Federal Ministry
of Economics
and Technology

Timetabling with TS-OPT

Thomas Schlechte

joint work with

Ralf Borndörfer and Berkan Erol

RailZurich2009

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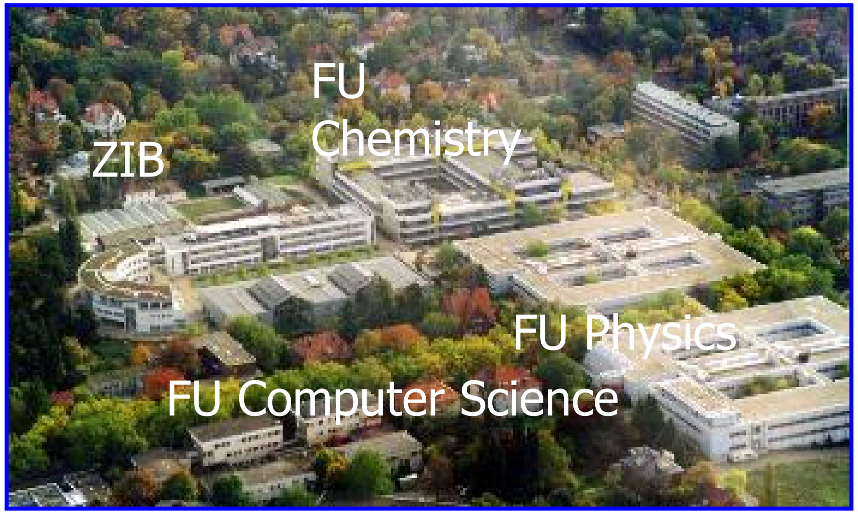
Zuse-Institute-Berlin (ZIB)

Konrad Zuse was the creator of the first fully automatic, programm controlled and freely programmable computer working in binary floating point arithmetic. The Z3 was finished in 1941.



Overview

1. Idea & Motivation
2. Train Timetabling Problem
3. Algorithmic Approach
4. TTPLib 2008

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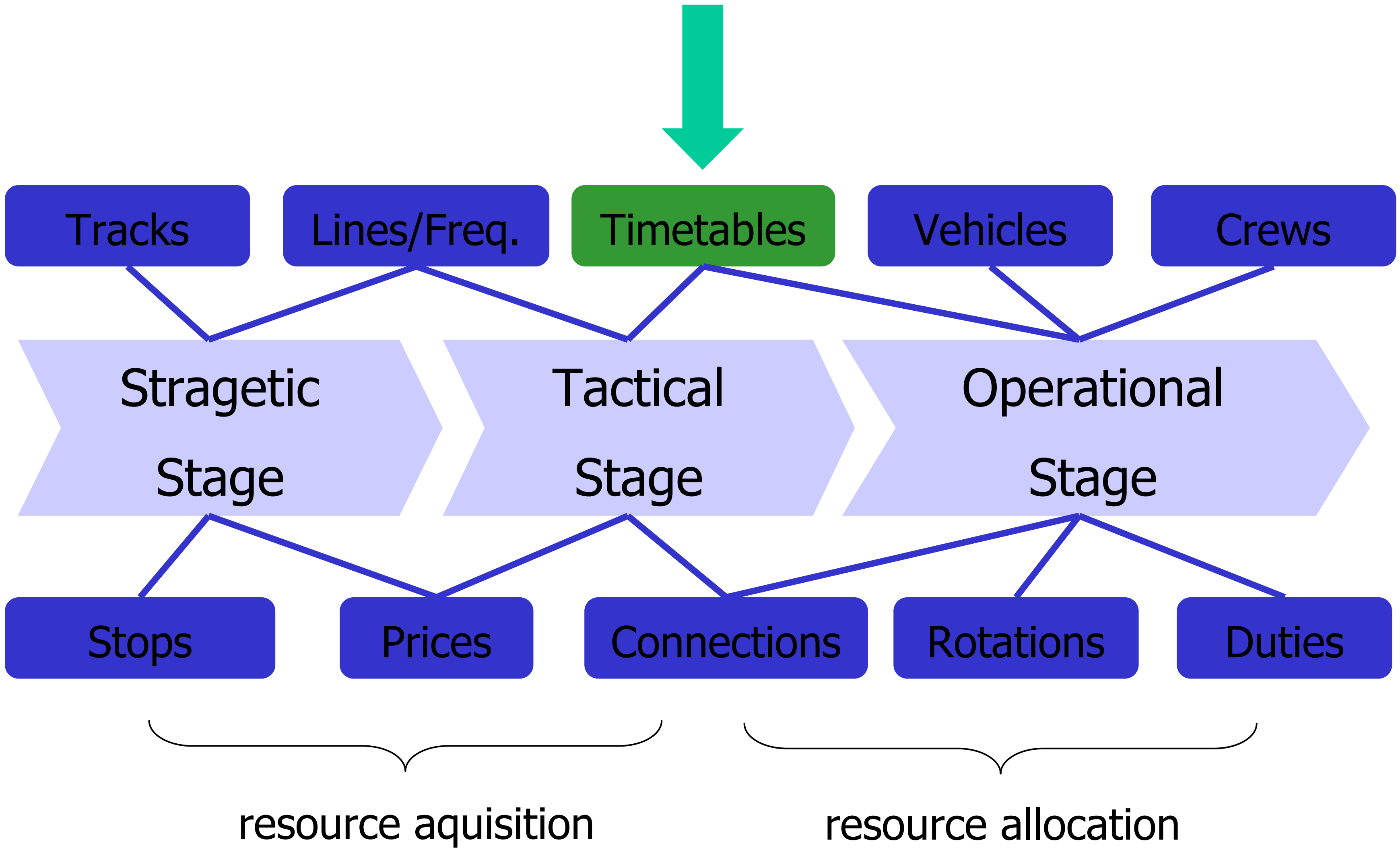
Planning in Public Transport



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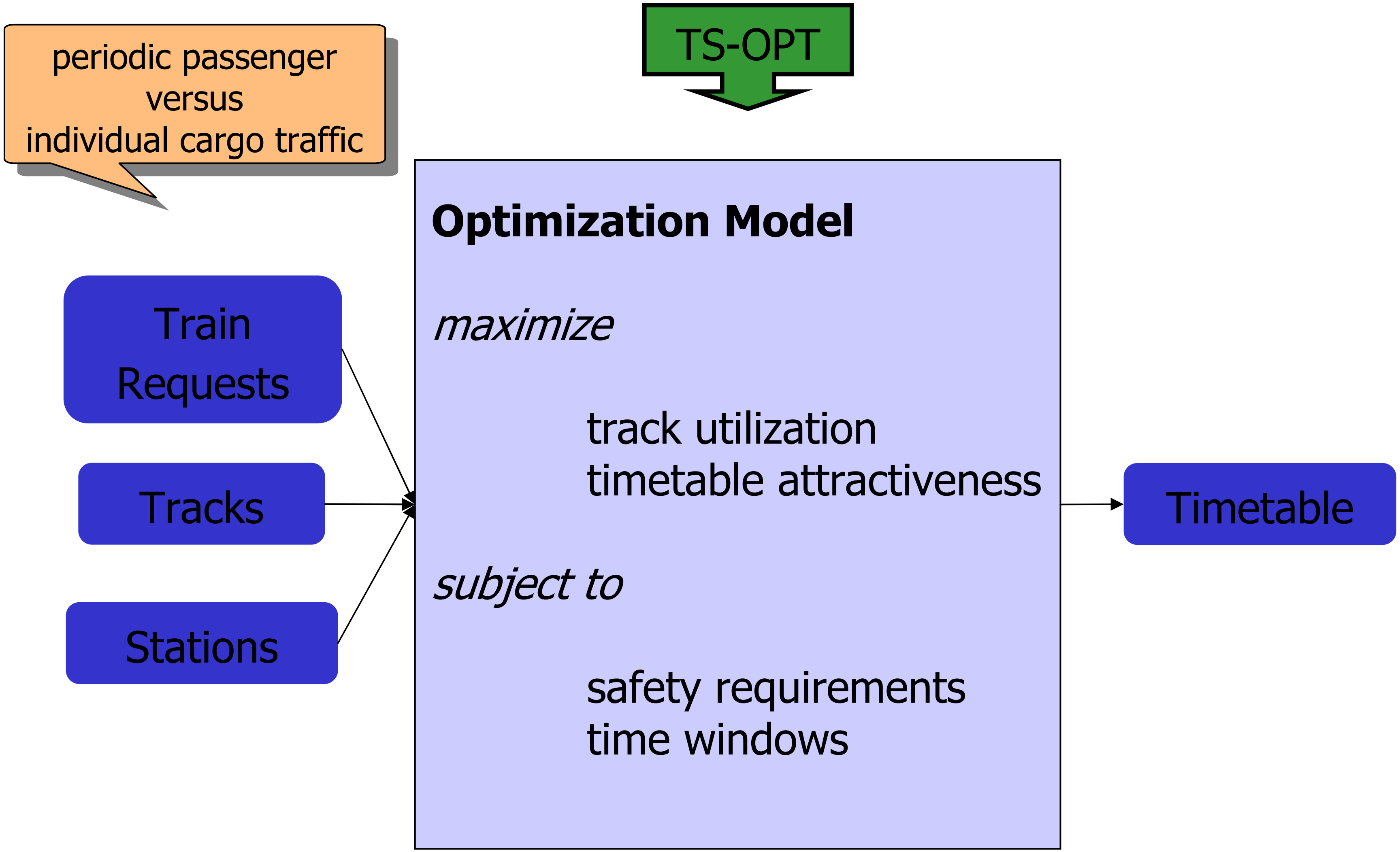
Timetabling (PESP & TTP & TPP)



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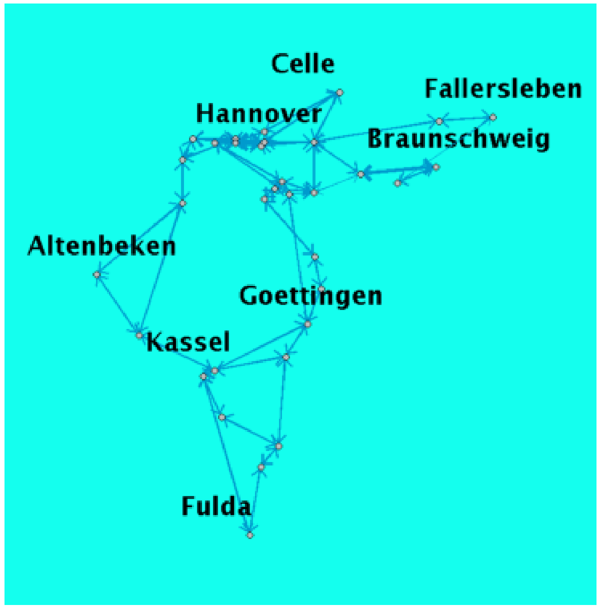
Railway Timetabling (TTP) - State of the Art



- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Cai and Goh (1994), Schrijver and Steenbeck (1994), Carey and Lockwood (1995)
- Nachtigall and Voget (1996), Odijk (1996) Higgings, Kozan and Ferreira (1997)
- **Brannlund, Lindberg, Nou, Nilsson (1998)**, Lindner (2000), Oliveira and Smith (2000)
- **Caprara, Fischetti and Toth (2002)**, Peeters (2003)
- Kroon and Peeters (2003), Mistry and Kwan (2004)
- Barber, Salido, Ingolotti, Abril, Lova, Tormas (2004)
- Semet and Schoenauer (2005),
- **Caprara, Monaci, Toth and Guida (2005)**
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006),
- **Cacchiani, Caprara, T. (2006), Cachhiani (2007)**
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- **Borndoerfer, Schlechte (2007)**
- **Fischer, Helmberg, Janßen, Krostitz (2008) ...**

non-cyclic timetabling literature

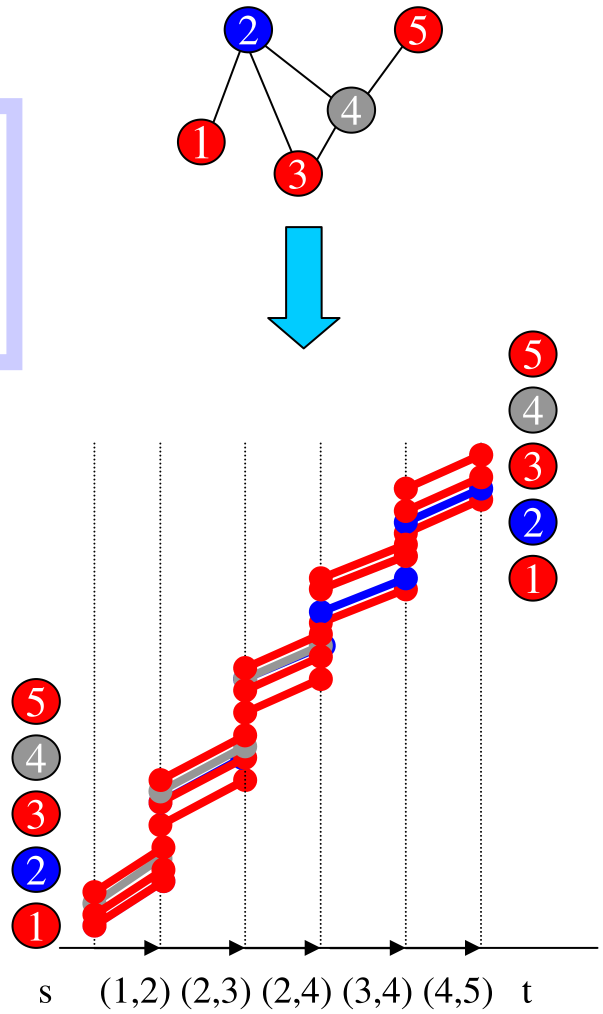
Computational Complexity

Proposition [Caprara, Fischetti, Toth (02)]:

OPTRA/TTP is $\mathcal{NP}$-hard.

Proof:

Reduction from Independent-Set.



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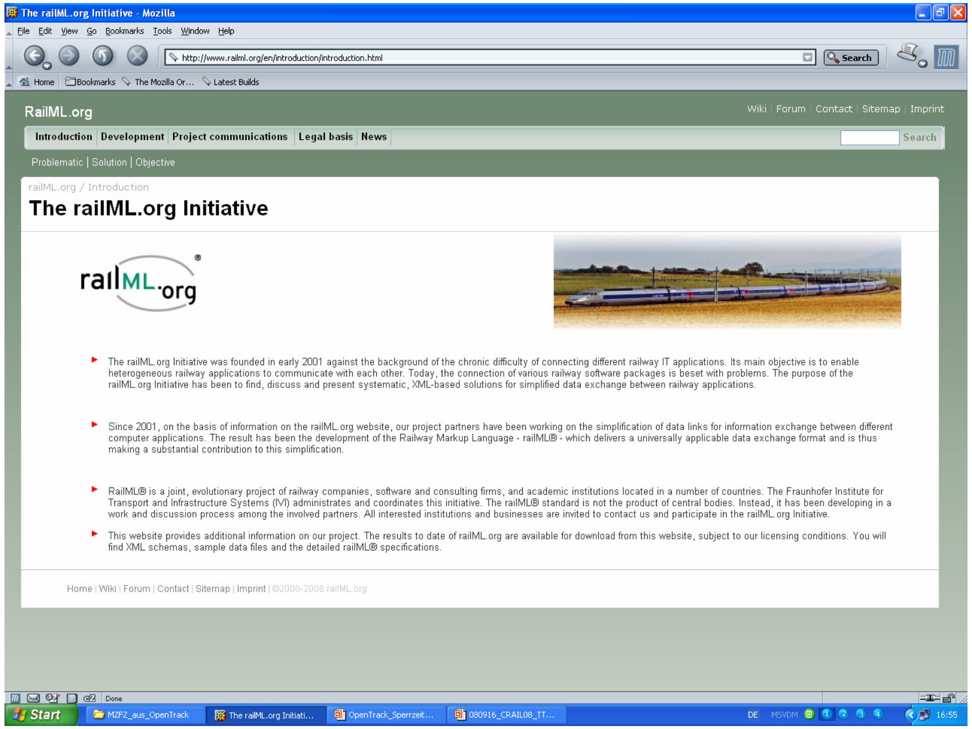
Microworld *(http:// www.railml.org)*

- RailML Standardization
  - Infrastructure (lines, switches, signals, gradient ...)
  - Rolling Stock (engine, brakes, wagon ...)
  - Timetable



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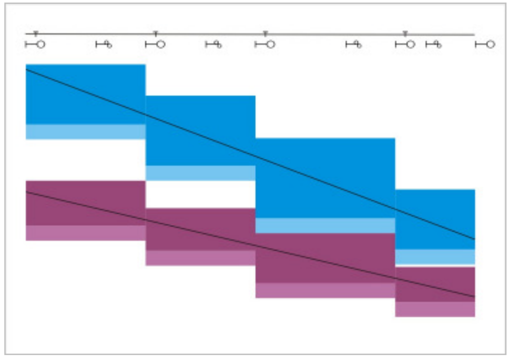
From Microscopy to Macroscopy

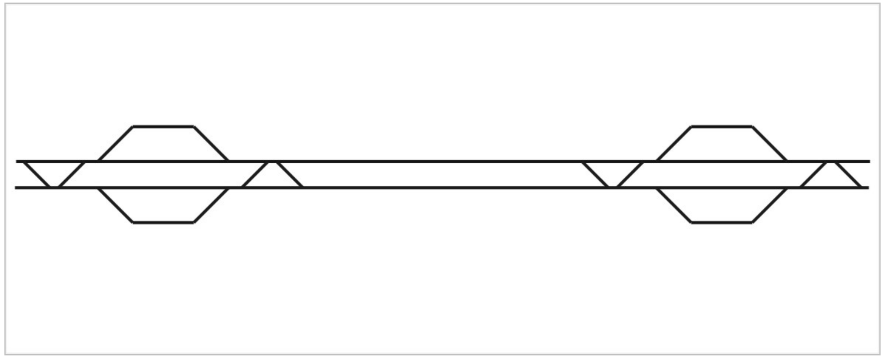


Block & Signal System

↓

Headways

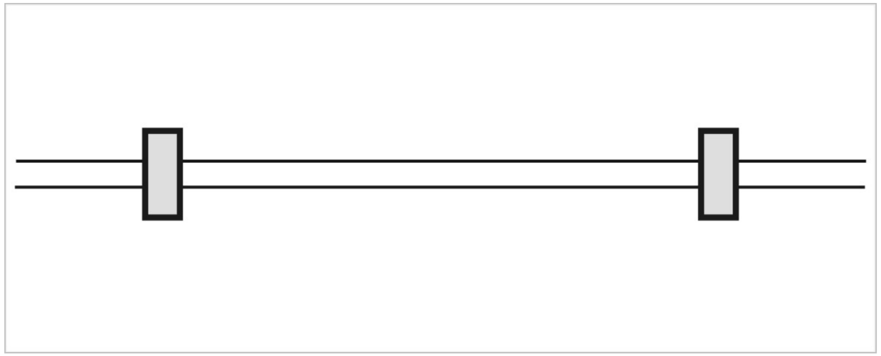
h	T_1	T_2	d
T_1	2	3	4
T_2	1	1	2



Station Layouts

↓

Station Capacities



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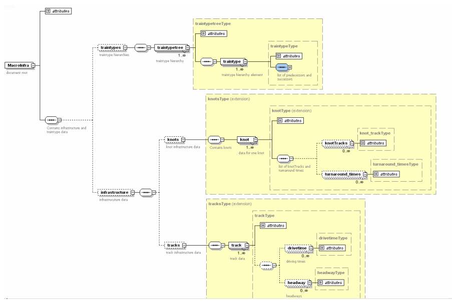
Macroscopic Infrastructure

- Traintypes
- Stations (standard, deadend or pseudo)
  - Capacities (per traintype, per mode)
  - Turnaroundtimes (per traintype)
- Tracks (connecting sides of stations)
  - Driving Times (per traintype, per mode)
  - Headway Times (per traintype combination)

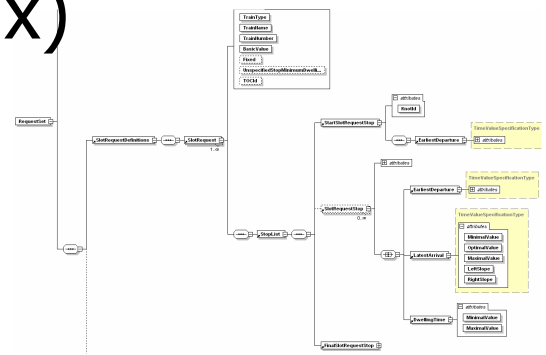
MacroInfra.xsd



Train Requests

- Traintype and utility value
- Departure Station
  - preferable time window (min,opt,max)
- Arrival Station
  - preferable time window (min,opt,max)
- Optional Stops
  - preferable arrival time window
  - preferable departure time window
  - preferable dwell time (min,max)

RequestSet.xsd



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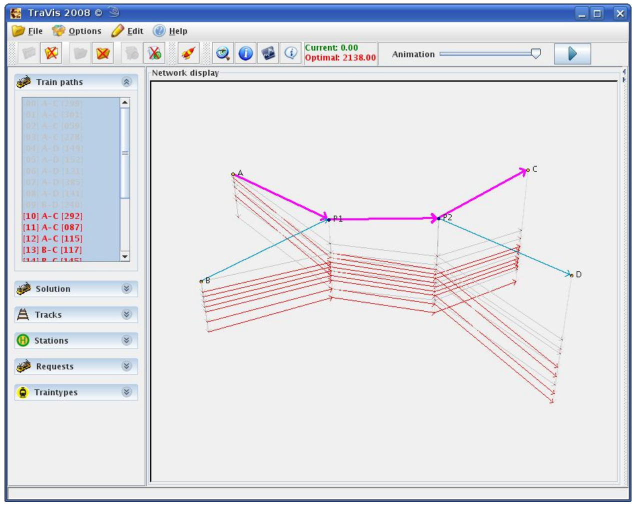
Train Timetabling Problem

