Recovery-Robust Platforming by Network Buffering

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Outline

Train Platforming Problem

- In and Out
- TPP deterministic model

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Problem definition

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The objective of train platforming is assigning trains to platforms in a railway station. Platforming is carried out:

- for a specific railway station
- after the timetable has been defined

In and Out

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- Train schedule : arrival, departure times, directions and allowed shifts
- Railway station topology: platforms, paths and directions

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- Train schedule : arrival, departure times, directions and allowed shifts
- Railway station topology: platforms, paths and directions

Output

- Assign each train a platform and two paths for arrival and departure s.t. no operational constraint is violated

Train schedule

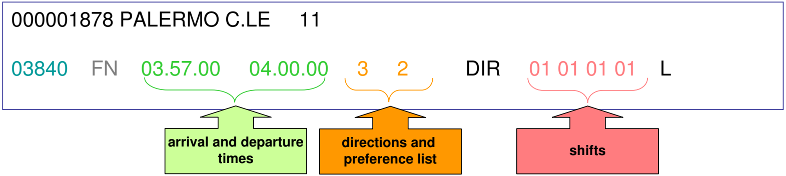

Figure: Train Schedule

The train schedule of a railway station contains info on arrival and departure times, directions and allowed shifts of each train passing through it.

Railway station topology

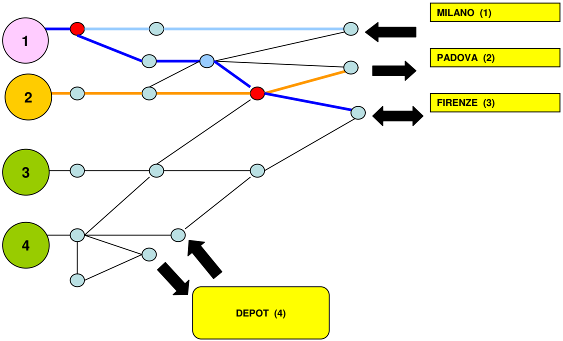

Figure: Topology

The topology of a railway station includes platforms, paths and directions.

Resources and operational constraints

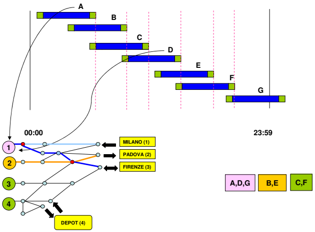

Figure: Platform and path.

Platform conflicts are forbidden, path conflicts are allowed to some extent.

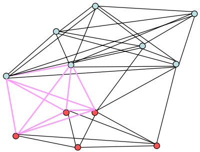

Figure: Incompatibility graph.

A pattern P for a train t is a 5-tuple defining: platform, arrival/departure paths and shifts. Operational constraints can be expressed using an incompatibility graph among patterns.

TPP deterministic model

$$\min \sum_{t \in T} \sum_{P \in \mathscr{P}_t} c_{t,P}\ x_{t,P} \tag{1}$$

s.t.

$$\sum_{P \in \mathscr{P}_t} x_{t,P} = 1, \qquad t \in T \tag{2}$$

$$\sum_{(t_1,P_1) \in K} x_{t_1,P_1} + \sum_{(t_2,P_2) \in K} x_{t_2,P_2} \leq 1, \qquad (t_1,t_2) \in T^2, K \in \mathscr{K}(t_1,t_2) \tag{3}$$

$$x_{t,P} \in \{0,1\}, \qquad t \in T,\ P \in \mathscr{P}_t \tag{4}$$

Details in Caprara *et al.* 2007 [2].

Robust Optimization

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Cons

Strict robustness is generally overconservative, because:

- solutions must cope with every likely scenarios without any recovery
- it is unable to account for limits to the sum of all disturbances

Recoverable Robustness

Informally speaking, a solution to an optimization problem is called *recovery robust* if it can be adjusted to all likely scenarios by limited recovery action. Thus a recovery-robust solution provides a service guarantee (Liebchen *et al.* 2007 [3]).

Robust Network Buffering

We are interested in the special case in which the recovery problem is a delay (y_i^a) propagation in some directed graph N, which is buffered on the arcs by means of f against disturbances on the arcs $a \in A(N)$. Denoting by $A(N)$ the set of arcs in N, we get:

$$\min_{f \in P} c(f)$$

$$\text{s.t. } \forall a \in A(N)\ \exists y^a \in \mathbb{R}^{|A(N)|} :$$

$$f_{(i,j)} + y_j^a - y_i^a \geq \Delta \cdot \chi_a((i,j)), \qquad a = (i,j) \in A(N)$$

$$D - d' y^a \geq 0$$

Details in Liebchen *et al.* 2007 [3].

Delay propagation network

The platforming gives rise in a natural way to a network in which the delay caused by disturbances propagates. This *delay propagation network* is a directed acyclic graph in which each vertex represents the delay of a particular train for a particular resource.

