Timetabling with TS-OPT

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joint work with
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Konrad Zuse was the creator of the first fully automatic, programm controlled and freely programmable computer working in binary floating point arithmetic. The Z3 was finished in 1941.
Overview

1. Idea & Motivation
2. Train Timetabling Problem
3. Algorithmic Approach
4. TTPLib 2008
Planning in Public Transport

Strategic Stage:
- Tracks
- Lines/Freq.
- Timetables

Tactical Stage:
- Stops
- Prices
- Connections

Operational Stage:
- Vehicles
- Rotations
- Duties
- Crews

resource acquisition

resource allocation
Timetabling (PESP & TTP & TPP)

periodic passenger versus individual cargo traffic

Train Requests
Tracks
Stations

Optimization Model

maximize

track utilization
timetable attractiveness

subject to

safety requirements
time windows

TS-OPT

Timetable
Overview

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3. Column Generation Approach
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Railway Timetabling (TTP) - State of the Art

- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Semet and Schoenauer (2005),
- Caprara, Monaci, Toth and Guida (2005)
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006),
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- Borndoerfer, Schlechte (2007)
- Fischer, Helmberg, Janßen, Krostitz (2008) ...

non-cyclic timetabling literature
**Proposition** [Caprara, Fischetti, Toth (02)]: OPTRA/TTP is $\text{NP}$-hard.

**Proof:**
Reduction from Independent-Set.
Microworld (http://www.railml.org)

- RailML Standardization
  - Infrastructure (lines, switches, signals, gradient ...)
  - Rolling Stock (engine, brakes, wagon ...)
  - Timetable
From Microscopy to Macroscopy

Block & Signal System

Headways

<table>
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<th>T₁</th>
<th>T₂</th>
<th>d</th>
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<td>3</td>
<td>4</td>
</tr>
<tr>
<td>T₁</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Station Layouts

Station Capacities
Macroscopic Infrastructure

- Traintypes
- Stations (standard, deadend or pseudo)
  - Capacities (per traintype, per mode)
  - Turnaroundtimes (per traintype)
- Tracks (connecting sides of stations)
  - Driving Times (per traintype, per mode)
  - Headway Times (per traintype combination)
Train Requests

- Traintype and utility value
- Departure Station
  - preferable time window (min,opt,max)
- Arrival Station
  - preferable time window (min,opt,max)
- Optional Stops
  - preferable arrival time window
  - preferable departure time window
  - preferable dwell time (min,max)

RequestSet.xsd
Train Timetabling Problem

\[ I \] - set of train request

\[ P_i \] - set of railway slots for request \( i \in I \)

\[ C \] - conflict sets, \( \{ (P_q \in 2^P, \kappa_q) \} \)

\[ u_p^i \] - utility of \( i \in I \) for \( p \in P \)
Optimization Approach

Train Requests → Scheduling Digraph → Timetable
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Packing Models

Conflict graph
Maximal cliques
Perfect graph

Lukac (2004) - conflict graphs of quadrangle linear headway matrices
Borndörfer, Schechte (2007) - conflict graphs of block occupation (interval graphs)
TTP as Path Packing Problem

\[\begin{align*}
\text{(PPP)} \\
\text{max} & \quad \sum_{i \in I} \sum_{p \in P_i} u_p^i x_p^i \\
\text{s.t.} & \quad \sum_{i \in I} x_p^i \leq 1 \quad \forall i \in I \quad (i) \\
& \quad \sum_{p \in C} x_p^i \leq \kappa_c \quad \forall c \in C \quad (ii) \\
& \quad x_p^i \in \{0, 1\} \quad \forall p \in P_i, \forall i \in I \quad (iii)
\end{align*} \]

Variables
- Path usage (request i uses path p)

Constraints
- Do no violated conflict sets

Objective
- Maximize proceedings/utility

PhD Thesis V. Cachhiani (2007) - details on TTP formulated as (APP) and (PPP)
Novel Model

Track Digraph
Timeline(s)
Config paths

Artificial arcs represent valid headways!
Path Coupling Problem

\[
\begin{align*}
\text{(PCP)} \\
\text{max} & \quad \sum_{i \in I} \sum_{p \in P_i} u^i_p x^i_p \\
\text{s.t.} & \quad \sum_{p \in P_i} x^i_p \leq 1 \quad \forall i \in I \\
& \quad \sum_{q \in Q_j} y^j_q \leq 1 \quad \forall j \in J \\
& \quad \sum_{a \in p \in P} x^a_p - \sum_{a \in q \in Q} y^a_q \leq 0 \quad \forall a \in A_f \cap A_j \\
& \quad y^a_q \in \{0, 1\} \quad \forall q \in Q_j, \forall j \in J \\
& \quad x^a_p \in \{0, 1\} \quad \forall p \in P_i, \forall i \in I
\end{align*}
\]

