



# **“An Approach to Rapid Transit Rolling Stock Assignment”**

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# Active Transportation Projects

**Project number TRA2008-06782-C02-01 : “*Transportation Network Design*”,  
Spanish Science and Innovation Ministry, 2009 to 2011.**

**Project number PT2007-003-08CCPP : “*Optimization models applied to the robust  
planification and the management of the transit network in case of  
emergencies*”,**

**Spanish “Ministerio de Fomento”, Cedex, 2008 to 2010.**

**Project number FP6-021235-2 : *ARRIVAL “Algorithms for Robust and on-line  
Railway optimization: Improving the Validity and reliability of Large-scale  
systems”.***

**European Commission. Sixth Frame Program, 2006 to 2008.**

# ROLLING STOCK MOTIVATION

- The increased demand for passenger transportation and the resulting traffic congestion in central areas have lead many cities to build rapid transit systems (RTS) to avoid the use of private cars.
- At operative level with a period planning of hours, the data and the decisions are considered in a spatial and temporal network
- Adequate rolling stock (RS) capacity especially for the rush periods; many passengers cannot be transported according to the usual service standards or an excess of material is used in any period.

# Which is new in our approach?

-The Rolling Stock (RS) problem studies the assignment of a given train fleet in a dense urban network to satisfy timetable and demand.

-The RTS\_RS in metropolitan context has different characteristics to Interurban Railway System: High density networks, where distances between nodes are relatively short and the frequencies have high values.

*“Considering a given timetable and demand, in a context where composition changes are considered, the goal is to determine each car type and units number to compose the trains that must be assigned to satisfy them, in the context of the urban dense metropolitan rapid transit networks”.*

# Rolling Stock for underground and suburban trains

-In this paper a model has been defined to find the optimal assignment of train compositions: aggregation and disaggregation of different car types in deposit stations to form convoys.

- The RS also will consider the optimization of empty trains and the optimal allocations of the convoys in the deposits, all considering the character and capacities of this type of networks.

# Operative Rolling Stock model

SPACE-TIME NETWORK: Given a directed space-time graph  $G(S,A)$ , where  $S$  is the station's set and  $T$  the period's set.  $A$  is the arc's set. Each arc  $a$  is defined by  $(s,t,s',t')$ , where  $s$  and  $s'$  are the nodes origin and destination,  $t$  is the depart time, and  $t'$  is the arrival time. This is,  $t'=t+t_a$ , where  $t_a$  is the arc train time to move from  $s$  to  $s'$ . It is assumed that this time is constant. This means that is denoted by "a" the arc  $(s,s',t)$ , or  $(a',t)$ , where  $a'$  is the arc in the physical network.

INPUT: Train characteristics, demand, scheduling,

OUTPUT: Train composition moving, train inventory in deposit, empty trains, maniouvre trains.

# RS Sets

$T(t)$ : period' set.

$S(s)$ : station' set.

$A(a)$ : arc' set.

$L(l)$ : service' set.

$M(m)$ : train composition type' set.

$Sc(s)$ : deposit' set.

