



Federal Ministry  
of Economics  
and Technology

# Timetabling with TS-OPT

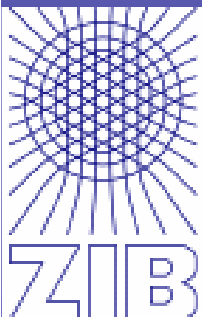
**Thomas Schlechte**

**joint work with**

**Ralf Borndörfer and Berkan Erol**

*RailZurich2009*

*12.02.2009*



Thomas Schlechte    Zuse-Institute-Berlin Berlin (ZIB)

[schlechte@zib.de](mailto:schlechte@zib.de)    <http://www.zib.de/schlechte>

# Zuse-Institute-Berlin (ZIB)

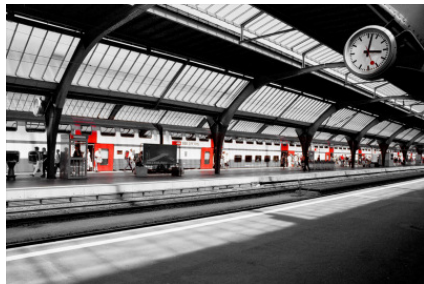
Konrad Zuse was the creator of the first fully automatic, programm controlled and freely programmable computer working in binary floating point arithmetic. The Z3 was finished in 1941.



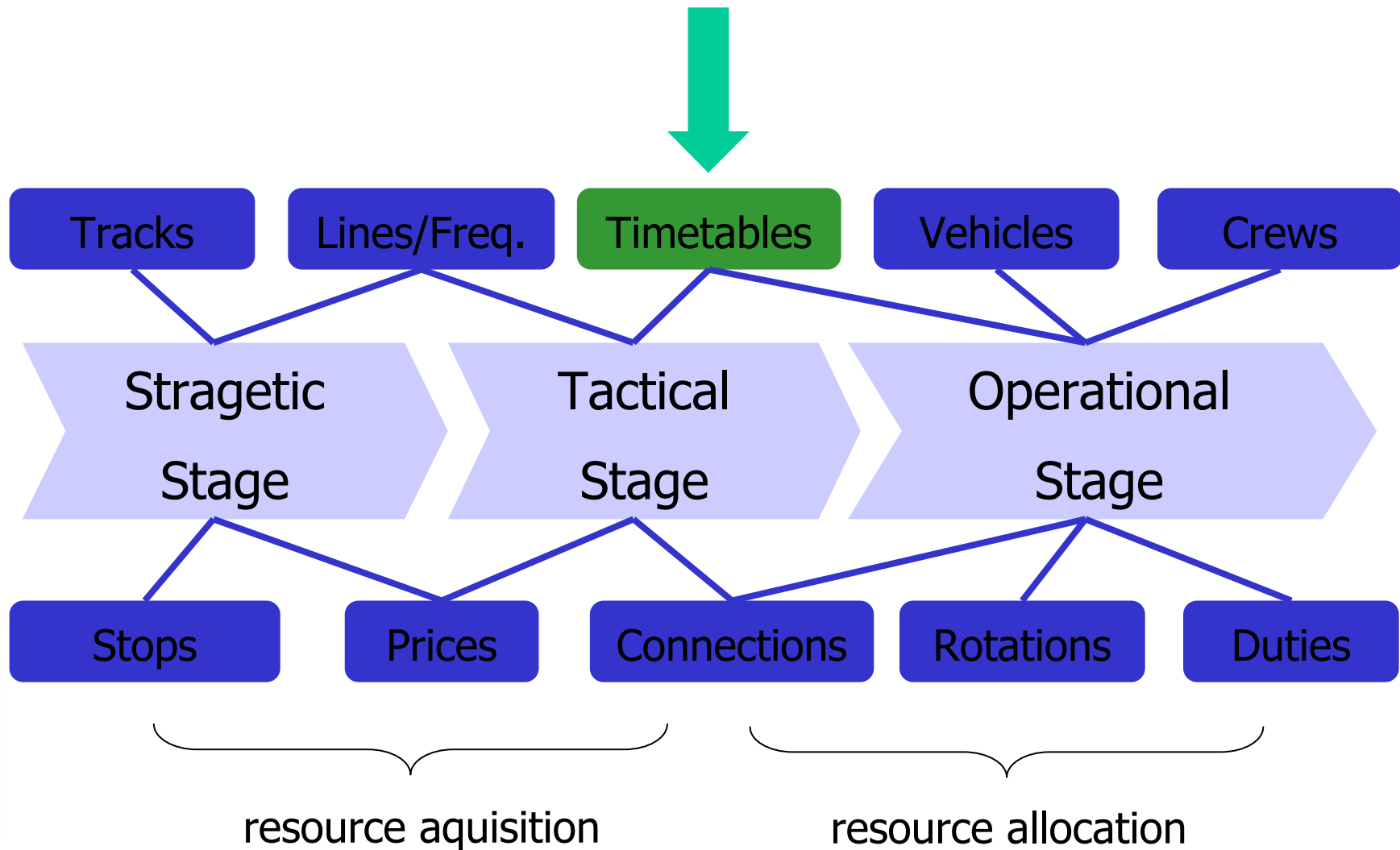
# Overview

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1. Idea & Motivation
2. Train Timetabling Problem
3. Algorithmic Approach
4. TTPLib 2008



# Planning in Public Transport



# Timetabling (PESP & TTP & TPP)

periodic passenger  
versus  
individual cargo traffic

TS-OPT

Train  
Requests

Tracks

Stations

## Optimization Model

*maximize*

track utilization  
timetable attractiveness

*subject to*

safety requirements  
time windows

Timetable

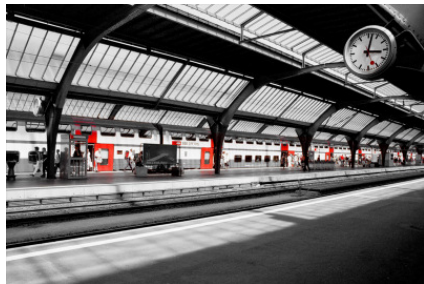




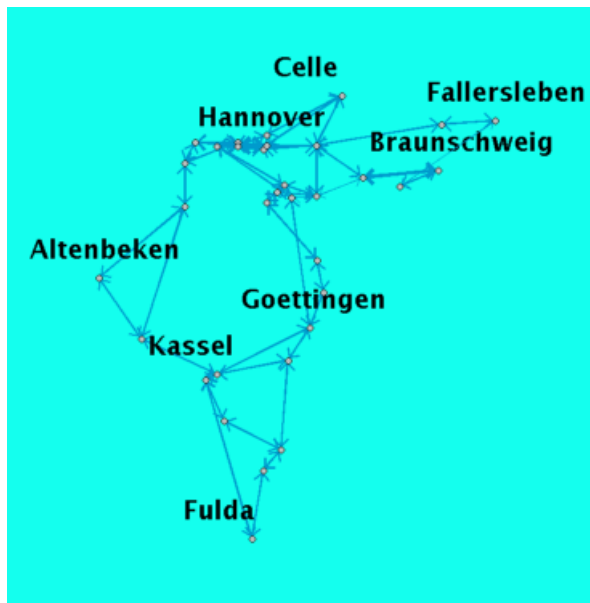
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# Railway Timetabling (TTP) - State of the Art



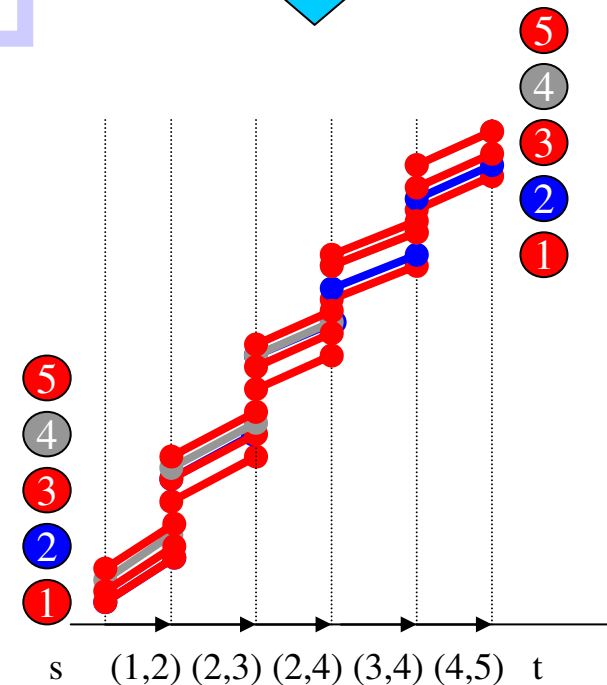
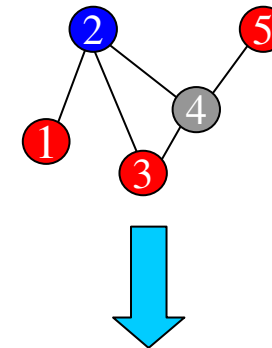
- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Cai and Goh (1994), Schrijver and Steenbeck (1994), Carey and Lockwood (1995)
- Nachtigall and Voget (1996), Odijk (1996) Higgings, Kozan and Ferreira (1997)
- **Brannlund, Lindberg, Nou, Nilsson (1998)**, Lindner (2000), Oliveira and Smith (2000)
- **Caprara, Fischetti and Toth (2002)**, Peeters (2003)
- Kroon and Peeters (2003), Mistry and Kwan (2004)
- Barber, Salido, Ingolotti, Abril, Lova, Tormas (2004)
- Semet and Schoenauer (2005),
- **Caprara, Monaci, Toth and Guida (2005)**
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006),
- **Cacchiani, Caprara, T. (2006), Cachhiani (2007)**
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- **Borndorfer, Schlechte (2007)**
- **Fischer, Helmberg, Janßen, Krostitz (2008) ...**

**non-cyclic timetabling literature**

# Computational Complexity

**Proposition** [Caprara, Fischetti, Toth (02)]:  
OPTRA/TTP is  $\mathcal{NP}$ -hard.

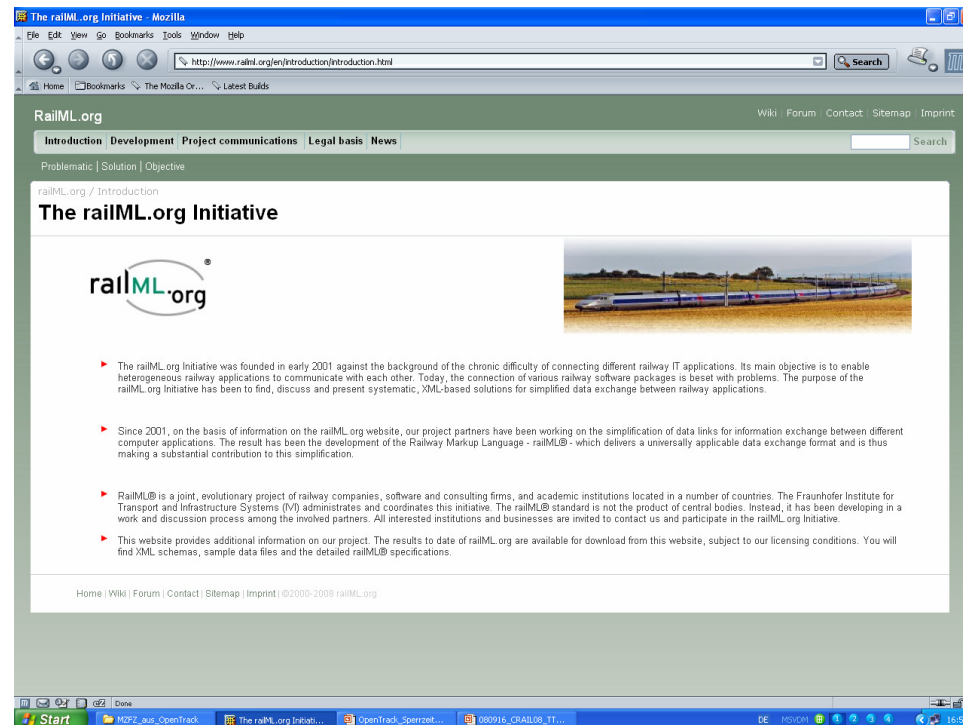
**Proof:**  
Reduction from Independent-Set.