**Variables**
- Path und config usage (request i uses path p, track j uses config q)

**Constraints**
- Path and config choice
- Path-config-coupling (track capacity)

**Objective Function**
- Maximize proceedings
Linear Relaxation of PCP

\[
\begin{align*}
(MLP) & \quad \text{max} & \sum_{i \in I} \sum_{p \in P_i} u^i_p x^i_p \\
\text{s.t.} & & \sum_{p \in P_i} x_p \leq 1 & \forall i \in I & (i) \\
& & \sum_{q \in Q_j} y_q \leq 1 & \forall j \in J & (ii) \\
& & \sum_{a \in P} x_p - \sum_{a \in Q} y_q \leq 0 & \forall a \in A_I \cup A_J & (iii) \\
& & 0 \leq y_q \leq 1 & \forall q \in Q & (iv) \\
& & 0 \leq x_p \leq 1 & \forall p \in P & 
\end{align*}
\]

<table>
<thead>
<tr>
<th>dual variable</th>
<th>information about</th>
<th>useful to</th>
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<tbody>
<tr>
<td>(\gamma_i)</td>
<td>bundle price</td>
<td>analyse request</td>
</tr>
<tr>
<td>(\pi_j)</td>
<td>track price</td>
<td>analyse network</td>
</tr>
<tr>
<td>(\lambda_a)</td>
<td>arc price</td>
<td>-</td>
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</table>
(DLP)

\[
\begin{align*}
\text{min} & \quad \sum_{j \in J} \pi_j + \sum_{i \in I} \gamma_i \\
\text{s.t.} & \quad \gamma_i + \sum_{a \in p} \lambda_a \geq \sum_{a \in p} u_a^i \quad \forall p \in P_i, \forall i \in I \\
& \quad \pi_j - \sum_{a \in q} \lambda_a \geq 0 \quad \forall q \in Q_j, \forall j \in J \\
& \quad \gamma_i \geq 0 \quad \forall i \in I \\
& \quad \lambda_a \geq 0 \quad \forall a \in A_I \cup A_J \\
& \quad \pi_j \geq 0 \quad \forall j \in J
\end{align*}
\]
Pricing of x-variables

\[ \gamma_i \]

\[ (\text{PRICE (x)}) \quad \exists \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a) \]

\[ c_a = -p_a + \lambda_a \]

Pricing Problem(x) :
Acyclic shortest path problems for each slot request i with modified cost function c !
Pricing of y-variables

\[(\text{PRICE (y)}) \exists \overline{q} \in Q_j : \pi_j < \sum_{a \in \overline{q}} \lambda_a\]

$C_a = -\lambda_a$

Pricing Problem (y):
Acyclic shortest path problem for each track j with modified cost function c!
PCP-Run of TS-OPT /LP Stage

scenario 570 trains

objective value

$10^5$

$10^4$

$10^3$

$10^2$

$10^1$

$10^0$

column generation iterations

$\beta(\gamma, \pi, \lambda)$

$\nu(RPLP)$
Linear Relaxation

solve linear program by primal, dual simplex or barrier (interior point method)
Cutting Planes

current solution has fractional variables improve by valid cutting planes
current solution is infeasible (at least one forbidden fractional value)
Branching on Variables

split problems into sub problems to cut off current solution
split problems into subproblems to cut off current solution
Two Step Approach

1. LP Solving
2. IP Solving

TS-OPT

- Duals by LP-Solver (CPLEX)
- Pricing by Dijkstra’s Shortest Path
- Column Generation
- Rapid Branching Heuristic

Pricing by Dijkstra’s Shortest Path

Duals by LP-Solver (CPLEX)

Column Generation

Rapid Branching Heuristic
Branch-Bound-Price or Dive-Generate

Evaluation of only few highly different sub-problems at iteration j to reach IP-Solutions fast.
Rapid Branching (S. Weider 2007)

Node selection of set of fixed to 1 variables by using perturbated cost function (bonus close to 1.0).

Update Upper Bound

Go on if target was reached, otherwise pseudo-backtrack.

Column Generation

\[ S_j^0 \]

\[ S_j^l \]
TTP-Models in TS-OPT

Theorem [BS08]: The optimal value ($v_{LP}(PCP)$) of the LP relaxation of PCP can be computed in polynomial time.