# Operative RS: Objective function

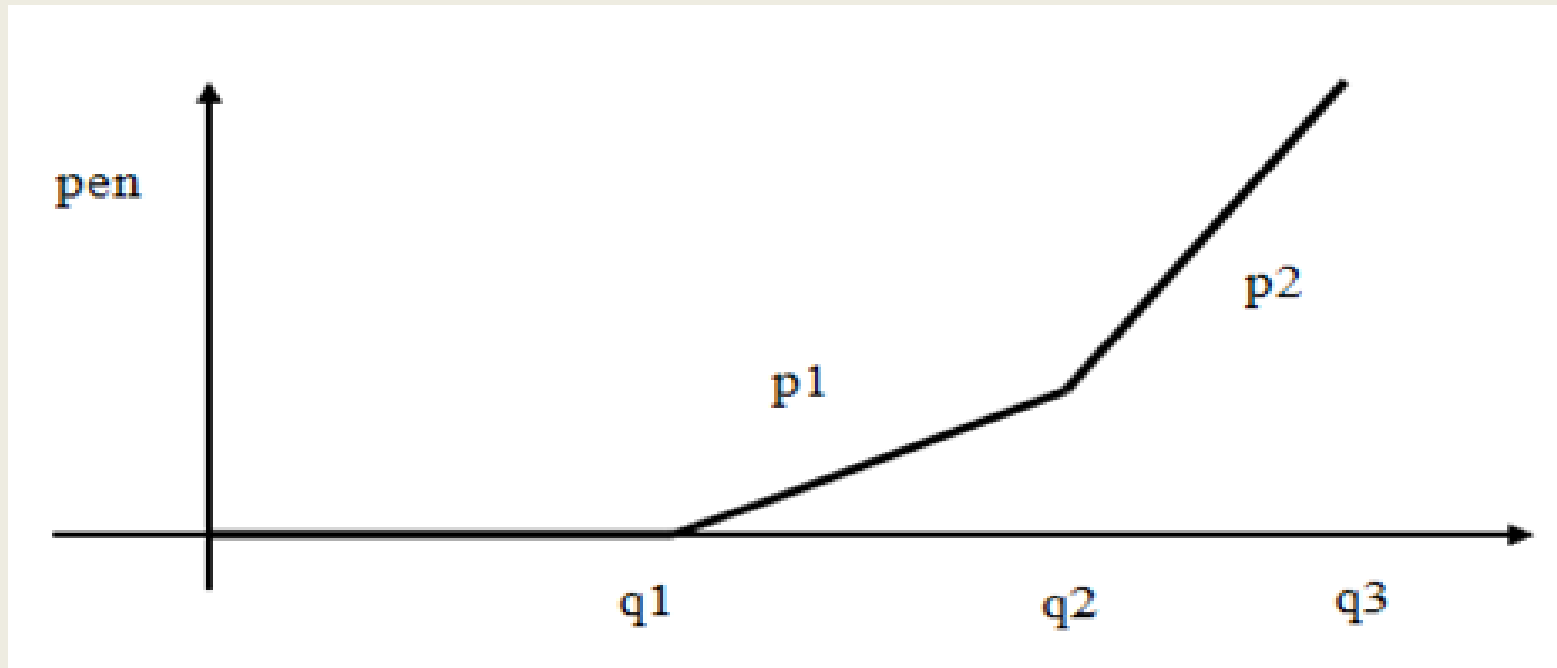
RSM is defined by an objective function, where the train operative costs, the train investment costs and some penalization costs are minimized

$$\begin{aligned} \text{Min} \quad & \sum_{\substack{m \in M \\ l \in L}} c_m x_{m,l} + \sum_{\substack{m \in M \\ l \in L}} c_m ex_{m,l} \\ & + \sum_{m \in M} ic_m y_n_m + \text{penalizations} \end{aligned}$$



# Piecewise Demand Penalization

In the flexible demand constraints the passenger capacity in arc “a” may be inferior to the demand but some extra demand “h1” and “h2” may be considered paying a penalization



# Flexible Demand constraints

In the flexible demand constraints the demand may be superior to the arc passenger capacity, but some extra demand “h1” and “h2” may be considered exceeding the capacity levels:

$$\sum_{m \in M} q1_m x_{m,l} \delta_a^l \geq g_{a,l} - h1_{a,l} - h2_{a,l}, \forall a \in A, \forall l \in L$$

The excess of demand is between q1 and q2:

$$h1_{a,l} \leq \sum_{m \in M} (q2_m - q1_m) x_{m,l} \delta_a^l, \forall a \in A, \forall l \in L$$

The excess of demand is between q2 and q3:

$$h2_{a,l} \leq \sum_{m \in M} (q3_m - q2_m) x_{m,l} \delta_a^l, \forall a \in A, \forall l \in L$$

