Generating timetables with partial periodicity

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RailZurich, 12. February 2009

Joint work with M.Laumanns, K.Schüpbach, S.Wörner, M.Fuchsberger
### Operated timetable in Switzerland

<table>
<thead>
<tr>
<th>Destination</th>
<th>Time</th>
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<tbody>
<tr>
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<td>17:41</td>
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<td>xx:25</td>
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<tr>
<td>Time</td>
<td>0:45</td>
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<tr>
<td></td>
<td>0:50</td>
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<tr>
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<td>0:58</td>
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<table>
<thead>
<tr>
<th>Wil</th>
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<tbody>
<tr>
<td></td>
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<td>St. Gallen</td>
<td>Time</td>
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<td></td>
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<td></td>
<td>23:17</td>
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**Stops:**
- Uzwil, Flawil, Gossau
- Uzwil, Flawil, Gossau
- Uzwil, Flawil, Gossau
- Uzwil, Flawil, Gossau
- Uzwil, Flawil, Gossau

**Time:**
- 0:28
- 0:23
- 0:28
- 0:21
- 0:28
- 0:21
- 0:23
## Operated timetable abroad

<table>
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<tr>
<th>City</th>
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<th>07:45</th>
<th>08:15</th>
<th>08:30</th>
<th>08:45</th>
<th>09:30</th>
<th>10:30</th>
<th>11:30</th>
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<tbody>
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<td>08:15</td>
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<tr>
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<td>03:30</td>
<td>03:30</td>
<td>03:59</td>
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<td>03:59</td>
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<table>
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<th>15:15</th>
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<th>16:10</th>
<th>17:20</th>
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<tbody>
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<td>12:10</td>
<td>14:10</td>
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<td>02:58</td>
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<table>
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<th>14:40</th>
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<th>16:42</th>
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<th>18:40</th>
<th>-:-</th>
</tr>
</thead>
<tbody>
<tr>
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<td>14:07</td>
<td>14:40</td>
<td>16:07</td>
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</tr>
<tr>
<td>Time</td>
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<td>02:09</td>
<td>02:45</td>
<td>02:09</td>
<td>02:43</td>
<td>02:09</td>
<td>02:45</td>
<td>-:-</td>
</tr>
</tbody>
</table>

### Standard offer
Arising questions

- How should we consider the presented timetables? Periodic or non-periodic?
- Why were these timetables so generated?
- How are they generated?
Remarks on periodicity

- Periodicity is important for the passengers
  - It should be part of the offer
- Irregularities are necessary to face changing demand over the day
  - Additional services in peak hours
  - Different demand in the evening
- Currently:
  - Manually, or
  - Manual postprocessing of automatic periodic timetabling
Current approaches

- **Periodic timetabling**
  - Good for regularity
  - Needs postprocessing for irregularities
  - Optimises only a part of the day

- **Non-periodic timetabling**
  - Good for irregularities
  - Loses offer of periodicity
  - Larger size

- **New approach: Partial periodic timetabling**
Partial periodic timetabling

1. Consider service intention for a whole day, with periodicity and exceptions as part of the offer
2. Formalise it in the partial periodic service intention
3. Generates partial periodic timetables

- Advantages:
  - No need of postprocessing
  - Allows optimisation all over the day
Partial periodic service intention

- Description of intended transport services for one day
- Set of services:
  - Train runs, connections, time dependencies
- Reference periodicity T
- Spatial-temporal graph
Train run

- Sequence of stations with:
  - Time slot for arrival / departure (at least one)
  - Lower/upper bounds for:
    - Trip time
    - Dwell time ( = 0 if train does not stop)
  - Periodicity
  - First recurrence
  - Number of recurrences

- Similar for connections and time dependencies

Service Intention
- Train run
- Connection
- Time dependency
Example
Solution approach: basic idea

Generating timetables with partial periodicity

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Projection: example
Model for projected problem

- Projected problem is modeled as a Periodic Event Scheduling Problem (PESP)
- Decision variables are event times (departure and arrival) of projected equivalence classes
- Train service constraints are easily modeled in PESP
- Headway constraints are different than classical PESP
Introduction (or not) of headway constraints

- If projection does not need headway
  → do not introduce **headway** constraint

- If projection needs headway
  → **check** original train service intention
Introduction (or not) of headway constraints

- Headway necessary in the original version?

No → No headway

Yes → headway

Once yes and once no → special situation (*)
Equivalence of the problem

- If (situation (*) does not occur) and (all time slots have size < T), then:

  Original Problem is equivalent to Projected Problem

- i.e. Solution Spaces are equivalent
Test scenario

- Central Switzerland: Zug – Lucerne – ArthGoldau
- Reverse-Engineering from 2008 SBB-Schedule
- Trains: intercity, local, cargo
- Compare with fully periodic variant without peak hours or late evening exceptions
## Computational results

<table>
<thead>
<tr>
<th>Scenario</th>
<th>T</th>
<th># variables</th>
<th># integer</th>
<th># constraints</th>
<th>CPU time [s]</th>
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<td>963</td>
<td>3449</td>
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<td>No</td>
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<td>16 920</td>
<td>70</td>
</tr>
</tbody>
</table>

# Situation (*) occurred, resulting in infeasible problem

In all other tests situation (*) did not occur
Conclusions

- Formalise partial periodicity, which is most common situation in practice
- Projection method enables the use of established methods for periodic timetabling
- The stronger the periodicity, the larger the size reduction
- Optimises over whole day, no need for post-processing
Thank You!

Time for questions!
Problem definition

**INPUT**
- Train service intention (incl. periodicity properties)
- Railway network
- Dynamic properties of rolling stock

**OUTPUT**
- Conflict-free train schedule
- Fulfilling service intention
Connection

- Connects 2 train runs at a common station
- **Minimum** changing time from station layout
- **Maximal** changing time from service intention
- Periodicity
  - First recurrence
  - Number of recurrences
Time dependency

- Between two 2 train runs
- Lower and upper bound for departure time difference
- e.g. to enhance the service during peak hours, or coordinate two different train runs on same (sub-)line
Solution approach: basic idea

1. Project all train runs on the periodic time \([0,T]\)
   - Create equivalence classes of train runs
2. Apply existing solvers for periodic scheduling
3. Roll out the created timetable on the complete day
   - Reduces problem size
Train service intention

- List of train services offered to the customers, including:
  - Train lines with stop and frequencies
  - Interconnection possibilities
  - Rolling stock

- TSI can be generated by planners manually or partially automatic (e.g. line planning)