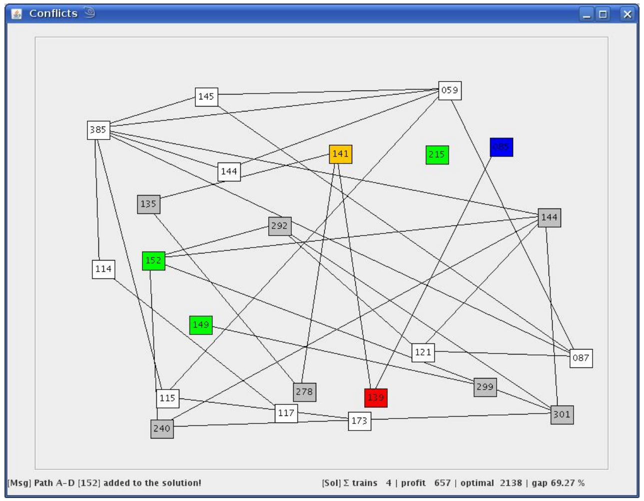
I - set of train request

P_i - set of railway slots for request $i \in I$

C - conflict sets, $\{(P_q \in 2^P, \kappa_q)\}$

u_p^i - utility of $i \in I$ for $p \in P$





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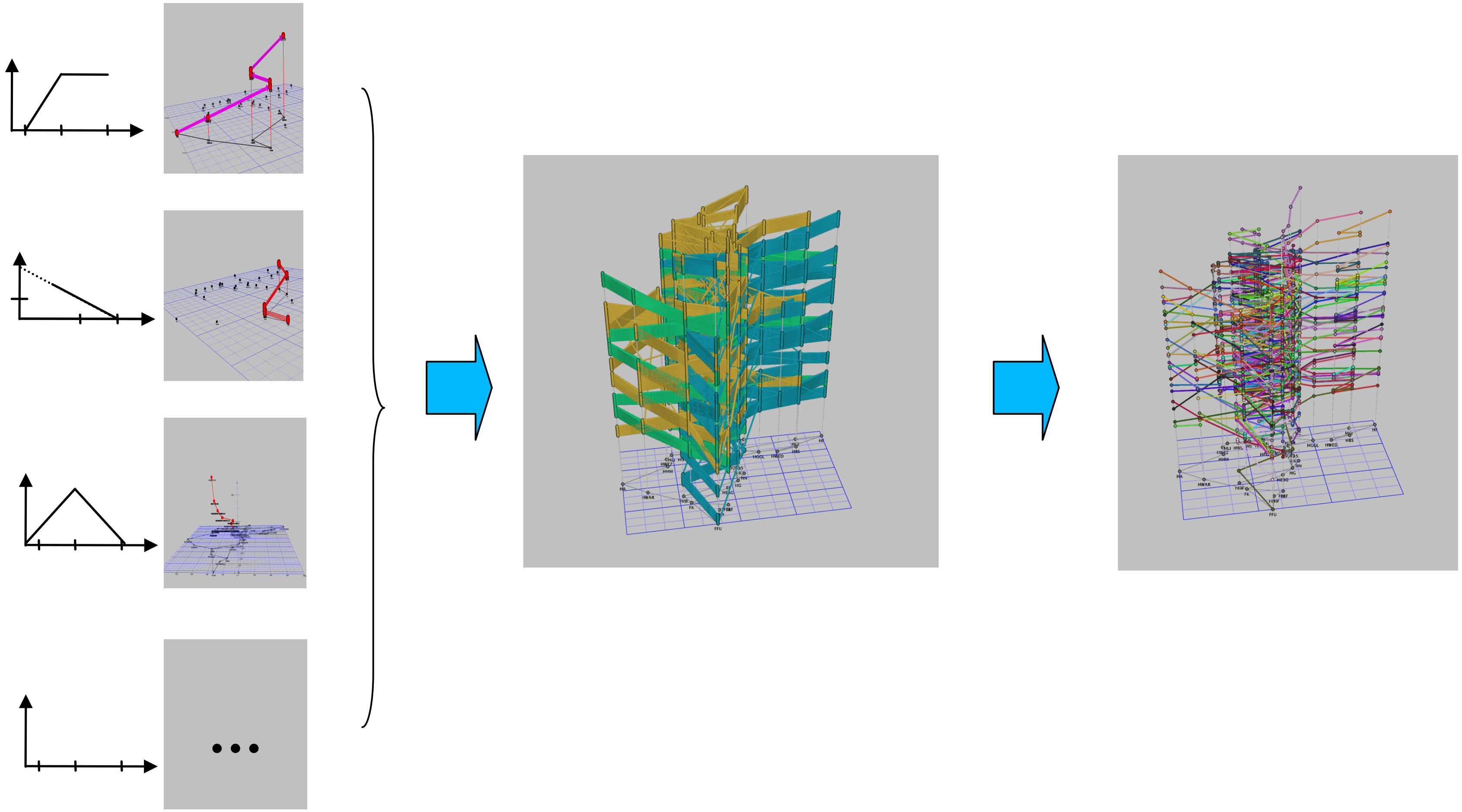
Optimization Approach



Train Requests → Scheduling Digraph → Timetable

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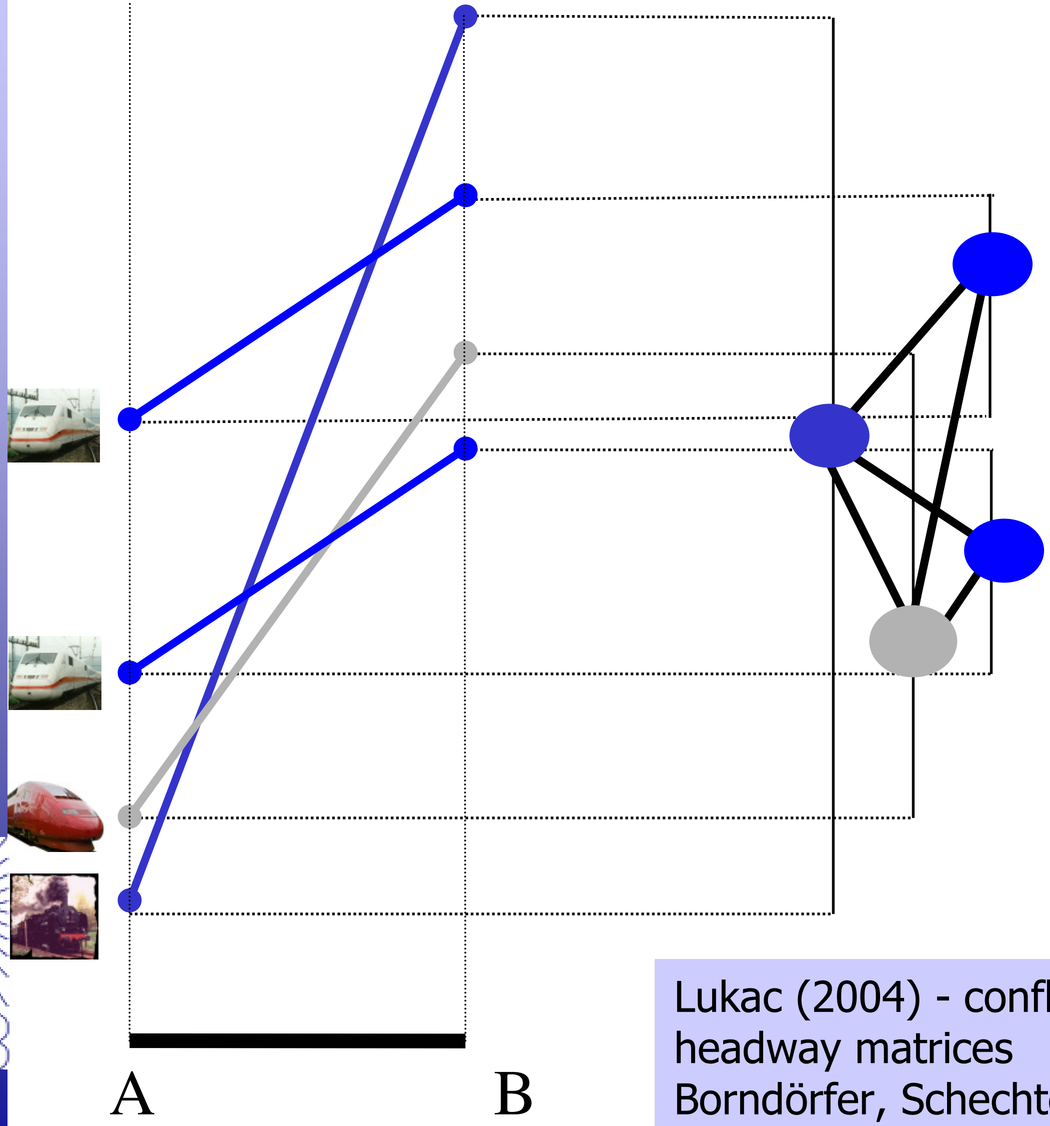
Packing Models



Conflict graph

Maximal cliques

Perfect graph

Lukac (2004) - conflict graphs of quadrangle linear headway matrices
Borndörfer, Schechte(2007) - conflict graphs of block occupation (interval graphs)

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TTP as Path Packing Problem

$$
\begin{array}{llll}
(PPP) & & & \\
\max & \sum_{i \in \mathcal{I}} \sum_{p \in P_i} u_p^i x_p^i & & \\
\text{s.t.} & \sum_{i \in \mathcal{I}} x_p^i \leq 1 & \forall i \in \mathcal{I} & \text{(i)} \\
& \sum_{p \in c} x_p^i \leq \kappa_c & \forall c \in C & \text{(ii)} \\
& x_p^i \in \{0, 1\} & \forall p \in P_i, \forall i \in \mathcal{I} & \text{(iii)}
\end{array}
$$

Variables

- Path usage (request i uses path p)

Constraints

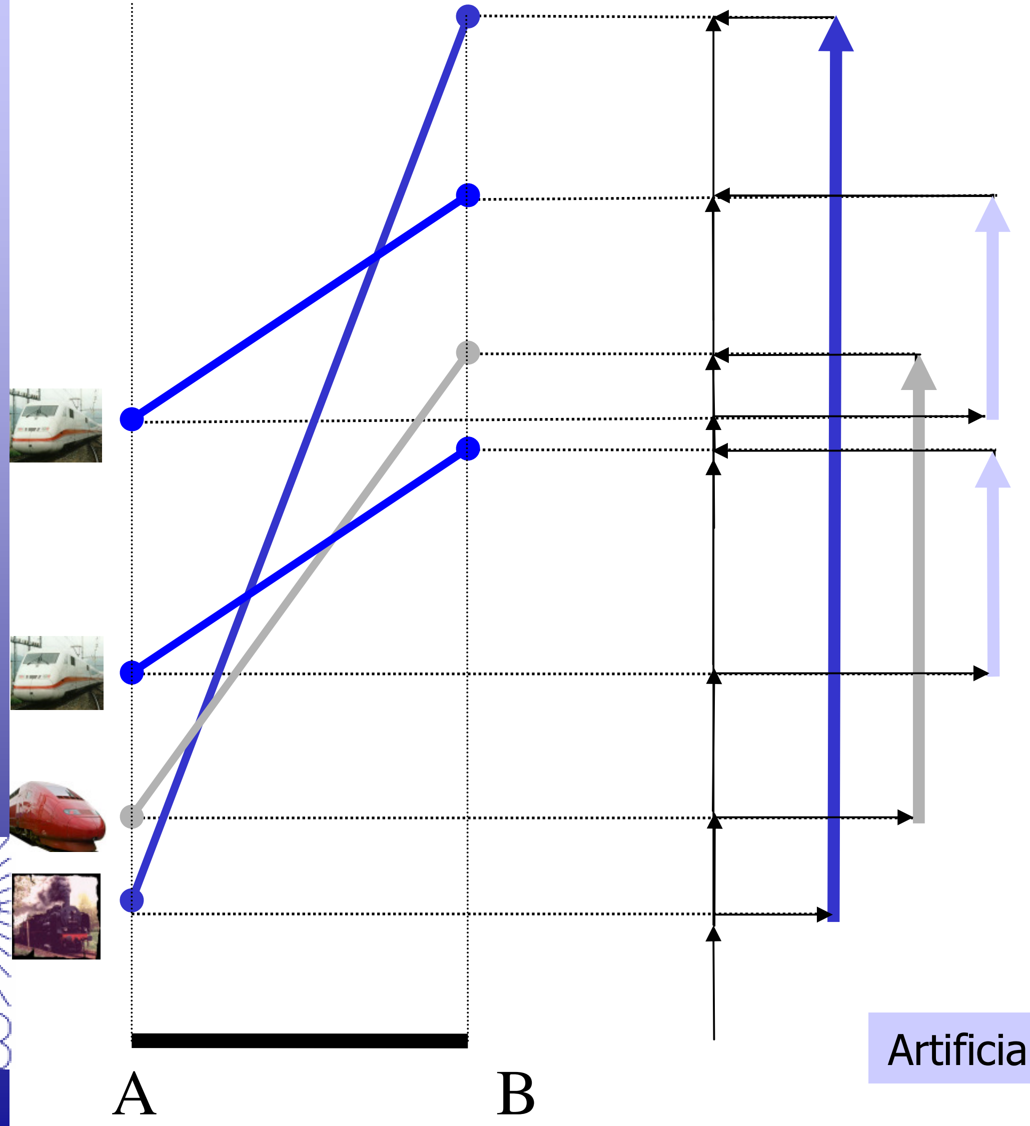
- Do no violated conflict sets

Objective

- Maximize proceedings/utility

PhD Thesis V.Cachhiani (2007) - details on TTP formulated as (APP) and (PPP)

Novel Model



Track Digraph

Timeline(s)

Config paths

Artificial arcs represent valid headways !

