14	0.099	188.2	722	0	0.100	38.4
	7	0.7	2095	8	0.097	138.8	561	0	0.100	28.9
25	3	0.3	2020	6	0.098	365.3	2845	8	0.000	338.0
	5	0.3	965	2	0.100	221.3	1709	6	0.100	268.8
	7	0.3	812	2	0.100	239.2	1601	6	0.000	257.5
	3	0.5	1745	6	0.097	361.2	2914	10	0.092	375.2
	5	0.5	2774	10	0.099	435.5	727	0	0.100	97.7
	7	0.5	201	0	0.082	33.0	1587	8	0.100	323.4
	3	0.7	765	4	0.076	121.7	3313	12	0.000	416.1
	5	0.7	7318	34	0.096	953.8	3126	12	0.099	457.8
	7	0.7	879	2	0.100	249.2	613	0	0.100	31.2

- N: number of nodes
- p: number of hubs
- Alpha: discount factor of inter-hub links

Summary of Algorithm Performance

- The Lagrangian relaxation and Branch-and-bound method is applied to solve SA and MA models.
- All the 144 cases can be solved to optimality within 1800s.
- When failure probability ***q*** is high, the cases are more difficult to solve

Impact of Hub Unavailability on System Design



(a) Configuration from classical model (b) Configuration from reliable model

- For most cases, the optimal hub locations and spoke node allocations of the classical and reliable models are different.
- The network configuration of the reliable model is more robust to random hub failures and can transport more passengers.

Performance of Reliable Hub-and-spoke Networks

Table 3: Comparison of the served passengers

		SA model			MA model		
$\|\mathbf{N}\|$	p	Classical Psg_c	Psg_r	Reliable $Improvement(\%)$	Classical Psg_c	Psg_r	Reliable $Improvement(\%)$
10	3	484653	499513	3.066	490297	499513	1.845
	5	487181	499513	2.531	494180	499513	1.068
	7	494730	499513	0.967	495343	499513	0.835
15	3	1155060	1182470	2.373	1162180	1182470	1.716
	5	1149840	1182470	2.838	1164140	1182470	1.550
	7	1154940	1182470	2.384	1169760	1182470	1.075
20	3	2781810	2877300	3.433	2820550	2877300	1.972
	5	2801900	2877300	2.691	2832790	2877300	1.547
	7	2803800	2877300	2.621	2845150	2877300	1.117
25	3	4135680	4270000	3.248	4163530	4270000	2.493
	5	4126900	4270000	3.467	4166670	4270000	2.420
	7	4133240	4270000	3.309	4210840	4270000	1.385

Verification with Multiple Simultaneous Disruptions

Table 5: Performance of reliable models under the multiple disruption assumption

$\lvert\mathbf{N}\rvert$	p	SA model Classical Psg'_c	SA model Psg'_r	SA model Reliable $Improvement(\%)$	MA model Classical Psg'_c	MA model Psg'_r	MA model Reliable $Improvement(\%)$
	3	484675	499122	2.981	490321	499136	1.798
10	5	487189	499188	2.463	494184	499272	1.030
	7	494733	499328	0.929	495343	499368	0.813
	3	1155100	1181860	2.317	1162190	1181790	1.686
15	5	1150030	1181610	2.746	1164190	1181600	1.495
	7	1155030	1181680	2.307	1169770	1181970	1.043
	3	2782650	2873540	3.266	2820660	2875360	1.939
20	5	2802410	2874540	2.574	2832880	2875490	1.504
	7	2804310	2874000	2.485	2845210	2876180	1.088
	3	4136760	4266010	3.124	4163920	4264990	2.427
25	5	4128160	4264950	3.314	4167180	4266400	2.381
	7	4134490	4267140	3.208	4211000	4268170	1.358

Sensitivity Analysis of Failure Rate

Table 6: Sensitivity analysis of failure rates under multiple disruptions

Model	p	q	Classical Psg'_c	Classical $Change(\%)$	Reliable Psg'_r	Reliable $Change(\%)$
SA	3	0.009	4226430	-0.114	4269660	-0.002
		0.04	4077820	-0.116	4264970	-0.006
	5	0.009	4226060	-0.115	4269710	-0.002
		0.04	4076470	-0.117	4264650	-0.006
	7	0.009	4239930	-0.079	4269690	-0.002
		0.04	4137320	-0.079	4264240	-0.007
MA	3	0.009	4248810	-0.055	4269810	-0.001
		0.04	4176050	-0.056	4266110	-0.005
	5	0.009	4246670	-0.061	4269800	-0.001
		0.04	4166690	-0.062	4266180	-0.004
	7	0.009	4249770	-0.053	4269780	-0.001
		0.04	4180320	-0.054	4266120	-0.004