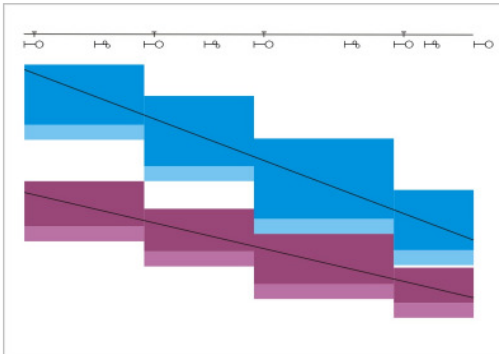


# Microworld (*http://www.railml.org*)

- RailML Standardization
  - Infrastructure (lines, switches, signals, gradient ...)
  - Rolling Stock (engine, brakes, wagon ...)
  - Timetable



# From Microscopy to Macroscopy

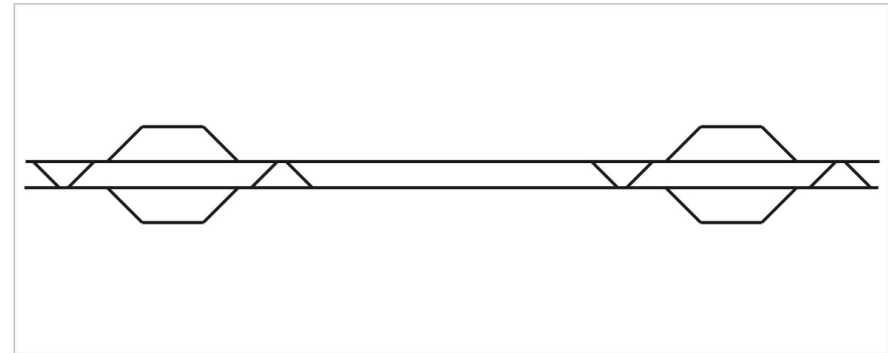


Block & Signal System



Headways

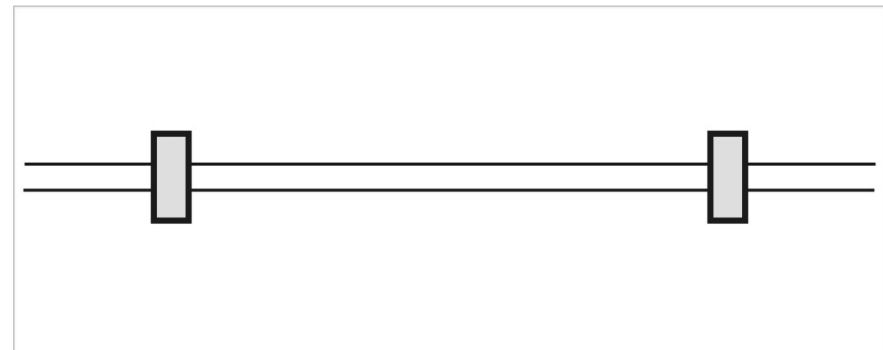
h	$T_1$	$T_2$	d
$T_1$	2	3	4
$T_2$	1	1	2






Station Layouts



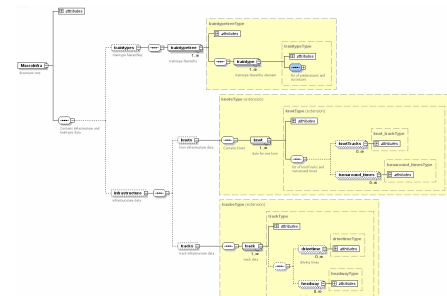
Station Capacities



# Macroscopic Infrastructure

- Traintypes 
- Stations (standard, deadend or pseudo)
  - Capacities (per traintype, per mode) 
  - Turnaroundtimes (per traintype)
- Tracks (connecting sides of stations) 
  - Driving Times (per traintype, per mode)
  - Headway Times (per traintype combination)

***MacroInfra.xsd***

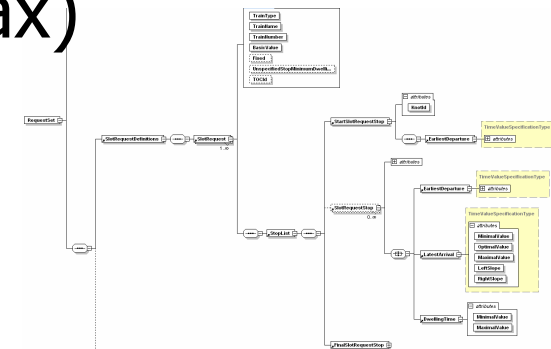


# Train Requests

- Traintype and utility value
- Departure Station
  - preferable time window (min,opt,max)
- Arrival Station
  - preferable time window (min,opt,max)
- Optional Stops
  - preferable arrival time window
  - preferable departure time window
  - preferable dwell time (min,max)

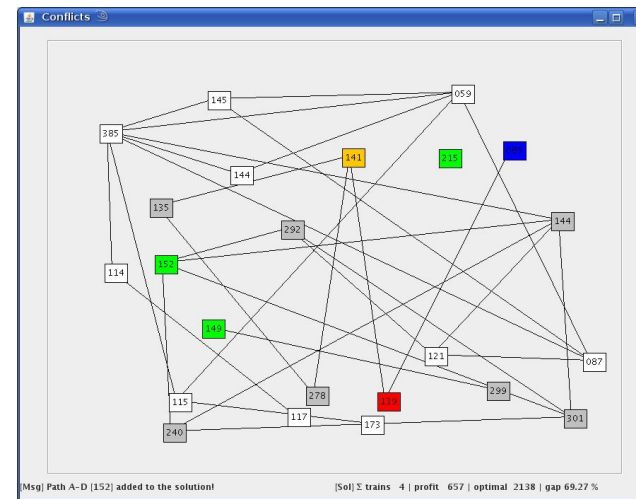
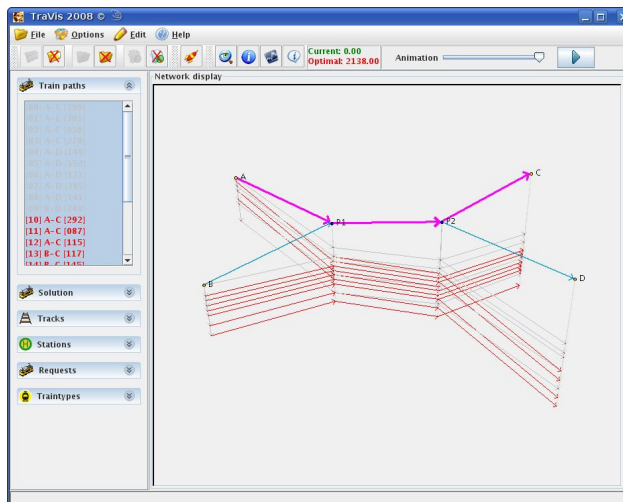


***RequestSet.xsd***



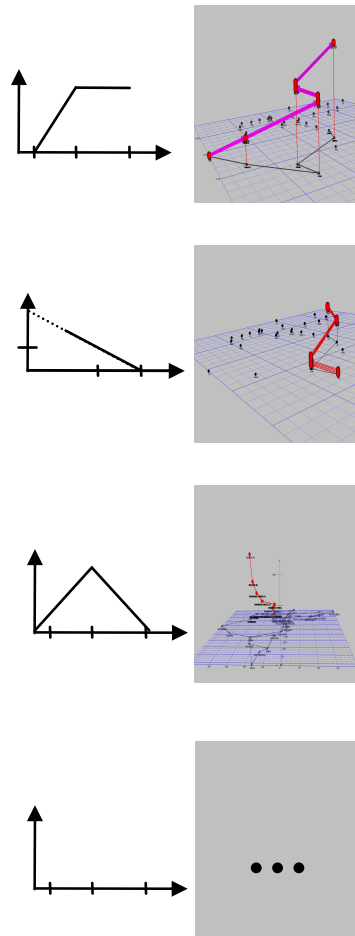
# Train Timetabling Problem

- $I$  - set of train request
- $P_i$  - set of railway slots for request  $i \in I$
- $C$  - conflict sets,  $\{(P_q \in 2^P, \kappa_q)\}$
- $u_p^i$  - utility of  $i \in I$  for  $p \in P$