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Path Coupling Problem

$$
\begin{array}{llllll}
(PCP) & & & & & \\
\max & \sum_{i\in I}\sum_{p\in P_i} u_p^i x_p^i & & & & \\
\text{s.t.} & \sum_{p\in\mathcal{P}_i} x_p & \leq 1 & \forall i \in I & & \text{(i)} \\
 & \sum_{q\in\mathcal{Q}_j} y_q & \leq 1 & \forall j \in J & & \text{(ii)} \\
 & \sum_{a\in p\in\mathcal{P}} x_p - \sum_{a\in q\in\mathcal{Q}} y_q & \leq 0 & \forall a \in A_I \cap A_J & & \text{(iii)} \\
 & y_q & \in \{0,1\} & \forall q \in \mathcal{Q}_j, \forall j \in J & & \text{(iv)} \\
 & x_p & \in \{0,1\} & \forall p \in \mathcal{P}_i, \forall i \in I & & \text{(v)}
\end{array}
$$

Variables

- Path und config usage (request i uses path p, track j uses config q)

Constraints

- Path and config choice
- Path-config-coupling (track capacity)

Objective Function

- Maximize proceedings

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Linear Relaxation of PCP

(MLP)

$$\begin{array}{llllll}
\text{max} & \sum_{i\in I}\sum_{p\in P_i} u_p^i x_p^i & & & & \\
\text{s.t.} & \sum_{p\in \mathcal{P}_i} x_p & \leq 1 & \forall i\in I & \text{(i)} & \\
 & \sum_{q\in \mathcal{Q}_j} y_q & \leq 1 & \forall j\in J & \text{(ii)} & \\
 & \sum_{a\in p\in \mathcal{P}} x_p - \sum_{a\in q\in \mathcal{Q}} y_q & \leq 0 & \forall a\in A_I\cup A_J & \text{(iii)} & \\
 & 0\leq y_q & \leq 1 & \forall q\in \mathcal{Q} & \text{(iii)} & \\
 & 0\leq x_p & \leq 1 & \forall p\in \mathcal{P} & \text{(iv)} &
\end{array}$$

γ_i π_j λ_a

dual variable	information about	useful to
γ_i	bundle price	analyse request
π_j	track price	analyse network
λ_a	arc price	-

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Dualization

$$
\begin{array}{llll}
(DLP) & & & \\
\text{min} & \sum_{j\in J} \pi_j + \sum_{i\in I} \gamma_i & & \\
\text{s.t.} & \gamma_i + \sum_{a\in p} \lambda_a \geq \sum_{a\in p} u_a^i & \forall p \in \mathcal{P}_i, \forall i \in I & \text{(i)} \\
& \pi_j - \sum_{a\in q} \lambda_a \geq 0 & \forall q \in \mathcal{Q}_j, \forall j \in J & \text{(ii)} \\
& \gamma_i \geq 0 & \forall i \in I & \text{(iii)} \\
& \lambda_a \geq 0 & \forall a \in A_I \cup A_J & \text{(iv)} \\
& \pi_j \geq 0 & \forall j \in J & \text{(v)}
\end{array}
$$

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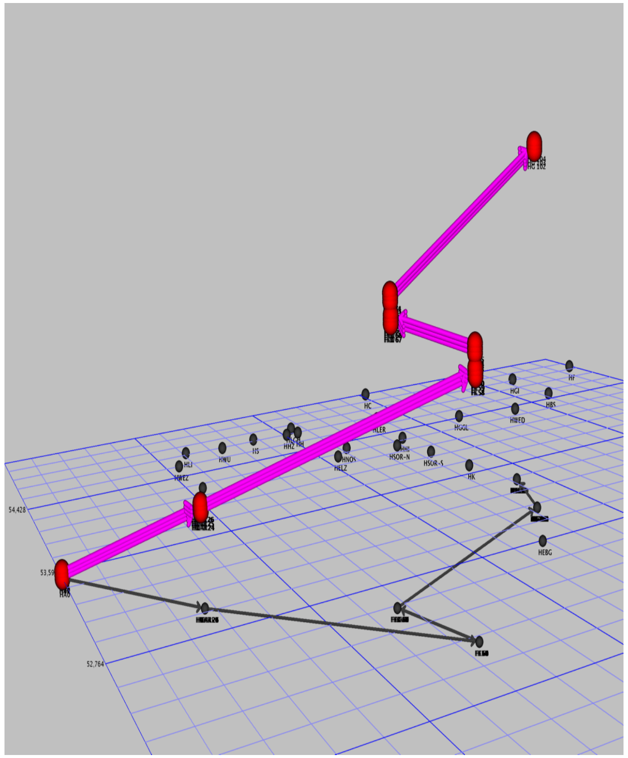
Pricing of x-variables

(PRICE (x)) $\exists\, \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \overline{p}} (p_a - \lambda_a)$



γ_i

$$c_a = -p_a + \lambda_a$$

<u>Pricing Problem(x) :</u>
Acyclic shortest path problems for each slot request i with modified cost function c !

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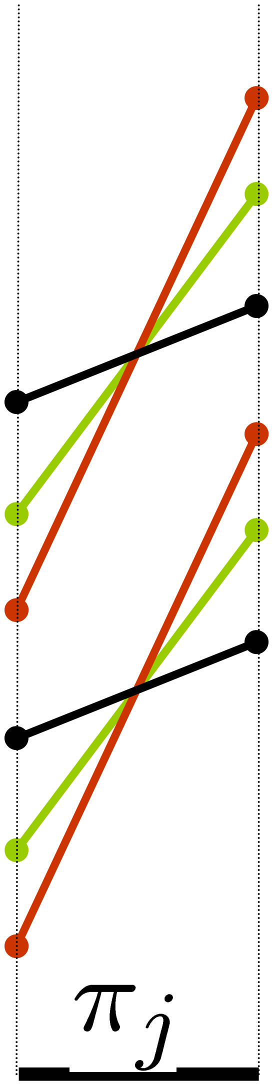
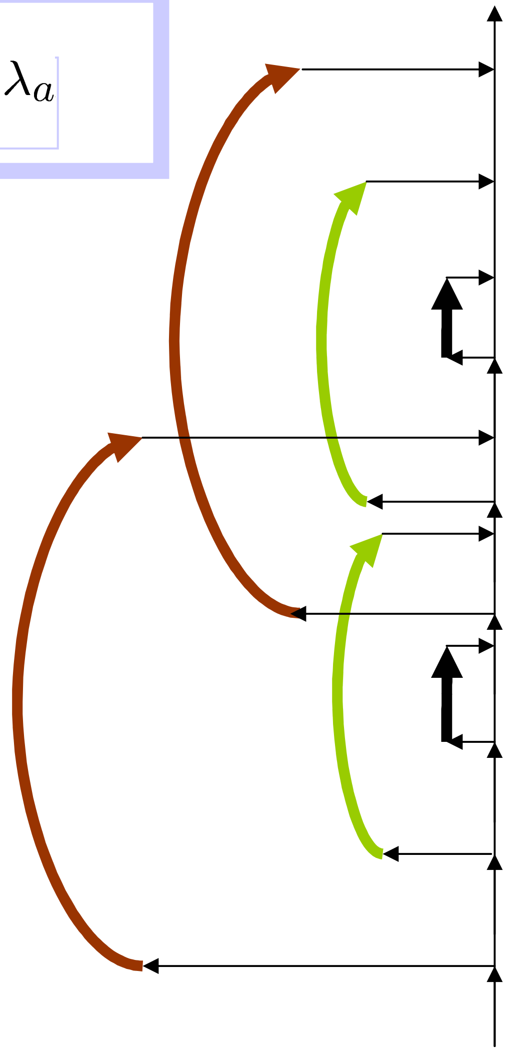
Pricing of y-variables



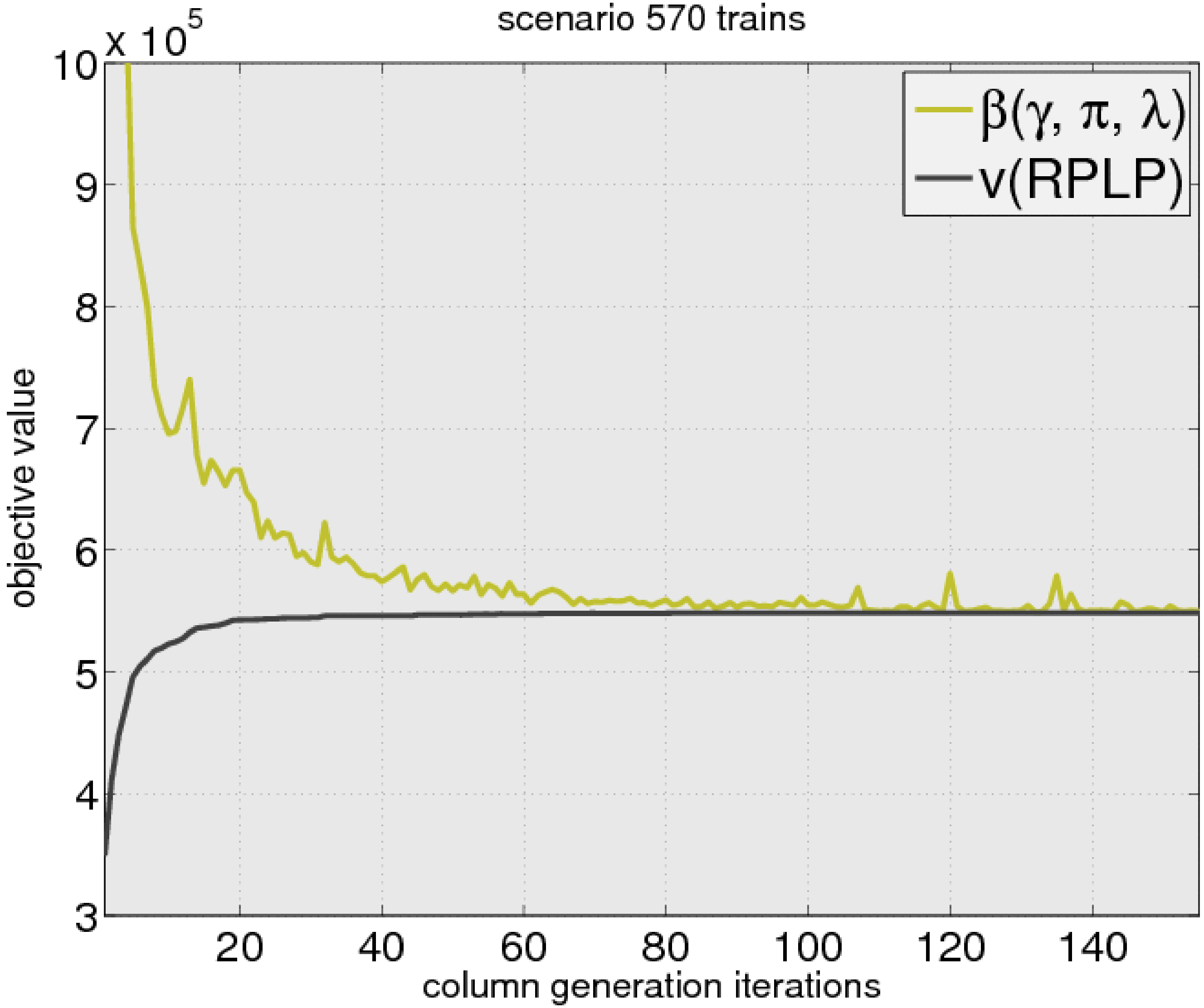
(PRICE (y)) $\exists\, \overline{q} \in Q_j : \quad \pi_j < \sum_{a \in \overline{q}} \lambda_a$

$$c_a = -\lambda_a$$



Pricing Problem(y) :
Acyclic shortest path problem for each track j with modified cost function c !