# Flexible Demand penalization

$$p1 \sum_{\substack{a \in A \\ l \in L}} h1_{a,l} + p2 \sum_{\substack{a \in A \\ l \in L}} h2_{a,l}$$

# Car section and length station constraints

The car capacity in the sections:

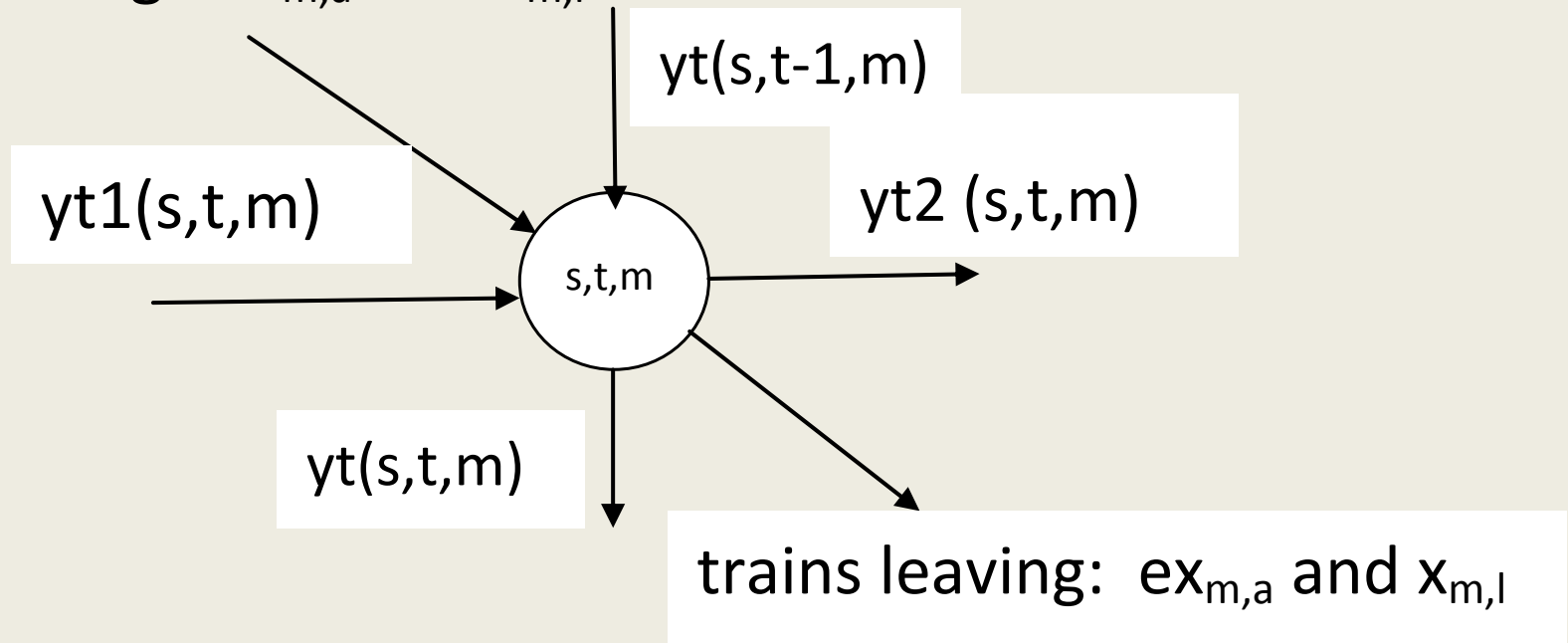
$$\sum_{\substack{m \in M, \\ l \in L}} N v_m x_{m,l} \delta_a^l \leq N_a, \forall a \in A$$

The length train capacity in stations:

$$\sum_{m \in M} l_m x_{m,l} \delta_{s,t}^l \leq l_s, \forall s \in S, \forall l \in L, \forall t \in T$$