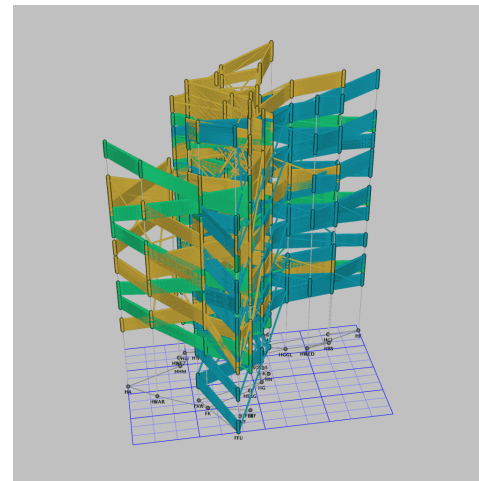




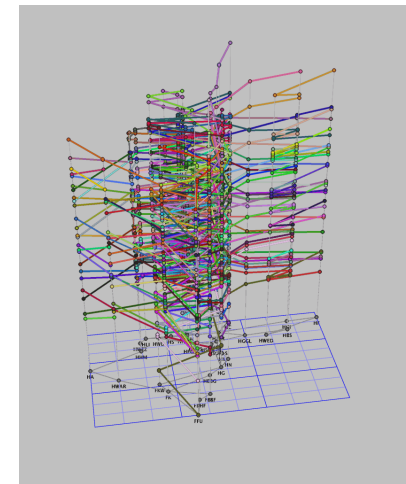
# Optimization Approach



Train Requests



Scheduling Digraph

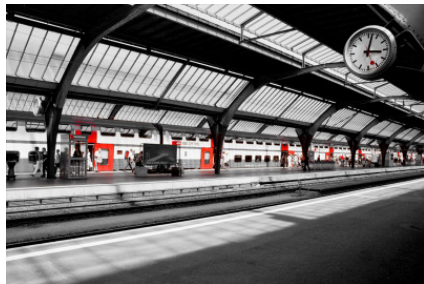


Timetable

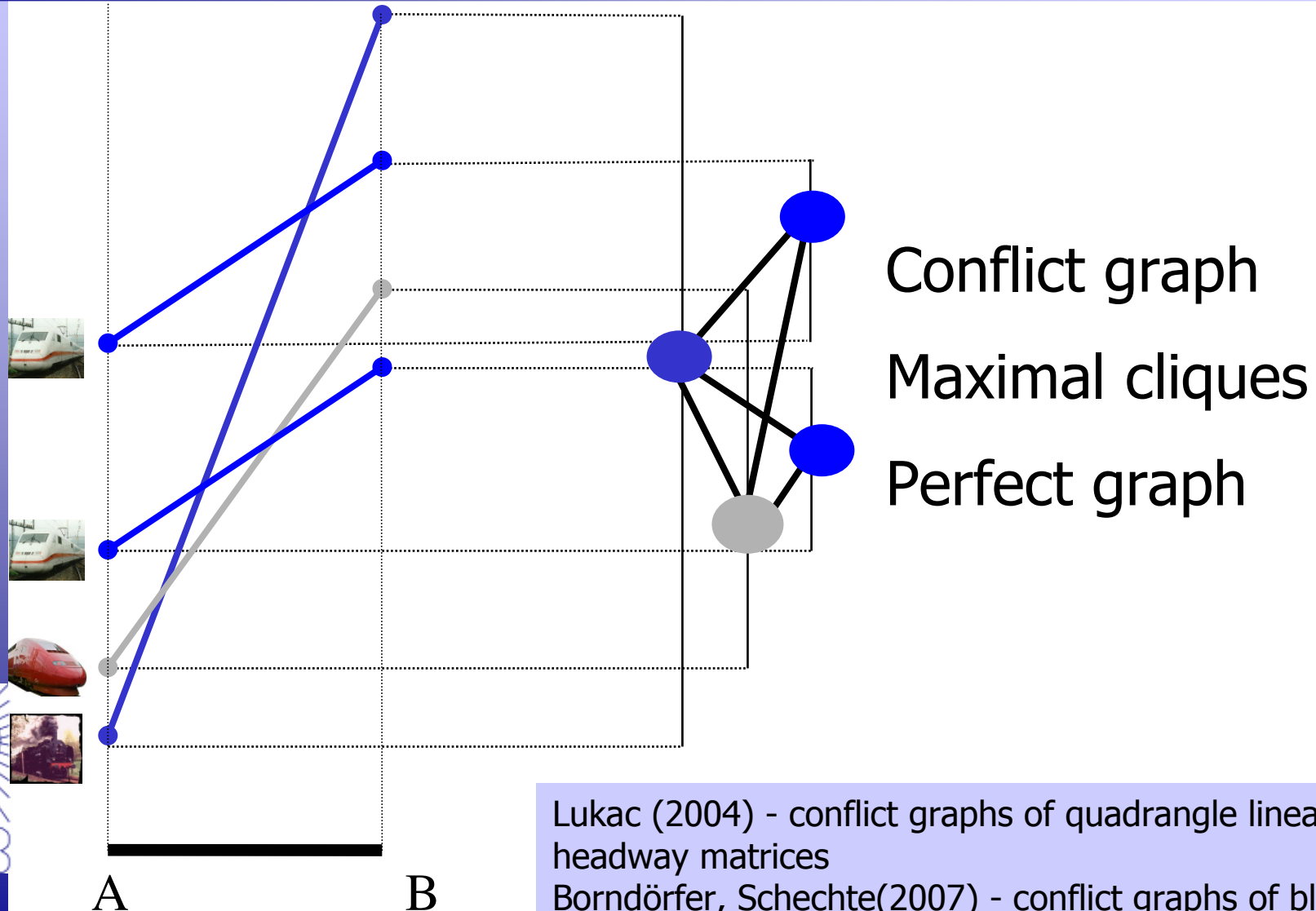
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# Packing Models



Lukac (2004) - conflict graphs of quadrangle linear headway matrices

Borndörfer, Schechte(2007) - conflict graphs of block occupation (interval graphs)

# TTP as Path Packing Problem

$$\begin{array}{ll}
 (PPP) & \\
 \max & \sum_{i \in \mathcal{I}} \sum_{p \in P_i} u_p^i x_p^i \\
 \text{s.t.} & \sum_{i \in \mathcal{I}} x_p^i \leq 1 \quad \forall i \in \mathcal{I} \quad (i) \\
 & \sum_{p \in c} x_p^i \leq \kappa_c \quad \forall c \in C \quad (ii) \\
 & x_p^i \in \{0, 1\} \quad \forall p \in P_i, \forall i \in \mathcal{I} \quad (iii)
 \end{array}$$

## Variables

- Path usage (request  $i$  uses path  $p$ )

## Constraints

- Do not violate conflict sets

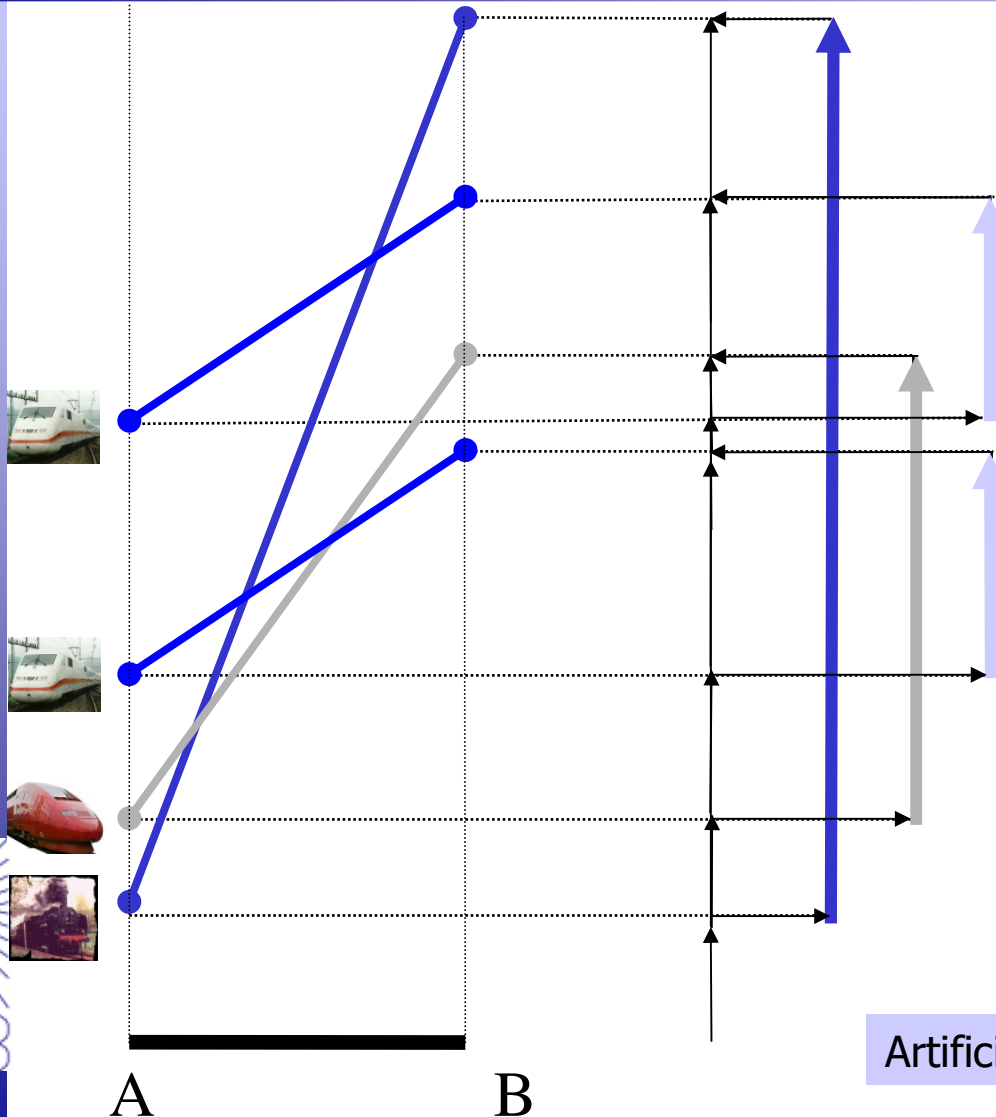
## Objective

- Maximize proceedings/utility

PhD Thesis V.Cachhiani (2007) - details on TTP formulated as (APP) and (PPP)



# Novel Model



Track Digraph

Timeline(s)

Config paths

Artificial arcs represent valid headways !



# Path Coupling Problem

(PCP)

$$\begin{array}{ll}
 \max & \sum_{i \in I} \sum_{p \in \mathcal{P}_i} u_p^i x_p^i \\
 \text{s.t.} & \sum_{p \in \mathcal{P}_i} x_p \leq 1 \quad \forall i \in I \quad \text{(i)} \\
 & \sum_{q \in \mathcal{Q}_j} y_q \leq 1 \quad \forall j \in J \quad \text{(ii)} \\
 & \sum_{a \in \mathcal{P}} x_p - \sum_{a \in \mathcal{Q}} y_q \leq 0 \quad \forall a \in A_I \cap A_J \quad \text{(iii)} \\
 & y_q \in \{0, 1\} \quad \forall q \in \mathcal{Q}_j, \forall j \in J \quad \text{(iv)} \\
 & x_p \in \{0, 1\} \quad \forall p \in \mathcal{P}_i, \forall i \in I \quad \text{(v)}
 \end{array}$$

## Variables

- Path und config usage (request  $i$  uses path  $p$ , track  $j$  uses config  $q$ )

## Constraints

- Path and config choice
- Path-config-coupling (track capacity)

## Objective Function

- Maximize proceedings



# Linear Relaxation of PCP

(MLP)

max

$$\sum_{i \in I} \sum_{p \in P_i} u_p^i x_p^i$$

s.t.

$$\sum_{p \in P_i} x_p \leq 1 \quad \forall i \in I \quad (i)$$

$$\sum_{q \in Q_j} y_q \leq 1 \quad \forall j \in J \quad (ii)$$

$$\sum_{a \in p \in \mathcal{P}} x_p - \sum_{a \in q \in \mathcal{Q}} y_q \leq 0 \quad \forall a \in A_I \cup A_J \quad (iii)$$

$$0 \leq y_q \leq 1 \quad \forall q \in \mathcal{Q} \quad (iii)$$

$$0 \leq x_p \leq 1 \quad \forall p \in \mathcal{P} \quad (iv)$$

$\gamma_i$   
 $\pi_j$   
 $\lambda_a$

dual variable	information about	useful to
$\gamma_i$	bundle price	analyse request
$\pi_j$	track price	analyse network
$\lambda_a$	arc price	-