PCP-Run of TS-OPT /LP Stage



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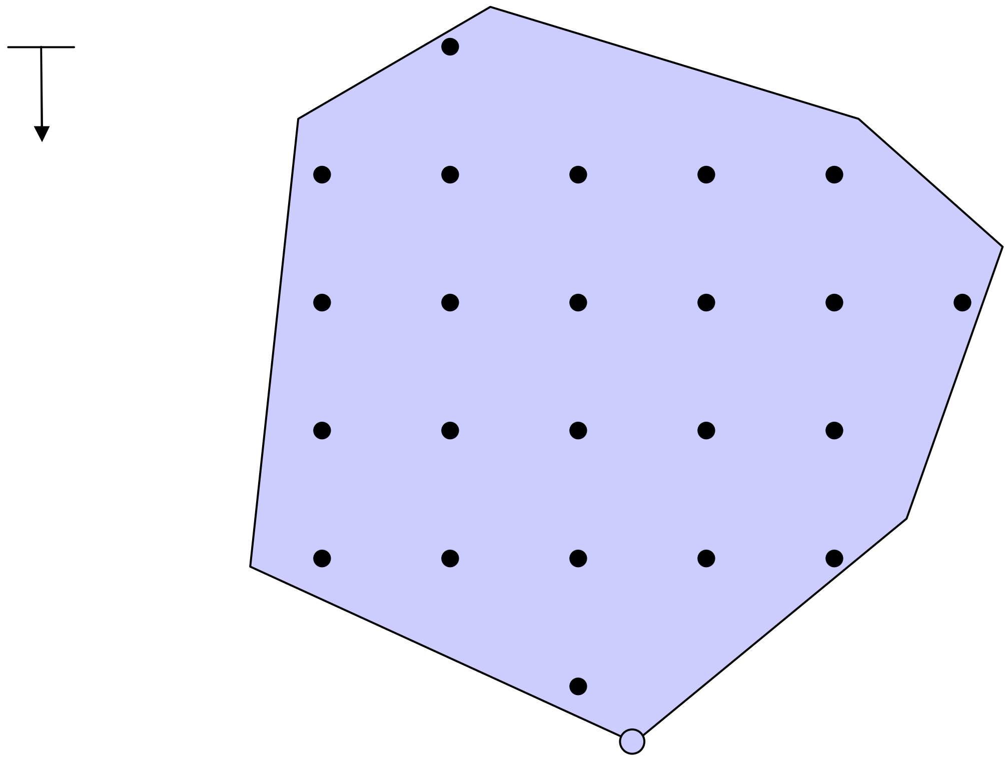
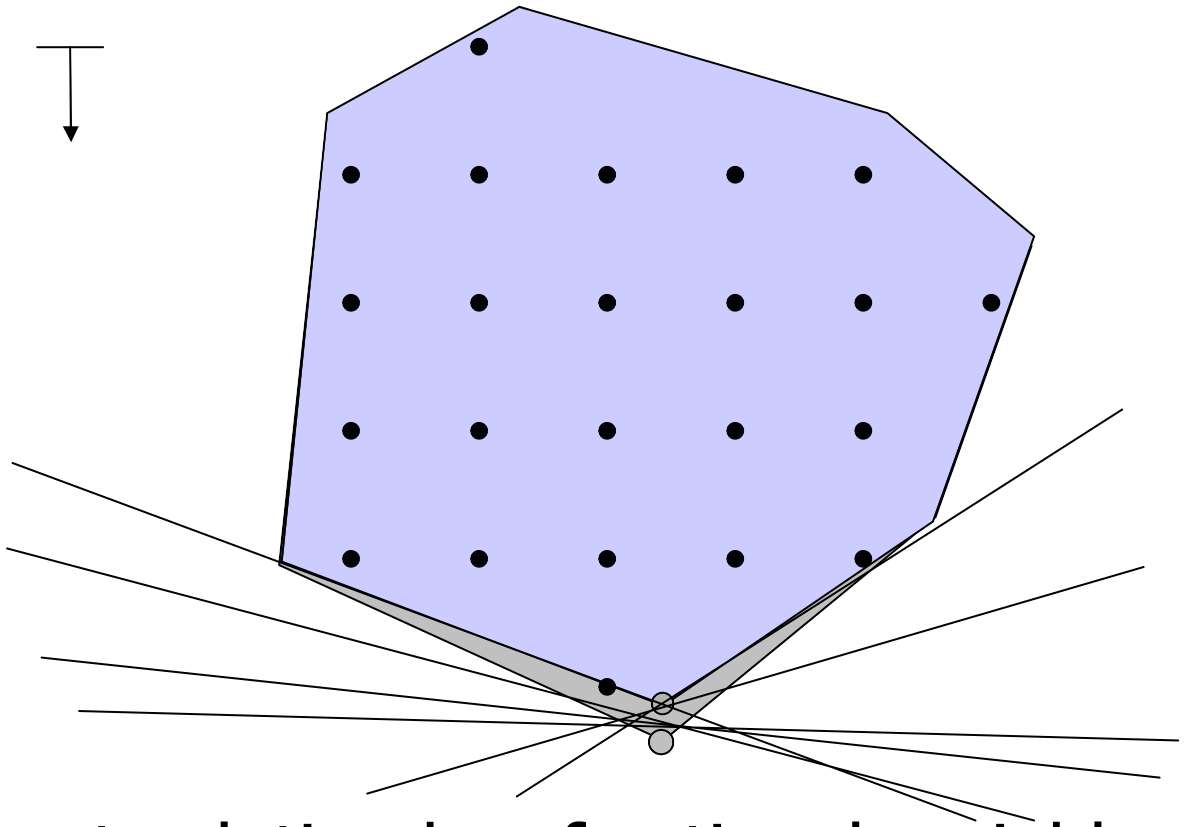
Linear Relaxation



- solve linear program by primal, dual simplex or barrier (interior point method)

Cutting Planes



current solution has fractional variables

improve by valid cutting planes

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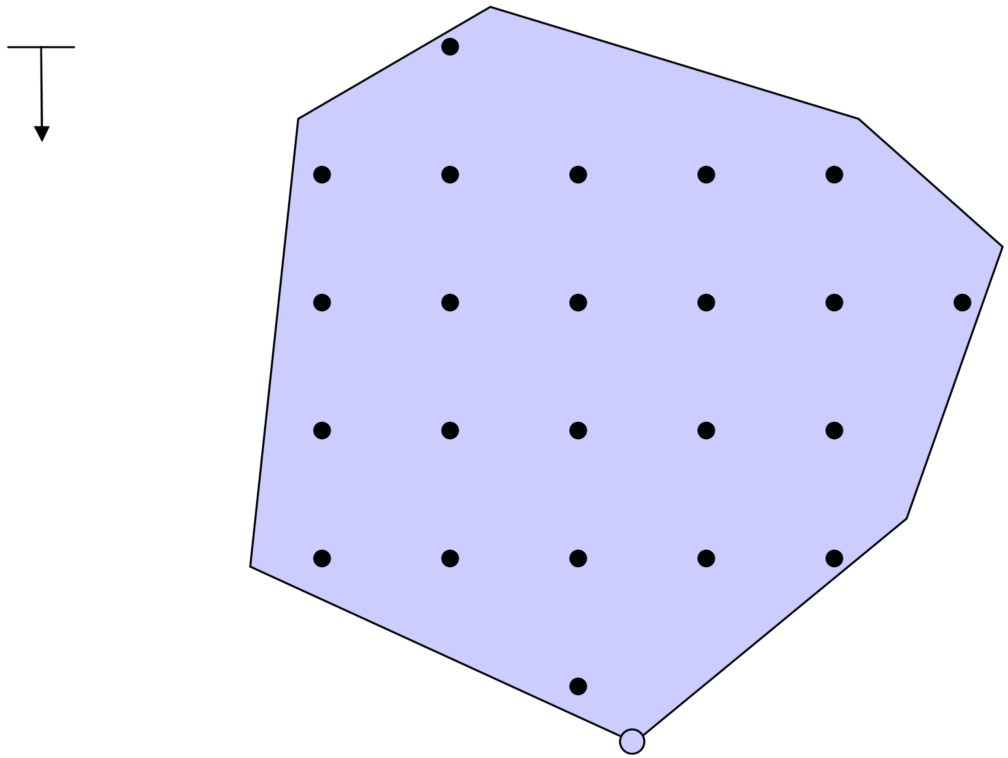
Branching



current solution is infeasible (at least one forbidden fractional value)

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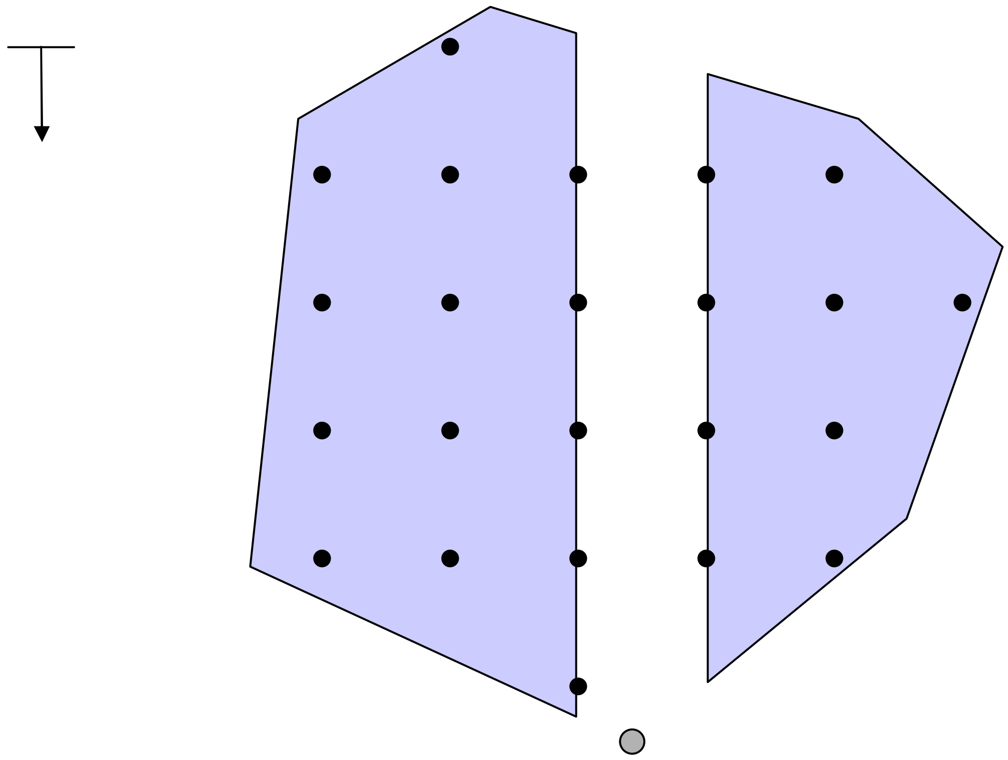
Branching on Variables