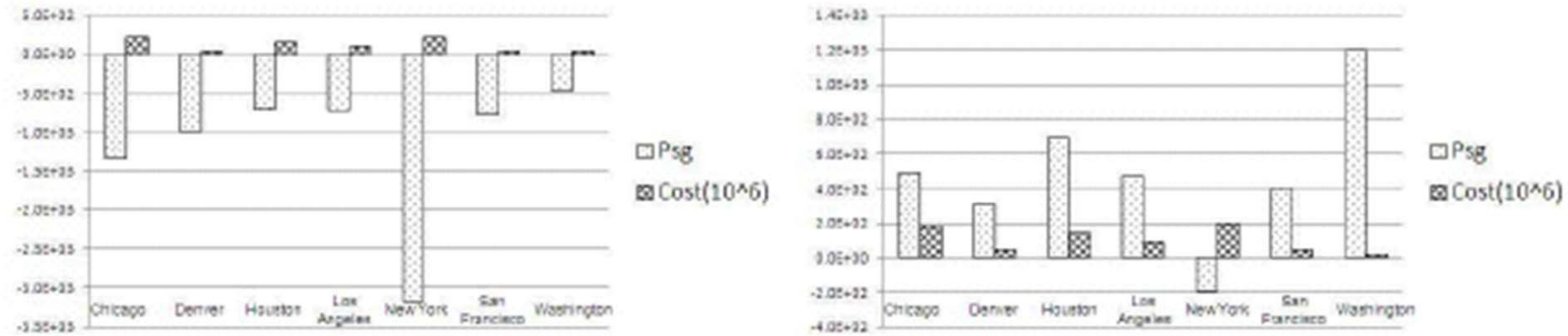
Application of Proposed Reliable Models to a Recent Airlines Merger

Disruption Probabilities of CAB Airports

Table A2: Disruption probabilities of potential hubs in reliable model

No.	City	q value	No.	City	q value	No.	City	q value
0	Atlanta	0.023	9	Houston	0.026	18	Phoenix	0.045
1	Baltimore	0.017	10	Kansas City	0.018	19	Pittsburgh	0.012
2	Boston	0.047	11	Los Angeles	0.049	20	St. Louis	0.035
3	Chicago	0.041	12	Memphis	0.024	21	San Francisco	0.043
4	Cincinnati	0.026	13	Miami	0.027	22	Seattle	0.020
5	Cleveland	0.047	14	Minneapolis	0.013	23	Tampa	0.036
6	Dallas-Fort Worth	0.012	15	New Orleans	0.019	24	Washington DC	0.050
7	Denver	0.015	16	New York	0.050			
8	Detroit	0.035	17	Philadelphia	0.024			

Relative Changes in the Expected Number of Passengers and Transportation Cost



(a) Results for $\mathbf{q}$ in Table A2 (b) Results for $\mathbf{q}$ in Table A2 with $q_5 = 0.025$

Conclusions

- A novel reliable Hub-and-spoke network design is proposed and is shown to be able to greatly improve the performance of the network system.
- Lagrangian relaxation method with variable fixing and Branch-and-bound technique are applied to solve the large scale optimization problem. Computational study demonstrates the effectiveness of these algorithms, as well as the superiority of the proposed models to classical models in terms of serving passengers and being robust subject to the variations of hub failure rates.
- It theoretically extends the existing literature on reliable network design and also has a clear practical impact on transportation and telecommunications systems.

Future Research

- Congestion effect (research-in-progress)
- Multiple simultaneous disruptions

Thanks

Yu Zhang, Ph.D.

Assistant Professor

Department of Civil and Environmental Engineering (CEE)

University of South Florida (USF)

Address: 4202 E. Fowler Ave. ENB 118| Tampa, FL 33620

Tel: 813-974-5846 | Fax: 813-974-2957

Office: ENC 3211

Email: yuzhang@usf.edu

URL: http://cee.eng.usf.edu/faculty/YuZhang