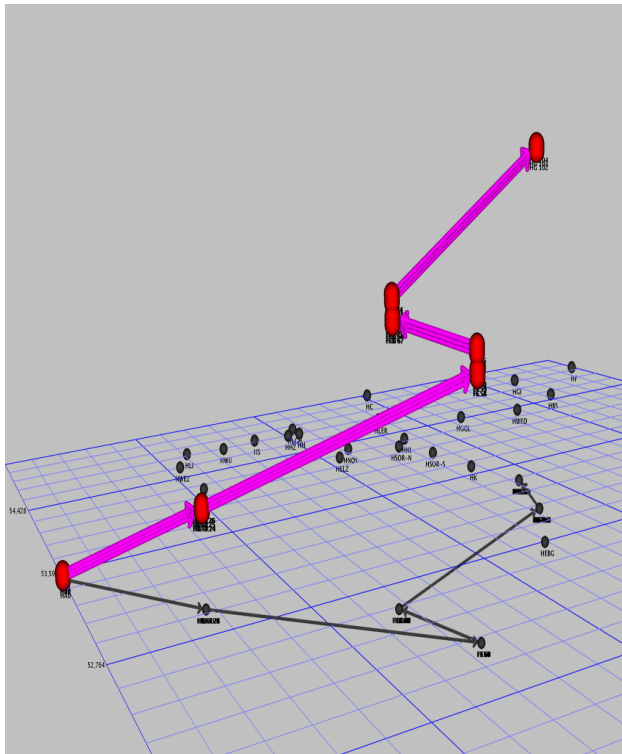
# Dualization

$$\begin{array}{ll}
 (DLP) & \\
 \min & \sum_{j \in J} \pi_j + \sum_{i \in I} \gamma_i \\
 \text{s.t.} & \gamma_i + \sum_{a \in p} \lambda_a \geq \sum_{a \in p} u_a^i \quad \forall p \in \mathcal{P}_i, \forall i \in I \quad (\text{i}) \\
 & \pi_j - \sum_{a \in q} \lambda_a \geq 0 \quad \forall q \in \mathcal{Q}_j, \forall j \in J \quad (\text{ii}) \\
 & \gamma_i \geq 0 \quad \forall i \in I \quad (\text{iii}) \\
 & \lambda_a \geq 0 \quad \forall a \in A_I \cup A_J \quad (\text{iv}) \\
 & \pi_j \geq 0 \quad \forall j \in J \quad (\text{v})
 \end{array}$$



# Pricing of x-variables

$$(\text{PRICE}(x)) \quad \exists \bar{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a \in \bar{p}} (p_a - \lambda_a)$$

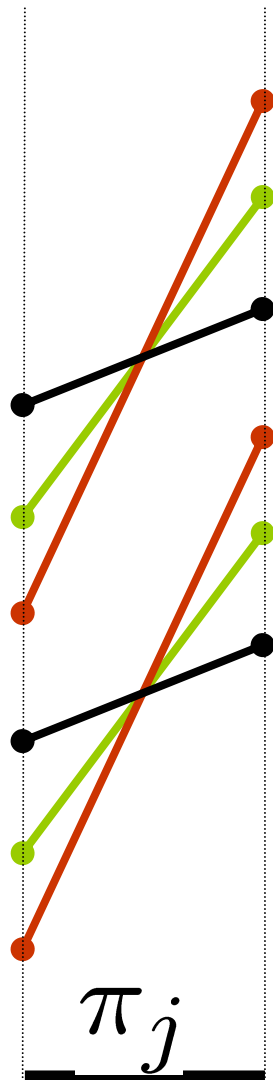


$$\gamma_i$$

$$c_a = -p_a + \lambda_a$$

Pricing Problem(x) :  
 Acyclic shortest path problems  
 for each slot request  $i$  with  
 modified cost function  $c$  !

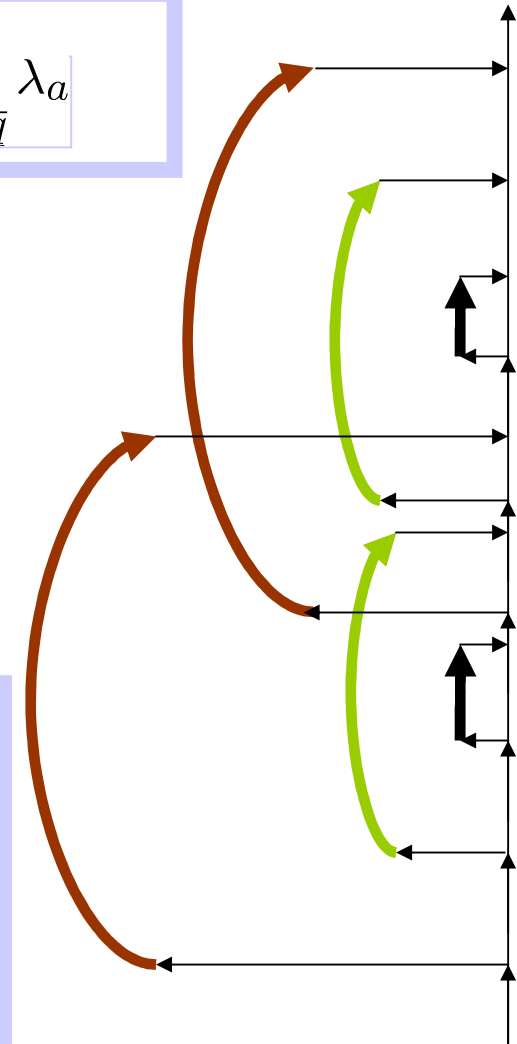
# Pricing of $y$ -variables



$$(\text{PRICE } (y)) \quad \exists \bar{q} \in Q_j : \pi_j < \sum_{a \in \bar{q}} \lambda_a$$

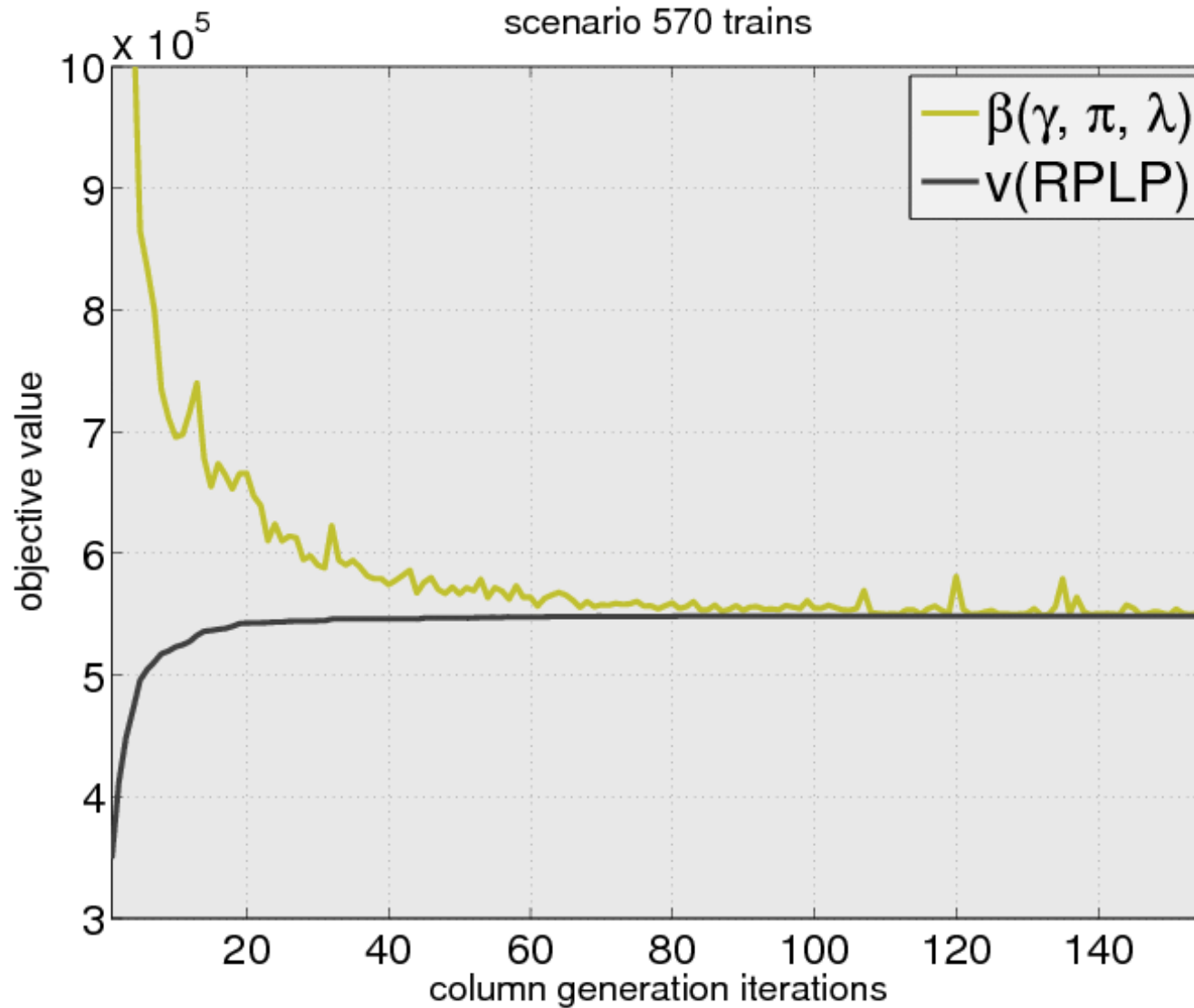
$$c_a = -\lambda_a$$

Pricing Problem(y) :  
Acyclic shortest path problem  
for each track  $j$  with modified  
cost function  $c$  !