Delay propagation network

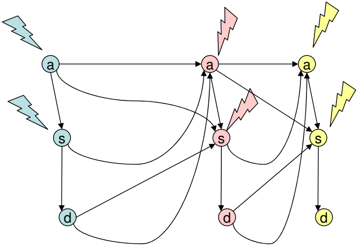

Figure: Delay propagation network

Each train has three associated vertices in this graph: (i) *a* for the arrival path, (ii) *s* for the stopping platform, and (iii) *d* for the departure path, corresponding to the delay (with respect to the nominal schedule) with which it will free up each of the three resources assigned to it.

Example

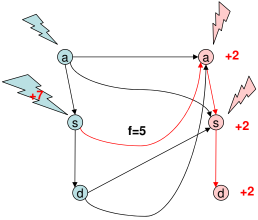

Figure: Example

Assume in the nominal schedule the first train frees up the platform at 10:00, the second train occupies it at 10:05 and frees it up at 10:10. Then a delay of more than 5 minutes for the first train results in a delay also for the second.

Delay propagation network model

$$D \geq \sum_{t \in T} (a_t^\xi + s_t^\xi + d_t^\xi), \qquad \xi \in \{\delta_t | t \in T\} \cup \{\delta'_t | t \in T\} \tag{5}$$

$$a_t^\xi \geq \delta_t^\xi, \qquad t \in T \tag{6}$$

$$s_t^\xi \geq a_t^\xi + \delta_t'^\xi, \qquad t \in T \tag{7}$$

$$d_t^\xi \geq s_t^\xi, \qquad t \in T \tag{8}$$

$$m_{t_2}^\xi \geq h_{t_1}^\xi - f(h_{t_1}, m_{t_2}), \qquad a = (h_{t_1}, m_{t_2}) \in A(N) \tag{9}$$

Buffers linking constraints

A straightforward link between the buffer value of a given arc $a \in A(N)$ associated with train pair $(t_1, t_2) \in T^2$ and the choice of patterns for the given pair of trains is the following:

$$\sum_{P_1 \in \mathscr{P}_{t_1}} \sum_{P_2 \in \mathscr{P}_{t_2}} c_{P_1,P_2,a} \; x_{t_1,P_1} \; x_{t_2,P_2}$$

where c_{a,P_1,P_2} is a constant associated to arc a and to the corresponding choice of patterns (P_1, P_2) for trains (t_1, t_2).

Buffers linking constraints

$$f_a \leq \sum_{P_1 \in \mathscr{P}_{t_1}} \alpha^a_{P_1} x_{t_1,P_1} + \sum_{P_2 \in \mathscr{P}_{t_2}} \beta^a_{P_2} x_{t_2,P_2} - \gamma^a, \qquad a \in A(N),\ (\alpha,\beta,\gamma) \in \mathscr{F}_a \quad (10)$$

Following Caprara *et al.* 2007 [2], the separation of Constraints (10) is done by a sort of polyhedral brute force, given that, for each pair of trains t_1, t_2, and for each arc $a \in A(N)$ the number of vertices in $Q_{t_1,t_2,a}$ is small. Specifically, $Q_{t_1,t_2,a}$ has $|\mathscr{P}_{t_1}||\mathscr{P}_{t_2}|$ vertices and lies in $\mathbb{R}^{|\mathscr{P}_{t_1}|+|\mathscr{P}_{t_2}|+1}$, so we can separate over it by solving an LP with $|\mathscr{P}_{t_1}||\mathscr{P}_{t_2}|$ variables and $|\mathscr{P}_{t_1}|+|\mathscr{P}_{t_2}|+1$ constraints.

Computational results: Palermo C.Le.

time window	# trains n.p.	D nom	CPU time nom (sec)	D RR	CPU time RR (sec)	Diff. D	Diff. D in %
A	0	646	7	479	46	167	25.85
B	2	729	7	579	3826	150	20.58
C	0	487	6	356	143	131	26.90
D	2	591	6	384	228	207	35.03
E	1	710	9	516	2217	194	27.32
F	1	560	7	480	18	80	14.29
G	3	465	11	378	64	87	18.71

Table: Results for Palermo Centrale

Computational results: Genova P.Princ.

time window	# trains n.p.	D nom	CPU time nom (sec)	D RR	CPU time RR (sec)	Diff. D	Diff. D in %
A	0	630	9	516	18190	114	18.10
B	0	838	11	624	3177	214	25.54
C	0	888	7	509	2495	379	42.68
D	4	895	8	657	9940	238	26.59
E	1	616	5	405	37	211	34.25
F	1	516	5	373	14	143	27.71
G	0	431	5	219	8	212	49.19

Table: Results for Genova Piazza Principe

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