# Node train conservation constraints and maneuvering penalization

trains arriving:  $ex_{m,a}$  and  $x_{m,l}$



# Node conservation constraints and maneuvering penalization

In the deposit balance constraints are considered: the convoys waiting in station, the train manoeuvring the station, the empty and commercial convoys:

$$yt_{s,t-1,m} + yt1_{s,t,m} + \sum_{a \in A} ex_{m,a} \bar{\delta}_{s,t}^a + \sum_{l \in L} x_{m,l} \bar{\delta}_{s,t}^l =$$

$$yt_{s,t,m} + yt2_{s,t,m}, \forall s \in Sc, \forall t \in T, \forall m \in M$$

$$\text{maneuvering penalization: } p3 \sum_{\substack{s \in S \\ m \in M \\ t \in T}} yt1_{s,t,m}$$

# Fleet capacity constraints

It ensures the cars used do not exceed the available ones.

The constraint counts the commercial and empty moving trains, the staying trains and the maneuvering trains.

$$\sum_{l \in L} x_{m,l} \delta_t^l + \sum_{a \in A} ex_{m,a} \delta_t^a + \sum_{s \in S} yt_{s,t,m} + \sum_{s \in S} yt1_{s,t,m} \leq N_m + yn_m; \forall m \in M, \forall t \in T$$

# Depart services and inventory cars constraints

Timetable is satisfied:

$$\sum_{m \in M} x_{m,l} = z_{s,t}^l, \forall s \in Sc, \forall t \in T, \forall l \in L$$

The train distribution is equal at each station at the initial and last periods of the planning period:

$$yt_{s,ti,m} = yt_{s,tf,m}, \forall s \in Sc, \forall m \in M$$





# Study Case: The Line C5 of Renfe-Madrid



# Case of study: Renfe Cercanias Madrid

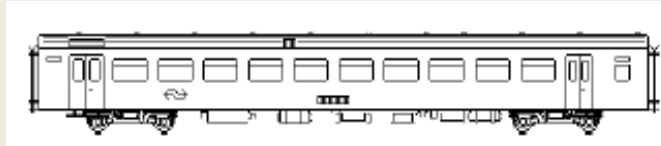
- The RS model tests have been done for the line C5 of the network. This line has four different deposits where the trains flow is studied.
- In C5 line there only is one car type (m1) and the trains can go in simple (one convoy) or double (two convoys) composition.
- We use a demand and a timetable given by Renfe. This demand is not symmetric and can vary much from an arc and the following one. That is why we introduce critic arcs in order to make smaller the model.

**m1:**  
**Simple or double?**



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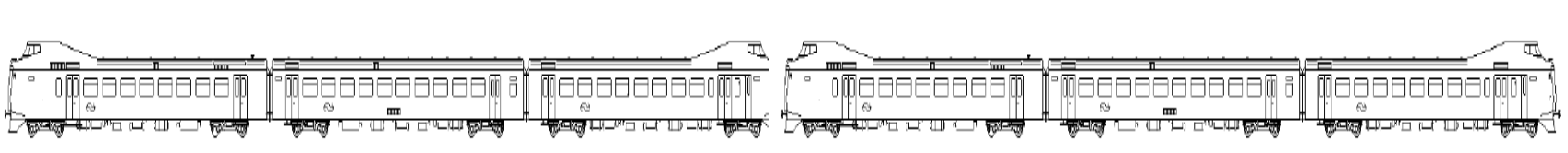
# Train Composition