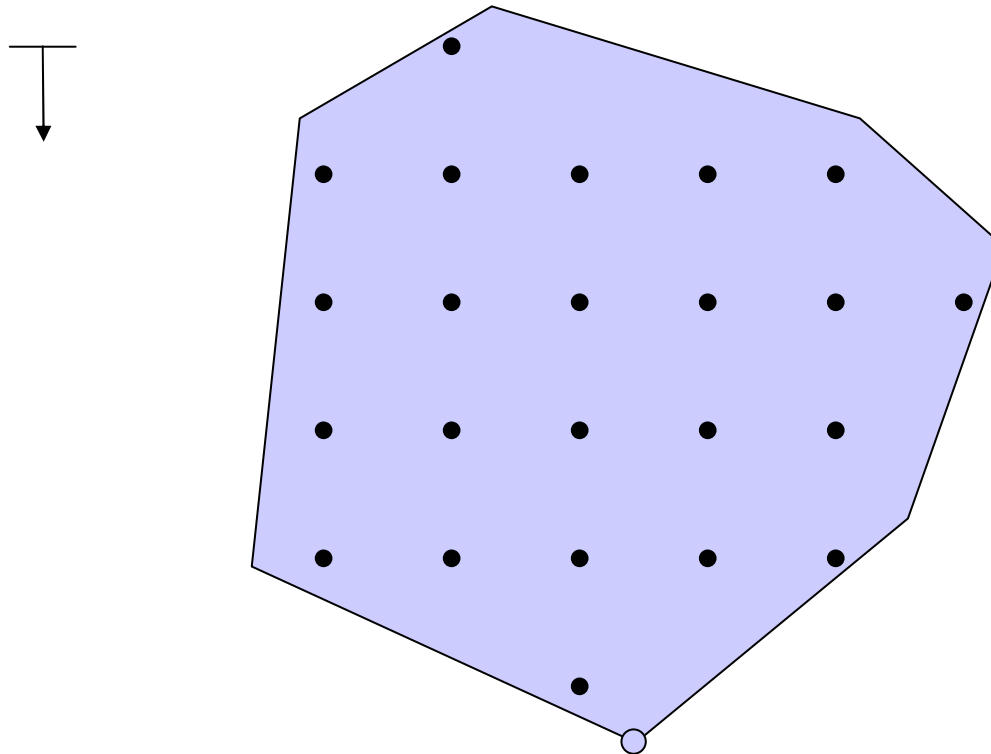


# PCP-Run of TS-OPT /LP Stage



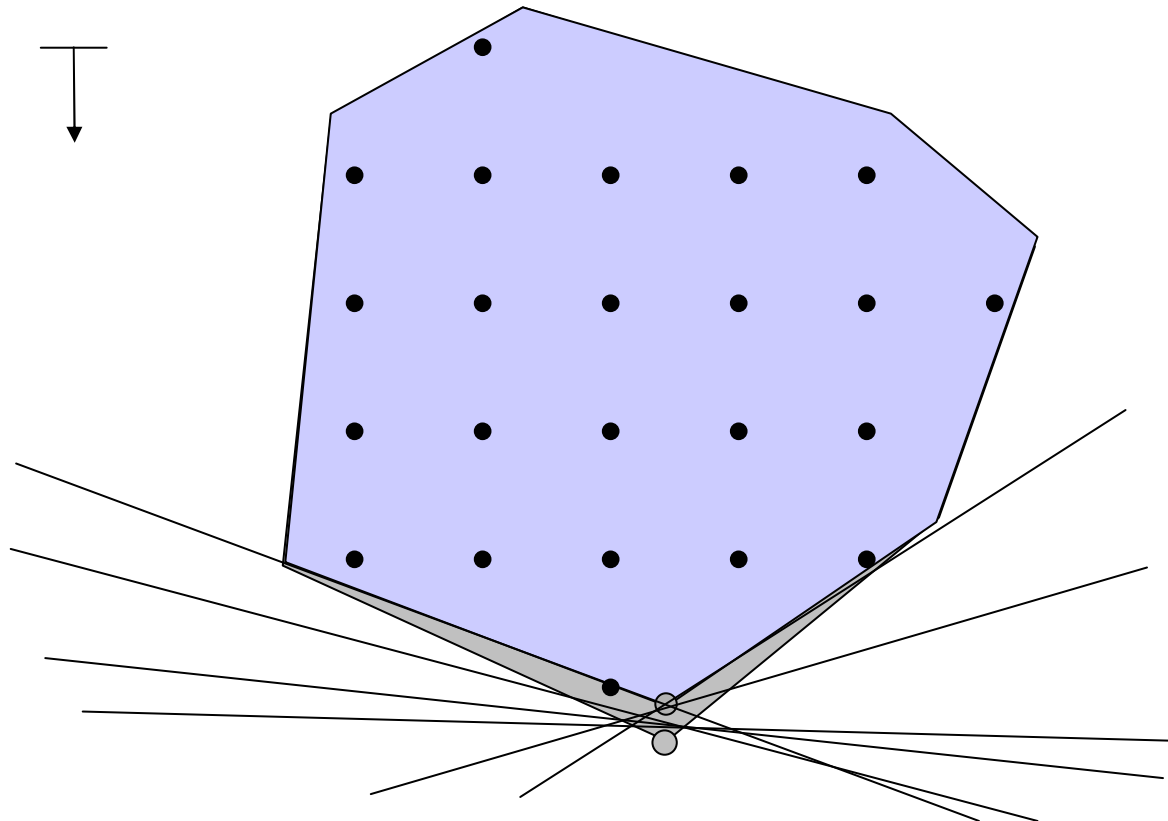
# Linear Relaxation

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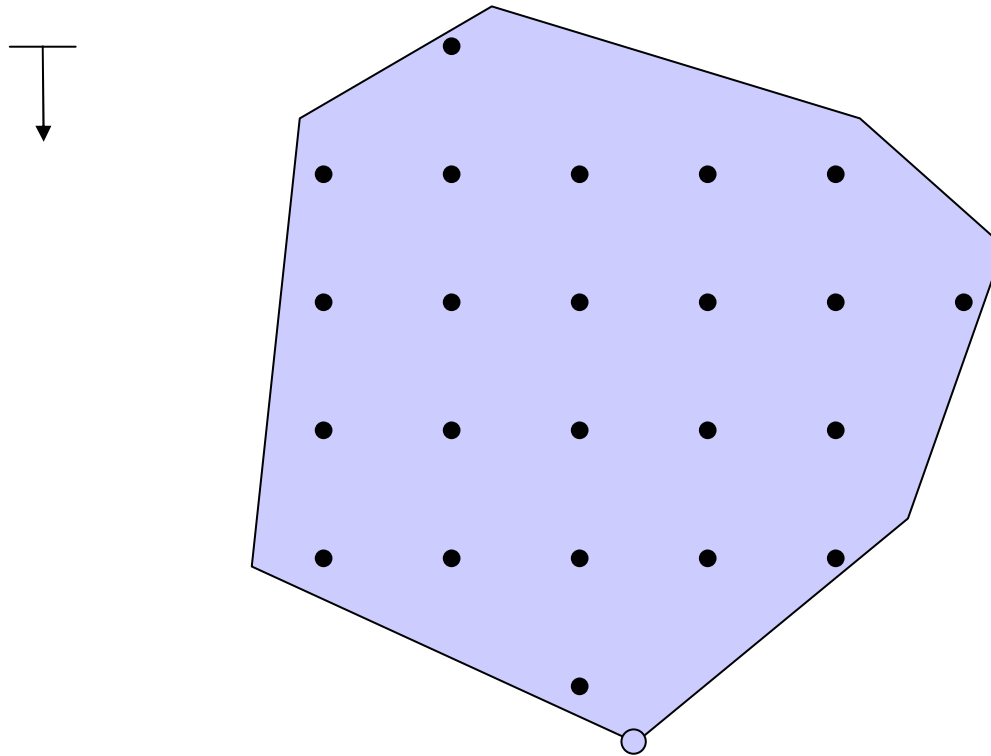
solve linear program by primal, dual simplex or barrier (interior point method)

# Cutting Planes



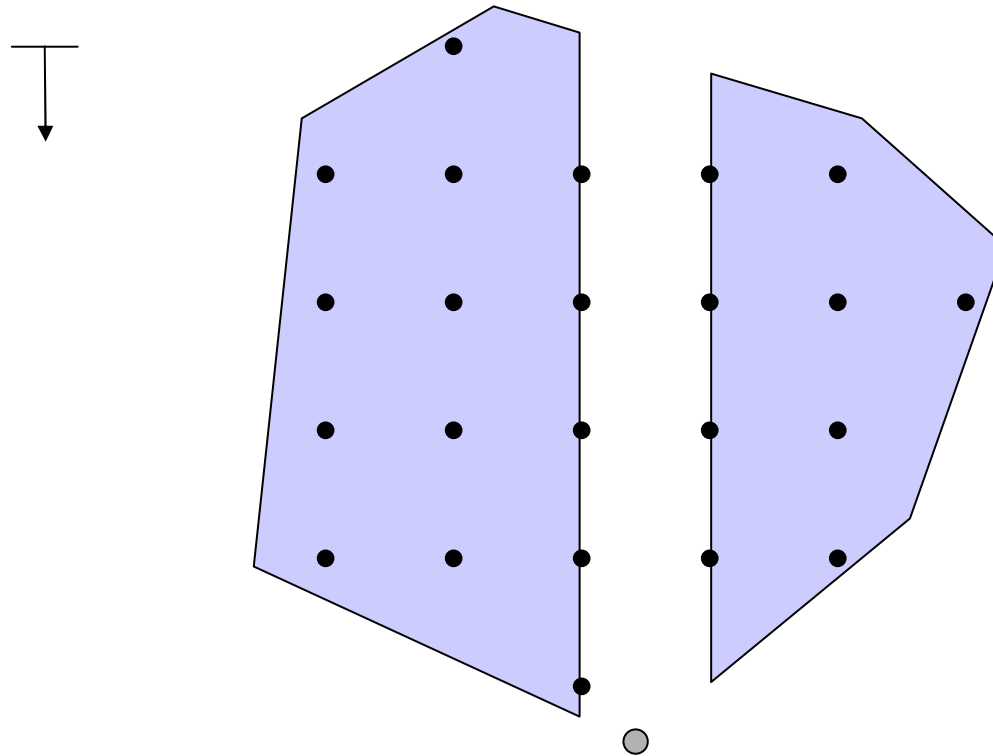
current solution has fractional variables  
improve by valid cutting planes

# Branching



current solution is infeasible (at least one forbidden fractional value)