- split problems into sub problems to cut off current solution

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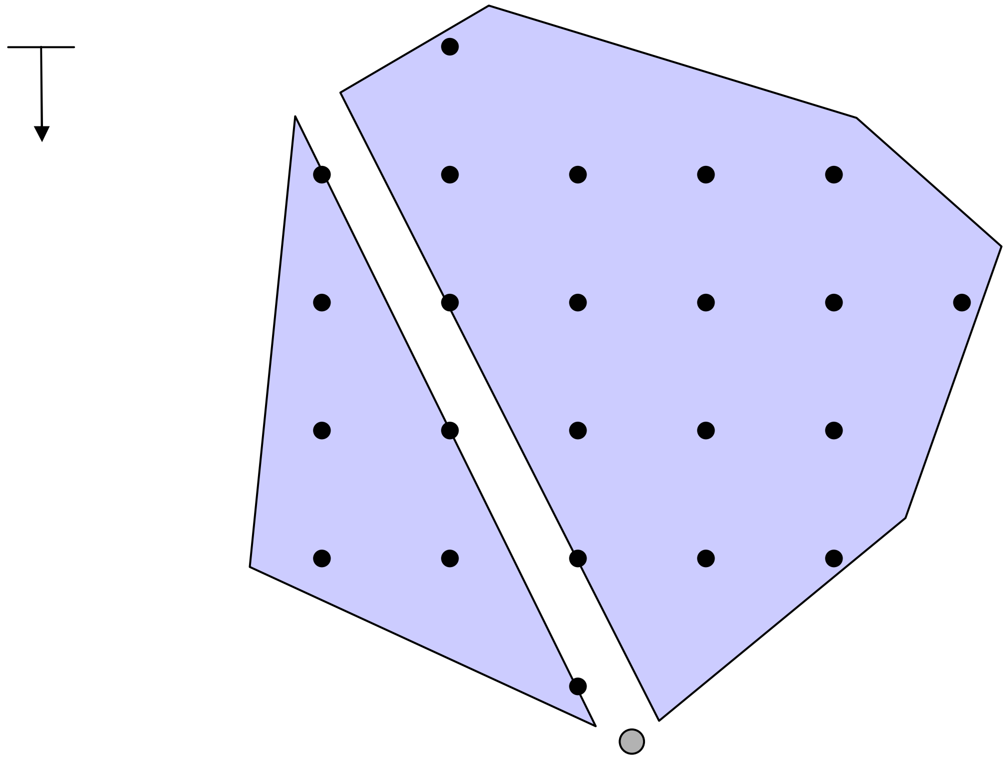
Branching on Constraints



split problems into subproblems to cut off current solution

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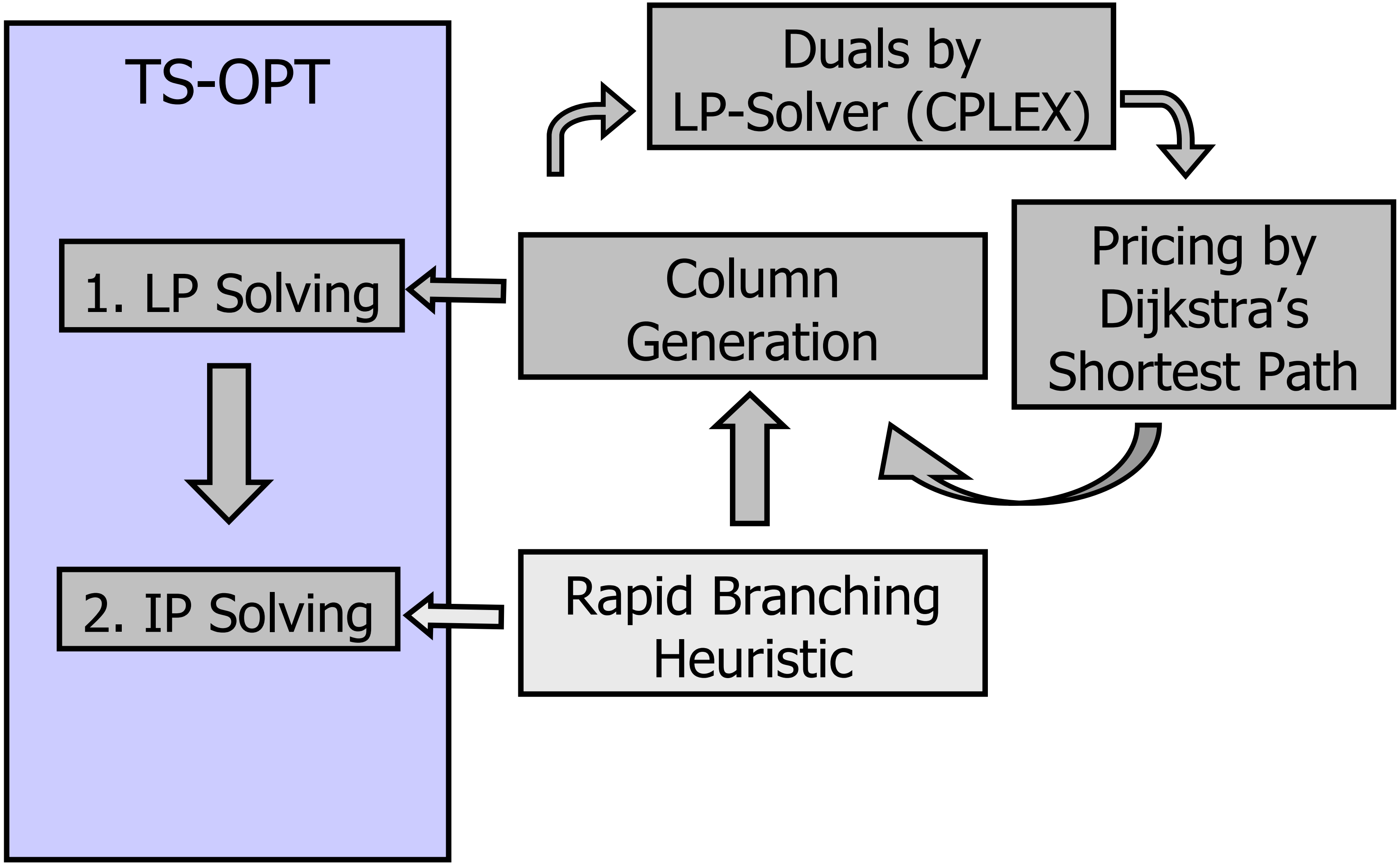
Two Step Approach



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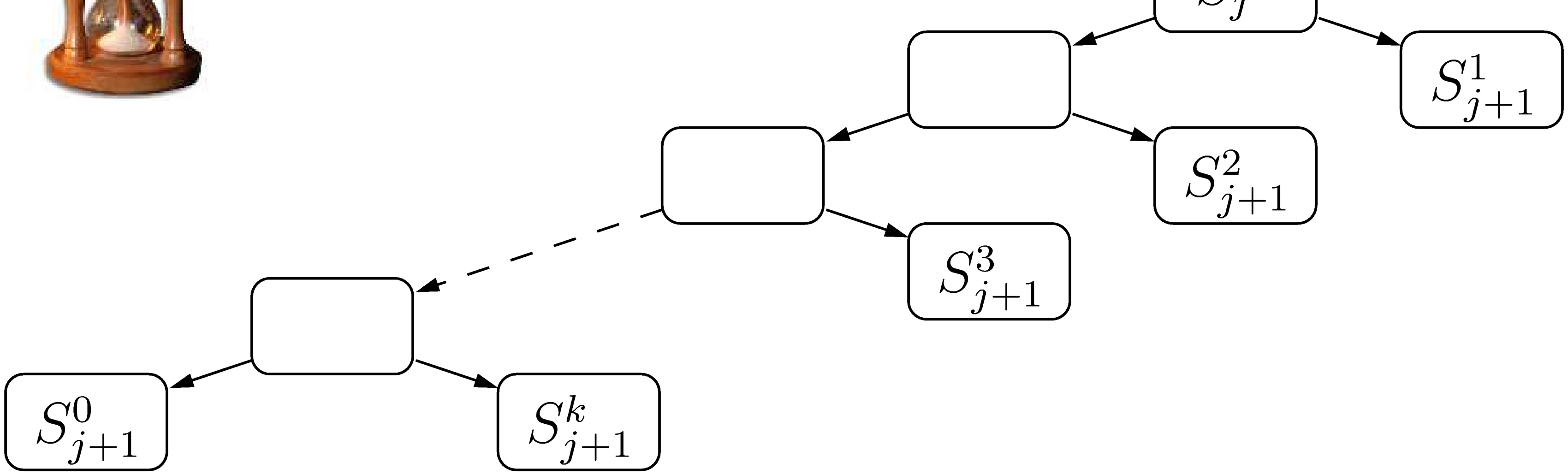
Branch-Bound-Price

or Dive-Generate



Evaluation of only few highly different sub-problems at iteration j to reach IP-Solutions fast.

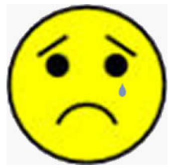
Rapid Branching (S.Weider 2007)

Node selection of set of fixed to 1 variables by using perturbated cost function (bonus close to 1.0).

S_j

Update Upper Bound

Column Generation



S_{j+1}^0

Go on if target was reached, otherwise pseudo-backtrack.

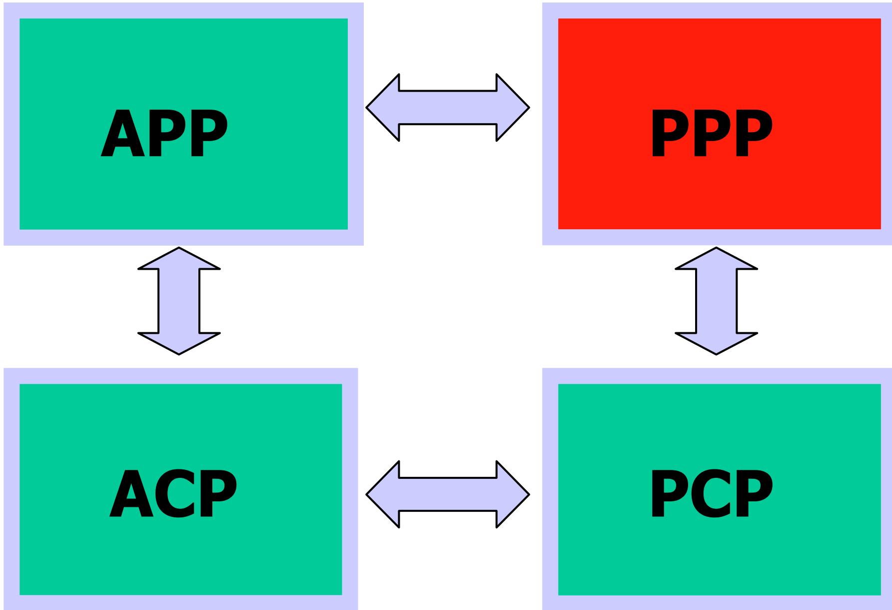
S_j^l

TTP-Models in TS-OPT

Theorem[BS08]: The optimal value (v_{LP}(PCP)) 0f the LP relaxation of PCP can be computed in polynomial time.