Carriage



Simple Convoy

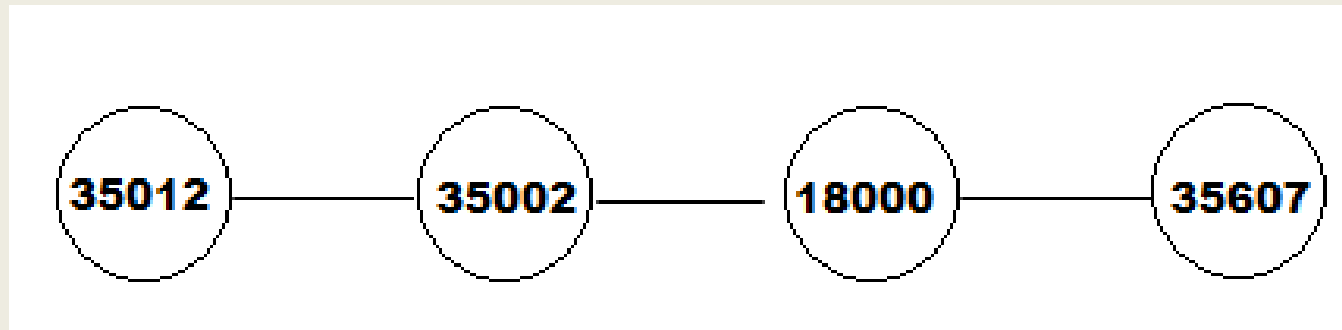


Double convoy

# Line C5



Line C5 of Suburban Madrid network



Deposit considered for the Line C5

# Car capacity and density

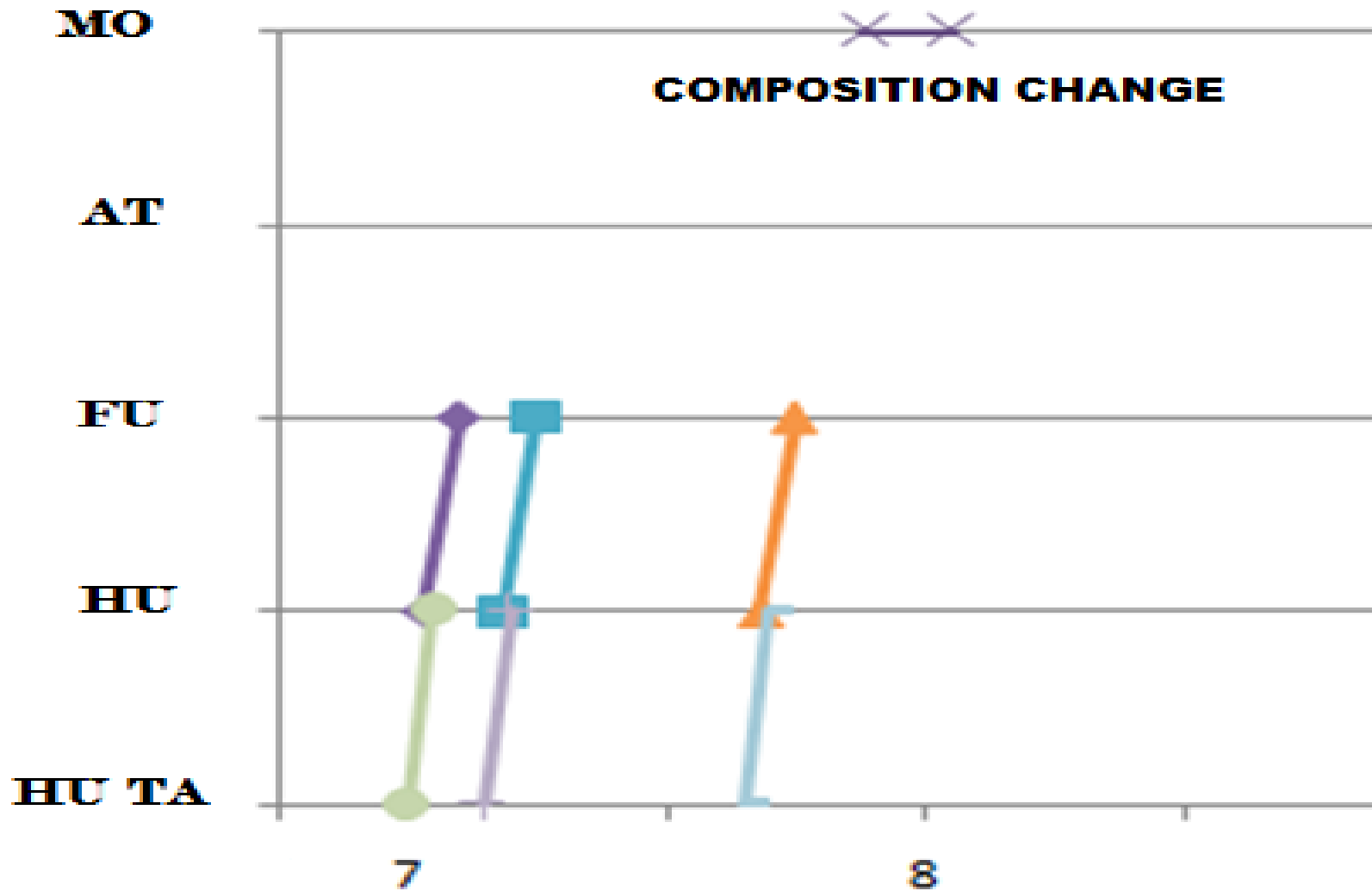
<b>Car Type</b>	<b>Capacity</b>	<b>Seats</b>	<b>Standing</b>	<b>Density (Pax/M2)</b>	<b>Length (m)</b>
m1	q1	240	261	3	80
m1	q2		348	4	
m1	q3		870	10	