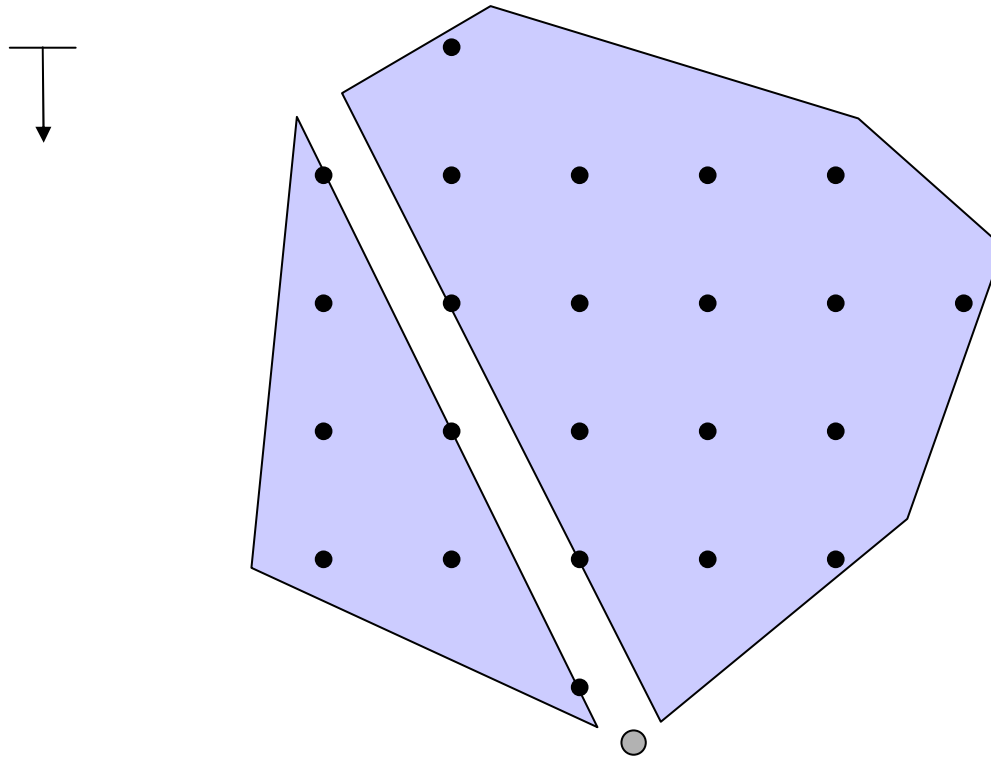
# Branching on Variables



split problems into sub problems to cut off current solution

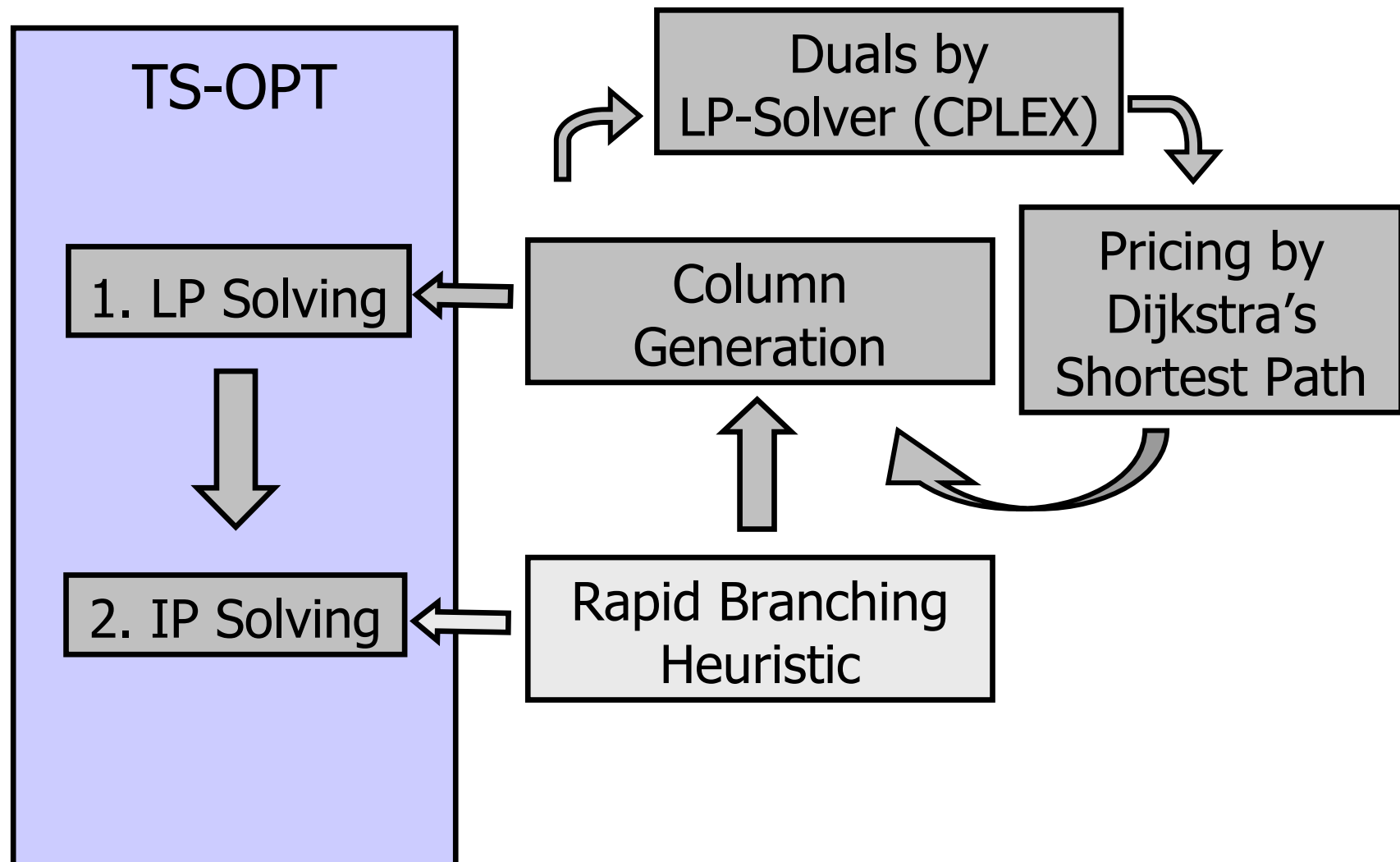


# Branching on Constraints



split problems into subproblems to cut off current solution

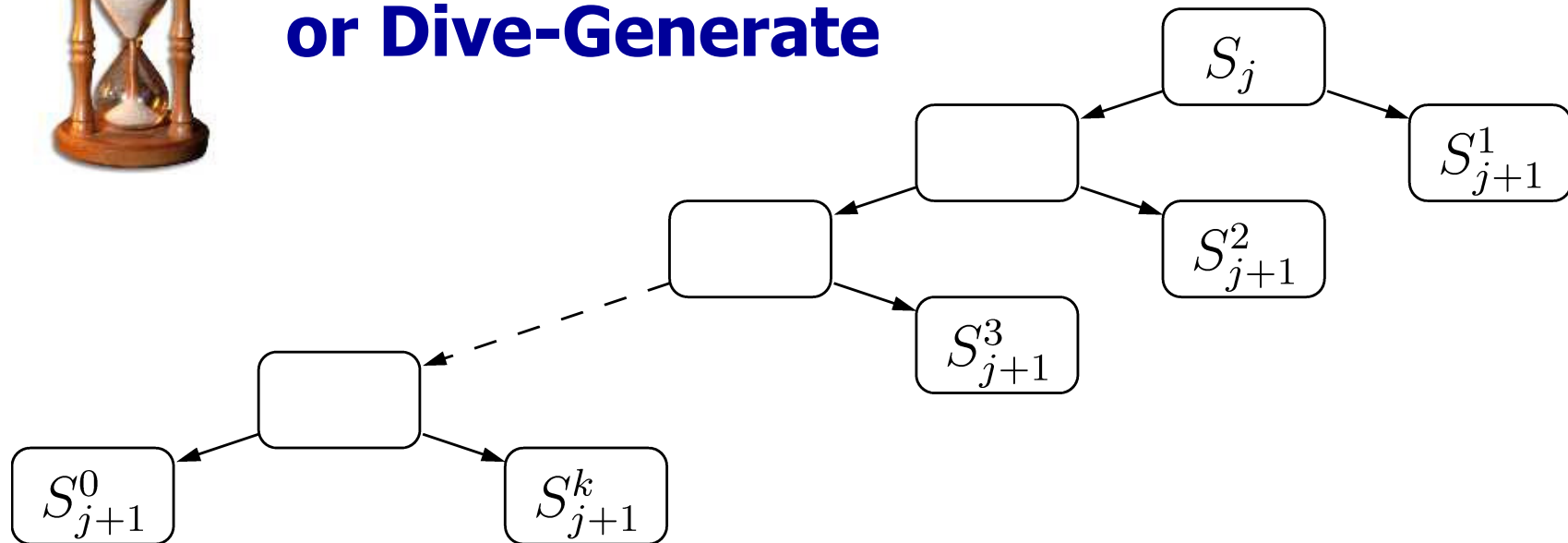
# Two Step Approach



# Branch-Bound-Price



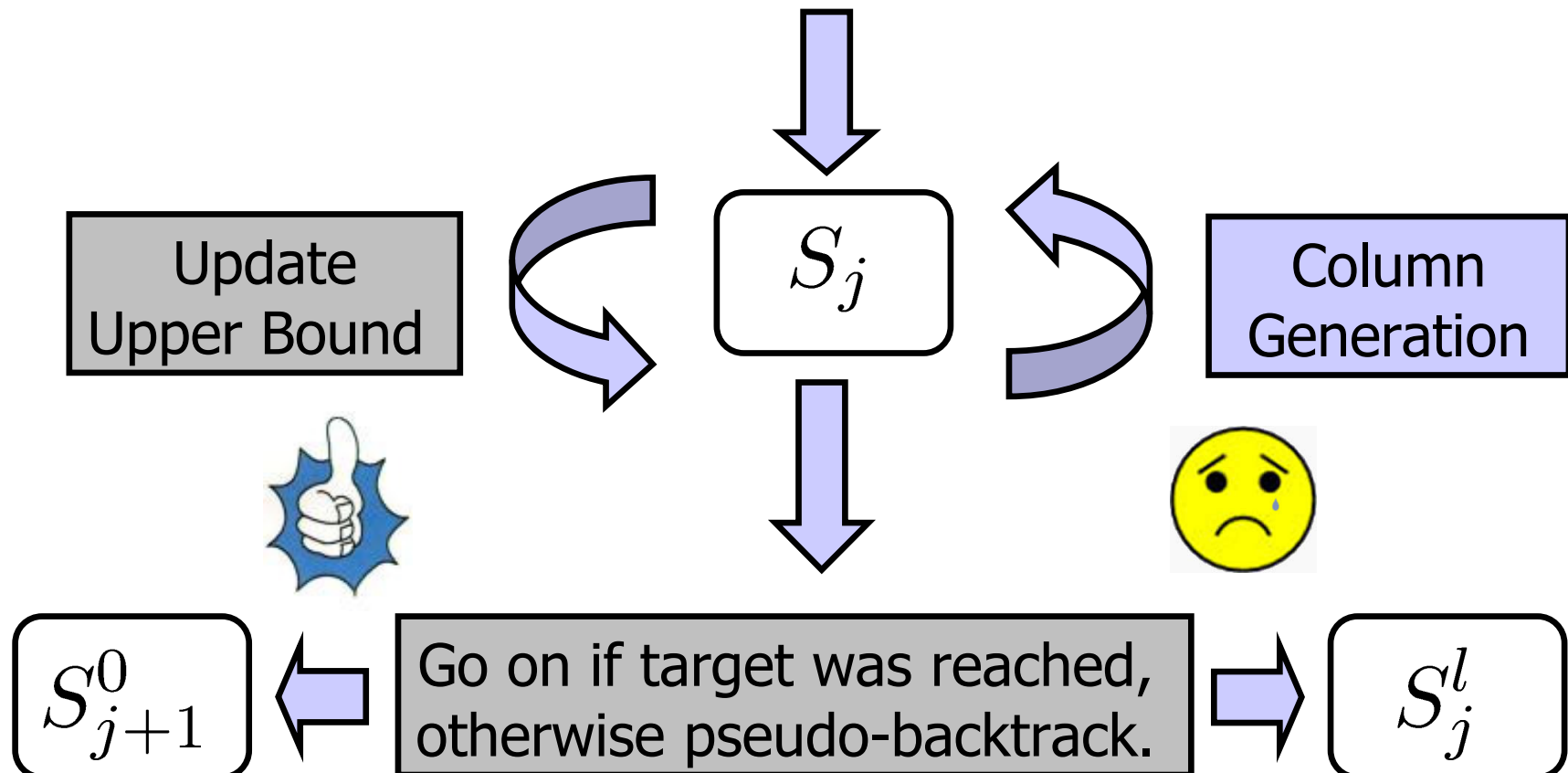
or Dive-Generate



Evaluation of only few highly different sub-problems at iteration  $j$  to reach IP-Solutions fast.

# Rapid Branching (S.Weider 2007)

Node selection of set of fixed to 1 variables by using perturbed cost function (bonus close to 1.0).