Theorem [BS08]:

$v_{LP}(PCP) = v_{LP}(ACP) = v_{LP}(APP) = v_{LP}(PPP)$. 
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TTPLib Concept

Infrastructure Provider

Macroscopic Network Data

Train Operators

Set of Train Requests

Train Timetabling Problem

Solver

Timetable
Solution - A Macroscopic Timetable

- List of scheduled train paths
- Sorted list of used macroscopic tracks
- Arrival times at visiting stations
- Departure times at visiting stations

MacroTimetable.xsd
3d Schedule by TraVis (by B.Erol)
Welcome to TTP-Lib

TTP-Lib is a library of test instances for the Track Allocation Problem initiated during the "Transrapidbooster" Project. Our vision is that TTP-Lib becomes an important portal for everyone working on the Track Allocation Problem also known as Train Timetabling Problem. For this purpose we want to establish an important library of test instances including informations like bounds, best known solutions, etc.

In addition we provide Traviis, a visualization tool based on JavaView. Mathias Kindler and Berkay Erol developed this tool at ZIB to visualize train timetabling problems in 3d, getting some useful insights. We cordially invite you to download and use this instrument.

TTP-Lib is maintained at ZIB by Ralf Boddeker, Berkay Erol and Thomas Schlechte and has been launched 05/2008. Special thanks to Sören Schulz, Andreas Tannier and Christian Weise for preparing the instances of version 1.0.

To attain this goal we require your help!

TTP-Lib is only useful if it is kept up-to-date and enhanced by its user. You can help us in various ways, e.g.

- submitting new test instances,
- submitting new solutions or dual bounds for existing test instances,
- using test instances from the TTP-Lib in your computational studies,
- using visualization tool Traviis,
- submitting new entries or corrections to the bibliography or the conference list,
- joining our E-mail list_shadow@de,
- referencing TTP-Lib and Traviis in your papers.

If you have any further information or comments to improve TTP-Lib, please do not hesitate and e-mail directly to ttplib.webmaster@zib.de.
TTPLib 2008 - Dummy Instances

- network wheel
- 11 train request scenarios
- contributed by A. Tanner (WIP – TU Berlin)
- 10/11 solved to optimality by TS-OPT
• network hakafu
• 50 train request scenarios
• contributed by S. Schultz, A. Tanner and A. Reuther
• 42/50 solved to optimality by TS-OPT
• network hakafu_stations
• 50 train request scenarios
• contributed by S.Schultz, A.Tanner and A. Reuther
• 42/50 solved to optimality by TS-OPT
APP Cplex:

0096: Tsopt_CPLEX_Msg(): 36000 2311 572.8283 734 568.3799 680.5584 7685482 19.74%
0096: Tsopt_CPLEX_Msg(): Elapsed real time =
0096: Tsopt_CPLEX_Msg(): 6335.98 sec. (tree size = 14.52 MB
0096: Tsopt_CPLEX_Msg(): , solutions = 38
0096: Tsopt_CPLEX_Msg():

0096: Tsopt_CPLEX_Msg(): 36050 2315 cutoff 568.3799 680.5584 7694860 19.74%
0096: Tsopt_CPLEX_Msg(): 36100 2317 cutoff 568.3799 680.5584 7701564 19.74%
0096: Tsopt_CPLEX_Msg(): 36150 2325 570.6950 530 568.3799 680.5584 7709180 19.74%
ACP Cplex:

0096: Tsopt_CPLEX_Msg():    400    93    595.5520 1602    595.2397  630.6793  243827    5.9 5%
0096: Tsopt_CPLEX_Msg():    450    87    597.7870  522    595.2397  630.6793  258572    5.95%
0096: Tsopt_CPLEX_Msg():    500   103    cutoff  595.2397  630.6793  278680    5.95%
0096: Tsopt_CPLEX_Msg(): Elapsed real time = 94.16 sec. (tree size = 0.24 MB
0096: Tsopt_CPLEX_Msg(): , solutions = 2

...
### ACP SCIP:

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</table>

SCP Status: problem is solved [optimal solution found]

Solving Time (sec): 177894.99

**Primal Bound**: +6.22365324690698e+02 (103 solutions)

**Dual Bound**: +6.22365324690698e+02

**Gap**: 0.00 %
Please Contribute - Improve TTPLib!

By submitting
- new test instances
- new solutions or upper bounds
- new entries to the bibliography and conference list

By using
- instances from the TTPLib in your computational studies
- free visualization tool TraVis
Thank you for your attention!