Theorem[BS08]:

v_{LP}(PCP) = v_{LP}(ACP) = v_{LP}(APP) = v_{LP}(PPP).



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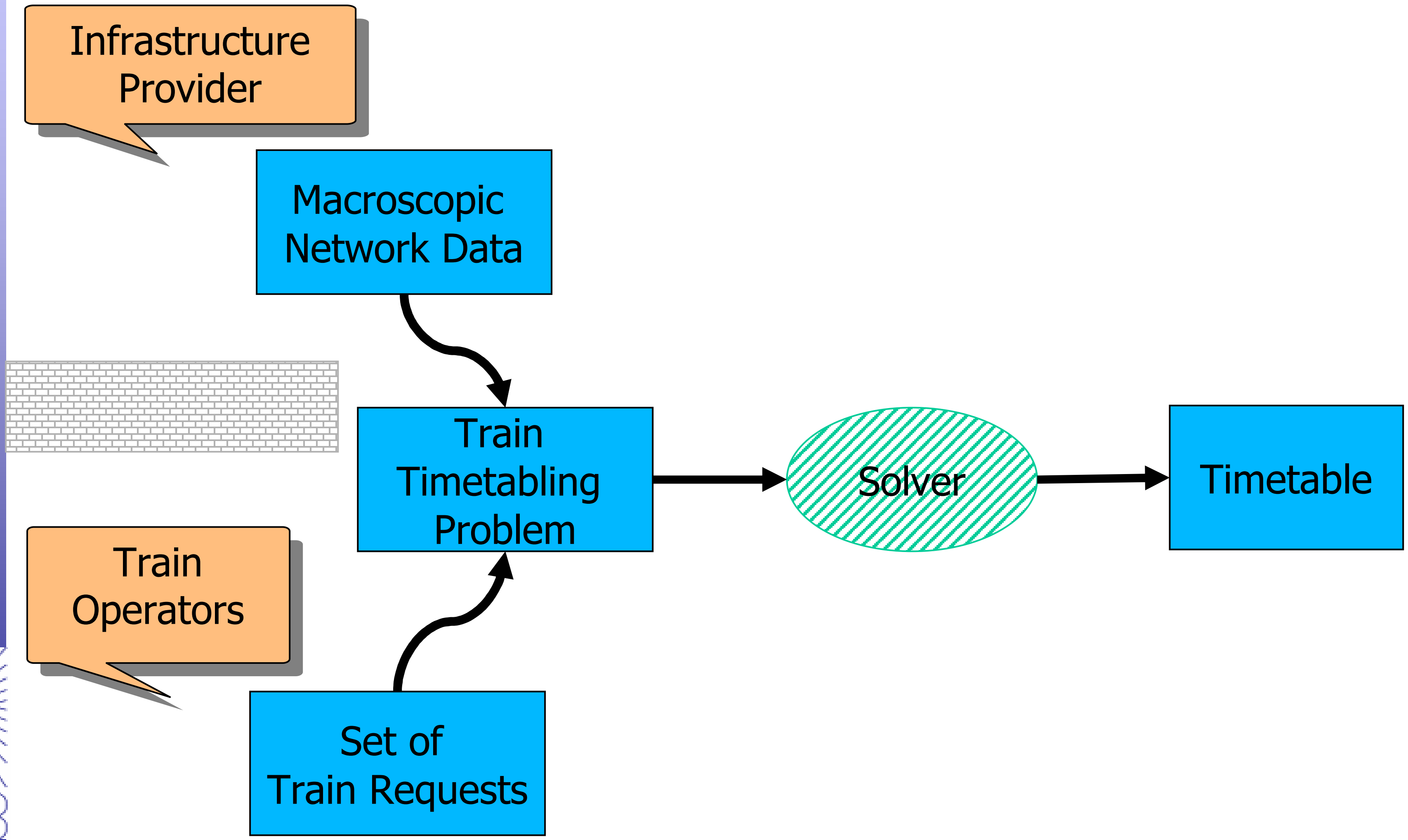
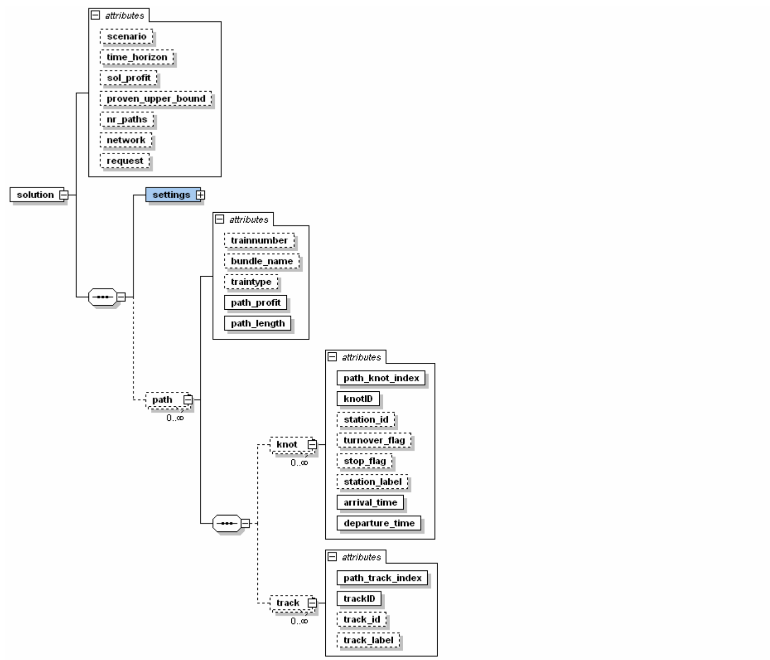
TTPLib Concept



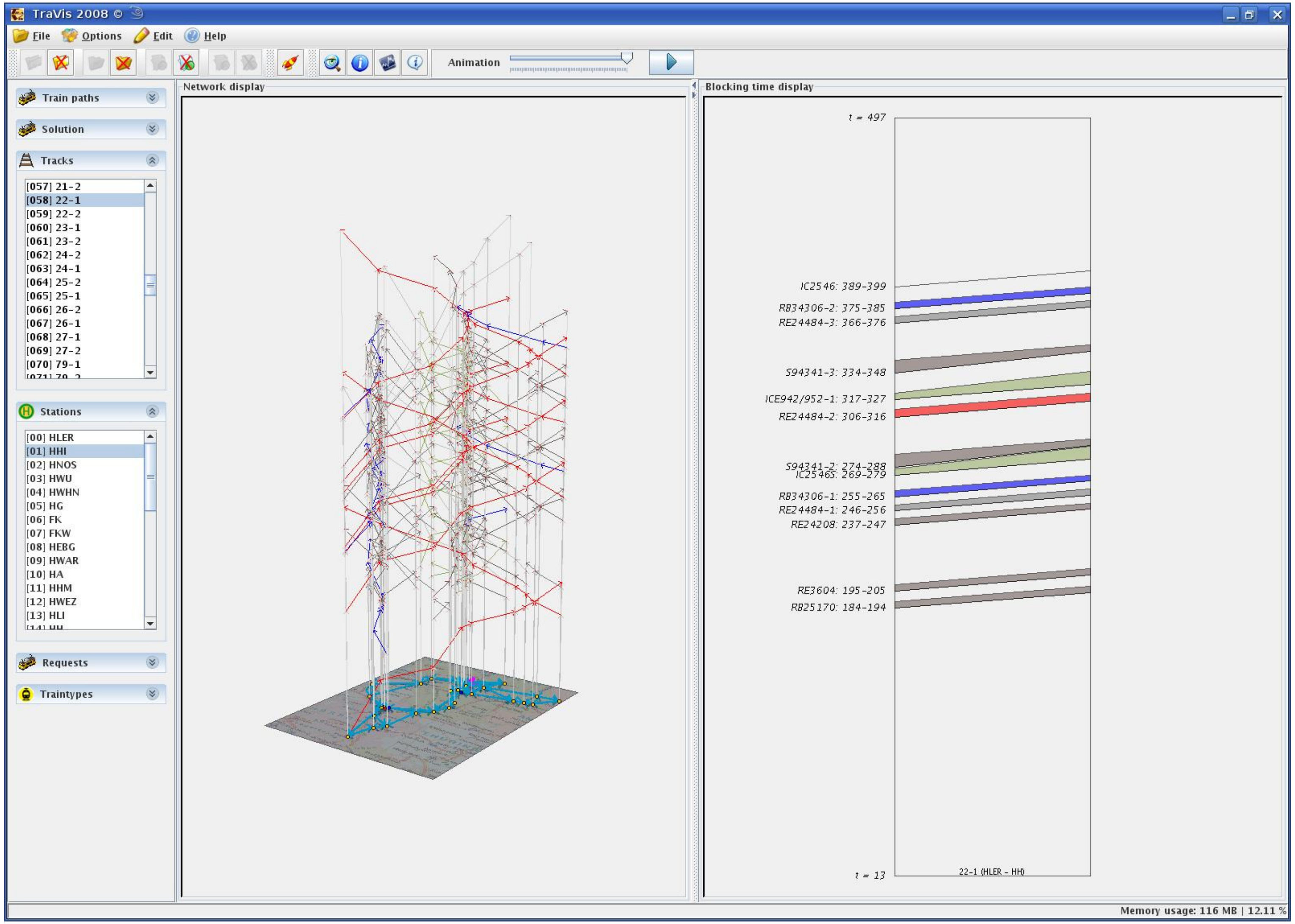
Solution - A Macroscopic Timetable

- List of scheduled train paths
  - sorted list of used macroscopic tracks
  - arrival times at visiting stations
  - departure times at visiting stations

MacroTimetable.xsd



3d Schedule by TraVis (by B.Erol)