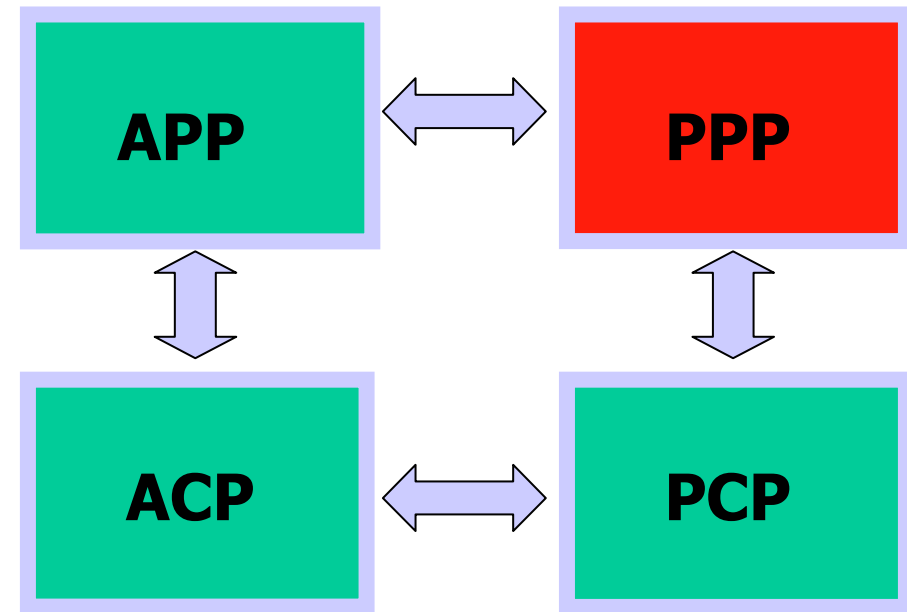
# TTP-Models in TS-OPT

**Theorem[BS08]:** The optimal value ( $v_{LP}(PCP)$ ) of the LP relaxation of PCP can be computed in polynomial time.

**Theorem[BS08]:**

$$v_{LP}(PCP) = v_{LP}(ACP) =$$

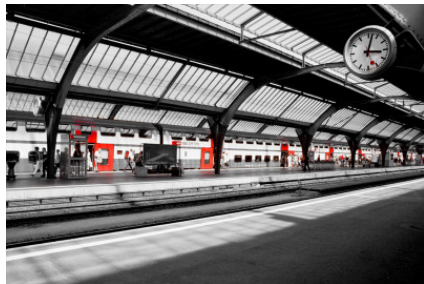
$$v_{LP}(APP) = v_{LP}(PPP).$$



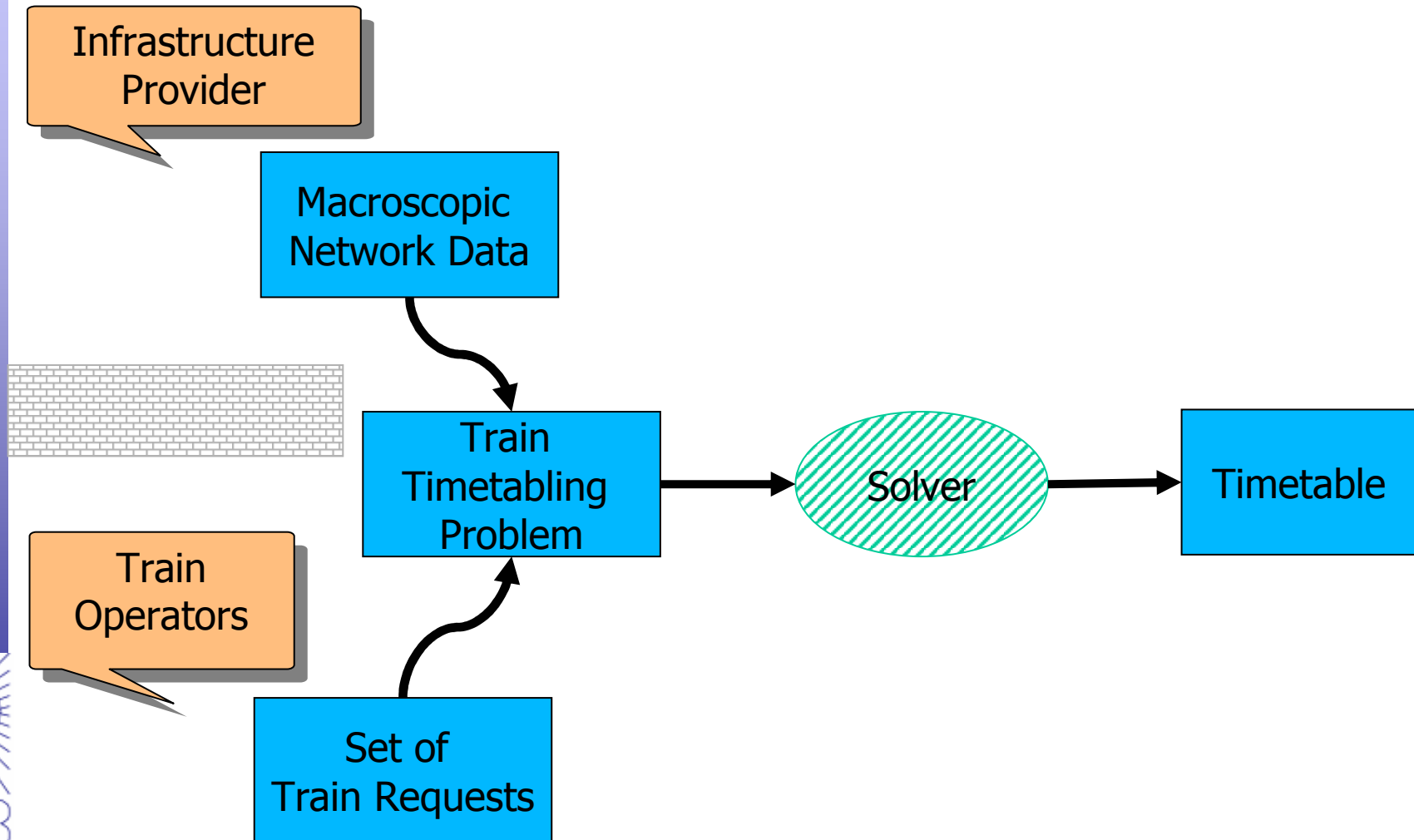
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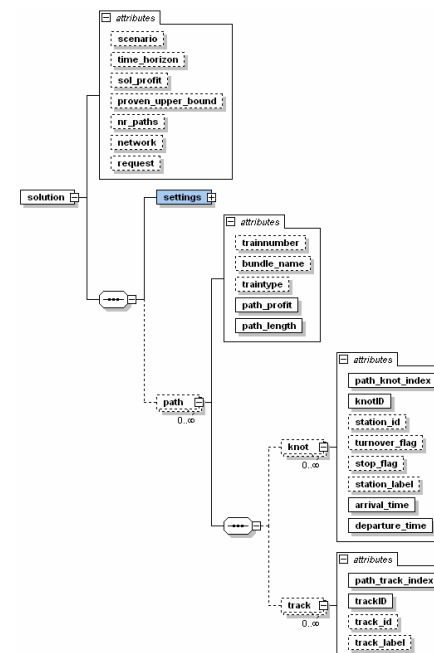


# TTPLib Concept



# Solution - A Macroscopic Timetable

- List of scheduled train paths
  - sorted list of used macroscopic tracks
  - arrival times at visiting stations
  - departure times at visiting stations

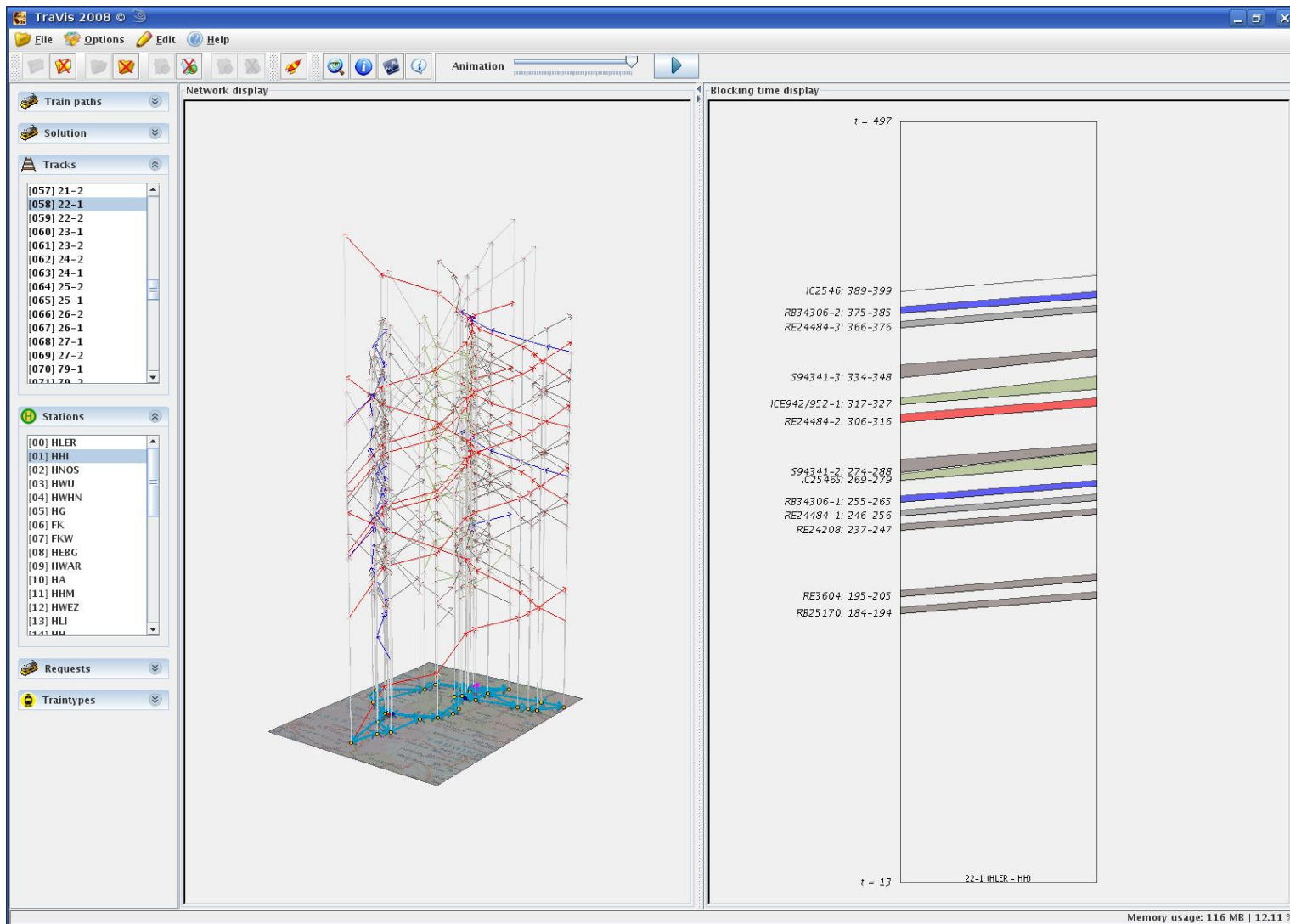


***MacroTimetable.xsd***





# 3d Schedule by TraVis (by B.Erol)



# TTPLib 2008 *http://ttplib.zib.de*

TTPLib: Train Timetabling Problem Library - Mozilla Firefox

Datei Bearbeiten Ansicht Chronik Lesezeichen Extras Hilfe

http://ttplib.zib.de/ Google

## TTP Lib

Train Timetabling Problem Library

Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)  
Division Scientific Computing  
Department Optimization

ZIB

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TraVis

**DOCUMENTATION**  
Problem Description  
Example  
File-formats  
Privacy Policy

### Welcome to TTP-Lib

**TTP-Lib** is a library of test instances for the Track Allocation Problem initiated during the "Trassenbörse"-Project. Our vision is that TTPLib becomes an important portal for everyone working on the Track Allocation Problem also known as Train Timetabling Problem. For this purpose we want to establish an library of test instances including informations like bounds, best known solutions et cetera.