# Train composition

Train	Origin Station	Departure Time	Arrival Station	Arrival Time	Composition	Occupation index (%)
19522	MO	7:02	HU	8:01	1	48.78
27320	MO	7:05	FU	8:00	1	43.27
27327	FU	7:14	MO	8:09	2	48.70
19526	MO	7:22	HU	8:21	2	25.09
27333	FU	7:28	MO	8:23	2	31.71
19528	MO	7:32	HU	8:31	1	44.30
27334	MO	7:39	FU	8:34	2	40.47
19530	MO	7:42	HU	8:41	2	27.62
27339	FU	7:44	MO	8:39	2	34.15
19525	HU	7:46	MO	8:45	1	35.52
19532	MO	7:52	HU	8:51	1	65.62
19527	HU	7:56	MO	8:55	1	45.74
27347	FU	8:04	MO	8:59	2	48.09
27348	MO	8:15	FU	9:10	2	27.97

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# Empty trains and car composition between 7 to 8 am.





# Rolling Stock results versus penalizations

p1-p2	Convoy	Commercial costs(€)	Empty train costs (€)	Savings (%)	Passenger excess(%)	Occupation index (%)	CPU TIME (in seconds)
0,5-4	66	76918,8	997,64	30,43	0,46	44,24	63.67
1-5	65	78530,16	1121,44	28,88	0,28	43,16	328.36
2-5,5	64	80085,36	1426,56	27,21	0,18	42,21	220.70
Actual	72	109765,2	2232,04	-	-	28,3	-

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# Conclusions

- The Rolling Stock model is a first approach in the new subject of the urban Rapid Transit network.
- The results do not only include the commercial movement: the empty movements, the adequate allocation of the material in the deposits and the optimal combining and splitting of the convoys to form the trains are also included.
- Other aspect considered in the model has been the inclusion in the planning period of all the rush hours in a daily period.
- The results obtained in the network tests have been satisfactory, not only because of the improved quality of the obtained allocations, but also by the reduction in the throughput time of the planning process that will be enabled by the application of the model.

# Futher researchs

- Influence of the car delay (respect to the planning timetable) in the Rolling Stock: Robust Rolling Stock in the Rapid Transit networks.
- The Rolling Stock solution in terms of the car successions.



Thanks for your attention

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