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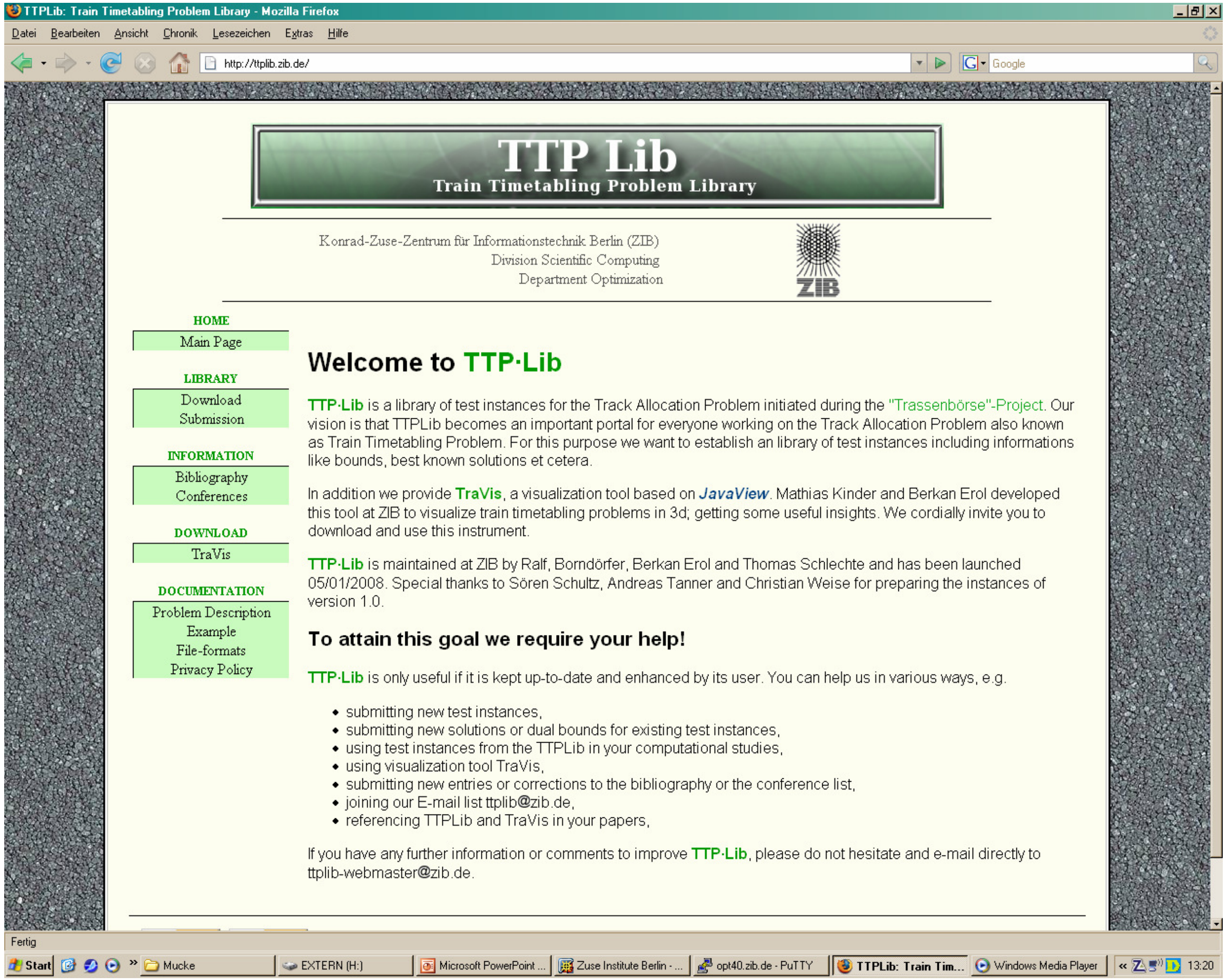
TTPLib 2008 *http://ttplib.zib.de*



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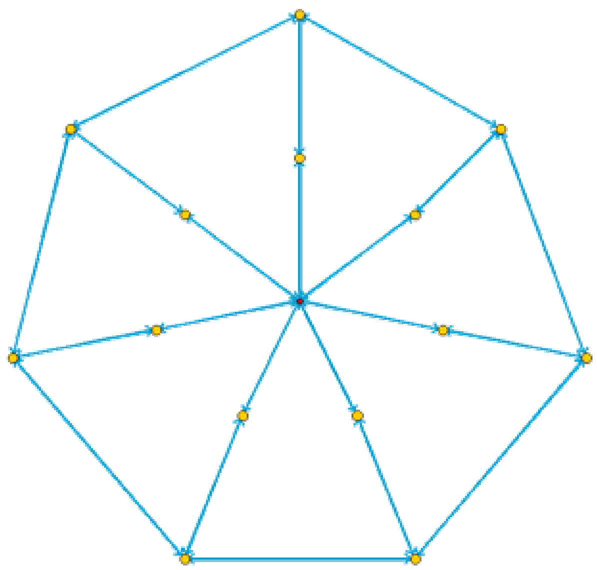
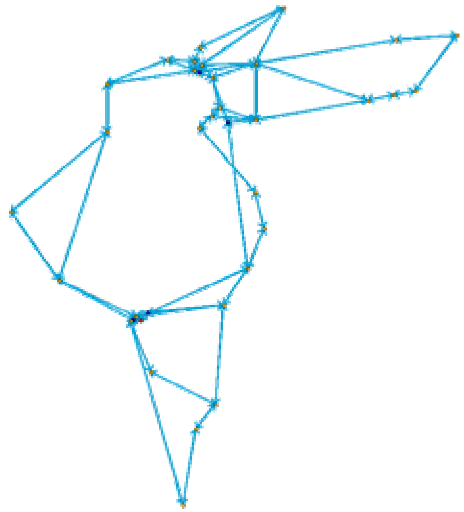
TTPLib 2008 - Dummy Instances



- network wheel
- 11 train request scenarios
- contributed by A.Tanner (WIP – TU Berlin)
- 10/11 solved to optimality by TS-OPT

TTPLib 2008 - Hanover/Kassel/Fulda



- network hakafu
- 50 train request scenarios
- contributed by S.Schultz, A.Tanner and A. Reuther
- 42/50 solved to optimality by TS-OPT

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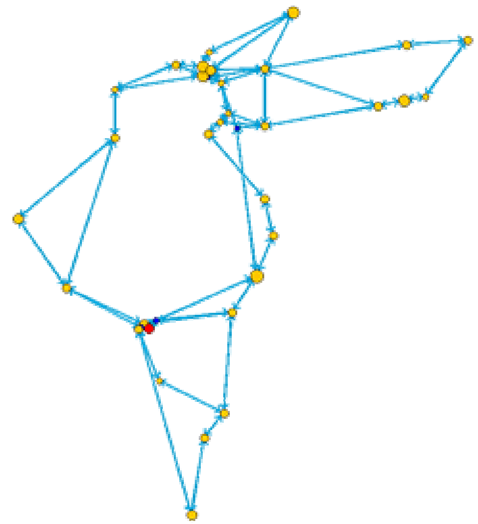
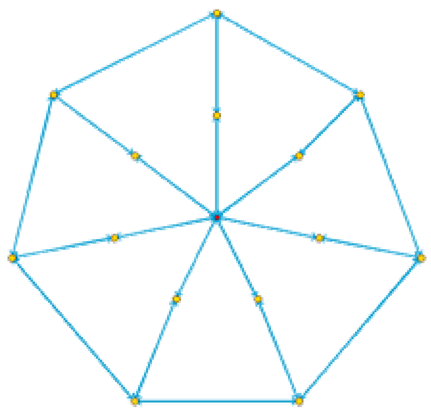
TTPLib 2008 - Hanover/Kassel/Fulda



- network hakafu_stations
- 50 train request scenarios
- contributed by S.Schultz, A.Tanner and A. Reuther
- 42/50 solved to optimality by TS-OPT

TTPLib 2008 – wheel_req10.xml



APP Cplex:

0096: Tsopt_CPLEX_Msg(): 36000 2311 572.8283 734 568.3799 680.5584 7685482 19.74%

0096: Tsopt_CPLEX_Msg(): Elapsed real time =

0096: Tsopt_CPLEX_Msg(): 6335.98 sec. (tree size = 14.52 MB

0096: Tsopt_CPLEX_Msg(): , solutions = 38

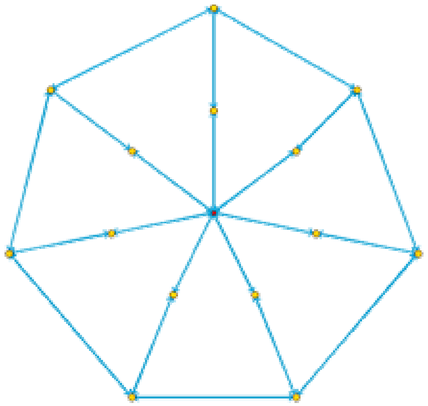
0096: Tsopt_CPLEX_Msg():)

0096: Tsopt_CPLEX_Msg(): 36050 2315 cutoff 568.3799 680.5584 7694860 19.74%

0096: Tsopt_CPLEX_Msg(): 36100 2317 cutoff 568.3799 680.5584 7701564 19.74%

0096: Tsopt_CPLEX_Msg(): 36150 2325 570.6950 530 **568.3799** 680.5584 7709180 **19.74%**

TTPLib 2008 – wheel_req10.xml



ACP Cplex:

0096: Tsopt_CPLEX_Msg(): 400 93 595.5520 1602 595.2397 630.6793 243827 5.95%

0096: Tsopt_CPLEX_Msg(): 450 87 597.7870 522 595.2397 630.6793 258572 5.95%

0096: Tsopt_CPLEX_Msg(): 500 103 cutoff 595.2397 630.6793 278680 5.95%

0096: Tsopt_CPLEX_Msg(): Elapsed real time =

0096: Tsopt_CPLEX_Msg(): 94.16 sec. (tree size = 0.24 MB

0096: Tsopt_CPLEX_Msg(): , solutions = 2

...

0096: Tsopt_CPLEX_Msg():)

0096: Tsopt_CPLEX_Msg(): 1900 35 620.3472 555 **620.1883** 630.0373 871767 **1.59%**

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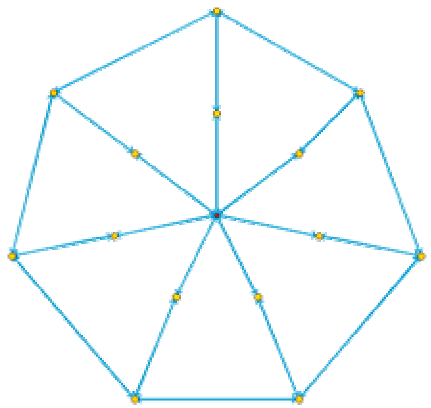
TTPLib 2008 – wheel_req10.xml



ACP SCIP:

time	node	left	LP iter	mem	mdpt	frac	vars	cons	ccons	cols	rows	cuts	confs	strbr	dualbound	primalbound	gap
2963m	294600	109	99418k	97M	58	-	11k	8692	8398	11k	8325	78	27k	105k	6.233377e+02	6.223653e+02	0.16%
2964m	294700	57	99439k	94M	58	-	11k	8683	8519	11k	8325	78	27k	105k	6.232824e+02	6.223653e+02	0.15%

SCIP Status : problem is solved [optimal solution found]

Solving Time (sec) : 177894.99

Solving Nodes : 294763

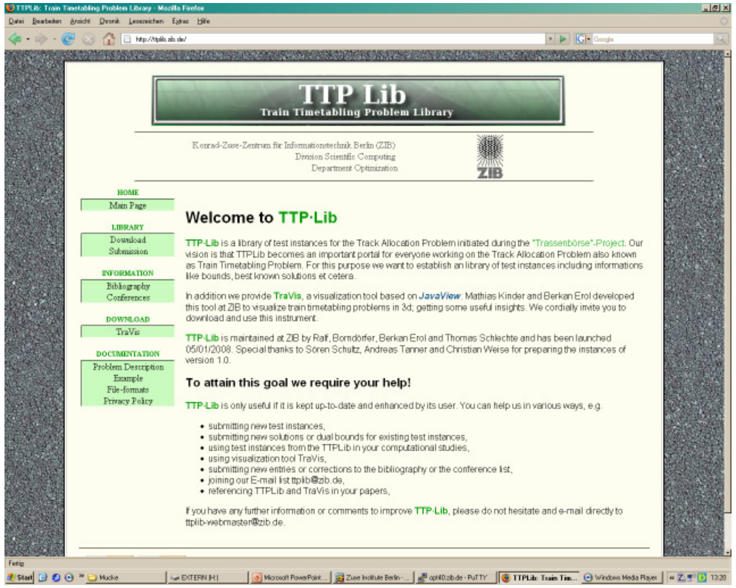
Primal Bound : +6.22365324690698e+02 (103 solutions)

Dual Bound : +6.22365324690698e+02

Gap : 0.00 %

Thomas Schlechte

Please Contribute - Improve TTPLib !



By submitting
- new test instances
- new solutions or upper bounds
- new entries to the bibliography and conference list

By using
- instances from the TTPLib in your computational studies
- free visualization tool TraVis

Thank you for your attention !

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