In addition we provide **TraVis**, a visualization tool based on **JavaView**. Mathias Kinder and Berkan Erol developed this tool at ZIB to visualize train timetabling problems in 3d, getting some useful insights. We cordially invite you to download and use this instrument.

**TTP-Lib** is maintained at ZIB by Ralf, Borndörfer, Berkan Erol and Thomas Schlechte and has been launched 05/01/2008. Special thanks to Sören Schultz, Andreas Tanner and Christian Weise for preparing the instances of version 1.0.

### To attain this goal we require your help!

**TTP-Lib** is only useful if it is kept up-to-date and enhanced by its user. You can help us in various ways, e.g.

- submitting new test instances,
- submitting new solutions or dual bounds for existing test instances,
- using test instances from the TTPLib in your computational studies,
- using visualization tool TraVis,
- submitting new entries or corrections to the bibliography or the conference list,
- joining our E-mail list [ttplib@zib.de](mailto:ttplib@zib.de),
- referencing TTPLib and TraVis in your papers,

If you have any further information or comments to improve **TTP-Lib**, please do not hesitate and e-mail directly to [ttplib-webmaster@zib.de](mailto:ttplib-webmaster@zib.de).

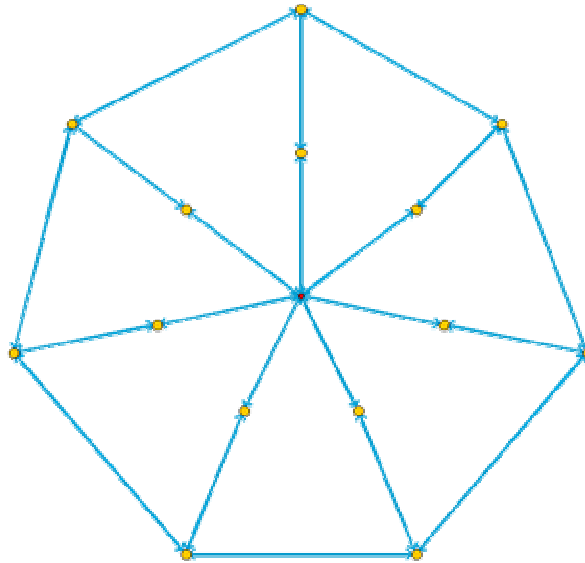
Fertig

Start Mucke EXTERN (H:) Microsoft PowerPoint... Zuse Institute Berlin... opt40.zib.de - PuTTY TTPLib: Train Tim... Windows Media Player 13:20



# TTPLib 2008 - Dummy Instances

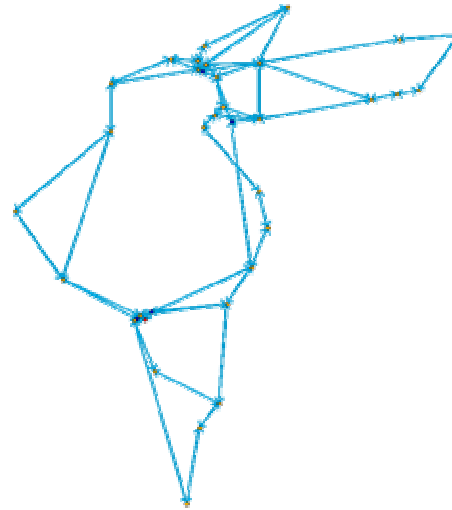
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- network wheel
- 11 train request scenarios
- contributed by A.Tanner ( WIP – TU Berlin)
- 10/11 solved to optimality by TS-OPT

# TTPLib 2008 - Hanover/Kassel/Fulda

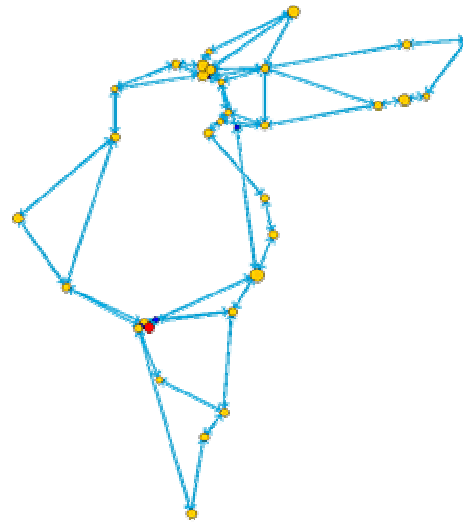
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- network hakafu
- 50 train request scenarios
- contributed by S.Schultz, A.Tanner and A. Reuther
- 42/50 solved to optimality by TS-OPT

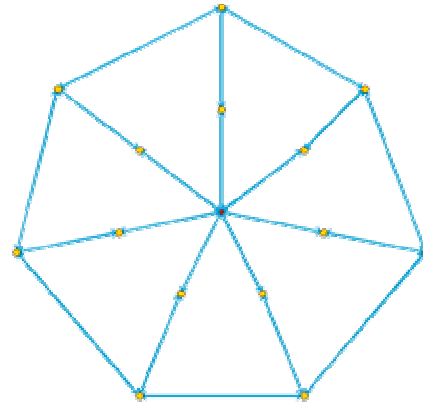
# TTPLib 2008 - Hanover/Kassel/Fulda

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- network hakafu\_stations
- 50 train request scenarios
- contributed by S.Schultz, A.Tanner and A. Reuther
- 42/50 solved to optimality by TS-OPT

# TTPLib 2008 – wheel\_req10.xml



APP Cplex:

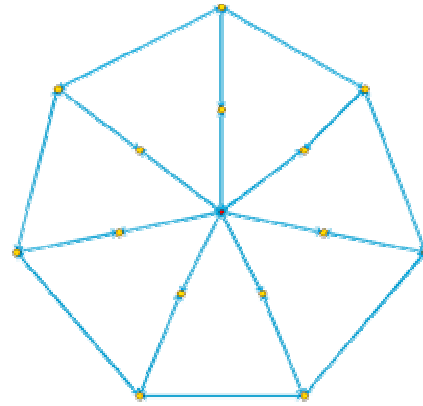
```

0096: Tsopt_CPLEX_Msg(): 36000 2311 572.8283 734 568.3799 680.5584 7685482 19.74%
0096: Tsopt_CPLEX_Msg(): Elapsed real time =
0096: Tsopt_CPLEX_Msg(): 6335.98 sec. (tree size = 14.52 MB
0096: Tsopt_CPLEX_Msg(): , solutions = 38
0096: Tsopt_CPLEX_Msg(): )
0096: Tsopt_CPLEX_Msg(): 36050 2315 cutoff 568.3799 680.5584 7694860 19.74%
0096: Tsopt_CPLEX_Msg(): 36100 2317 cutoff 568.3799 680.5584 7701564 19.74%
0096: Tsopt_CPLEX_Msg(): 36150 2325 570.6950 530 568.3799 680.5584 7709180 19.74%

```



# TTPLib 2008 – wheel\_req10.xml



ACP Cplex:

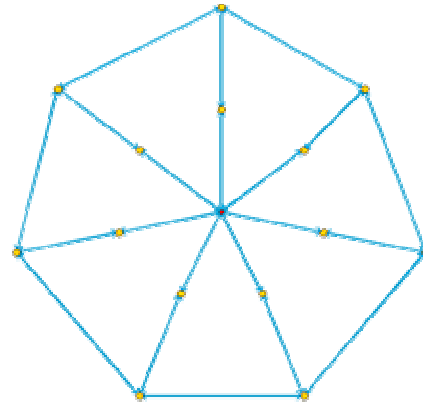
```

0096: Tsopt_CPLEX_Msg(): 400 93 595.5520 1602 595.2397 630.6793 243827 5.95%
0096: Tsopt_CPLEX_Msg(): 450 87 597.7870 522 595.2397 630.6793 258572 5.95%
0096: Tsopt_CPLEX_Msg(): 500 103 cutoff 595.2397 630.6793 278680 5.95%
0096: Tsopt_CPLEX_Msg(): Elapsed real time =
0096: Tsopt_CPLEX_Msg(): 94.16 sec. (tree size = 0.24 MB
0096: Tsopt_CPLEX_Msg(): , solutions = 2
...
0096: Tsopt_CPLEX_Msg(): )
0096: Tsopt_CPLEX_Msg(): 1900 35 620.3472 555 620.1883 630.0373 871767 1.59%

```



# TTPLib 2008 – wheel\_req10.xml



## ACP SCIP:

time	node	left	ILP iter	mem	lmdpt	lfrac	lvars	lcons	lconslcols	lrows	lcuts	lconfs	lstrbrl	dualbound	primalbound	gap	
2963m	294600	109	199418k	97M	58	-	11k	8692	8398	11k	8325	78	27k	105k	6.233377e+02	6.223653e+02	0.16%
2964m	294700	57	199439k	94M	58	-	11k	8683	8519	11k	8325	78	27k	105k	6.232824e+02	6.223653e+02	0.15%

SCIP Status : problem is solved [optimal solution found]

Solving Time (sec) : 177894.99

Solving Nodes : 294763

**Primal Bound** : +6.22365324690698e+02 (103 solutions)

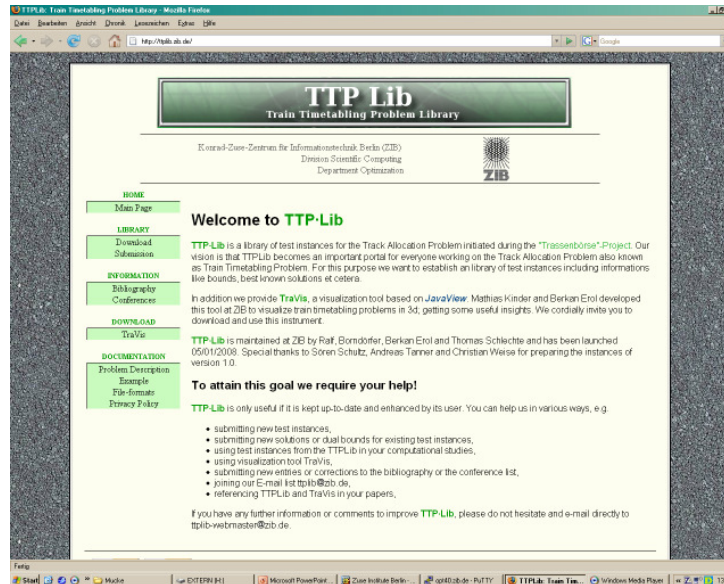
**Dual Bound** : +6.22365324690698e+02

**Gap** : 0.00 %





# Please Contribute - Improve TTPLib !



By submitting

- new test instances
- new solutions or upper bounds
- new entries to the bibliography and conference list

By using

- instances from the TTPLib in your computational studies
- free visualization tool TraVis





**Thank you  
for your attention !**

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