“An Approach to Rapid Transit Rolling Stock Assignment”
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The increased demand for passenger transportation and the resulting traffic congestion in central areas have lead many cities to build rapid transit systems (RTS) to avoid the use of private cars.

At operative level with a period planning of hours, the data and the decisions are considered in a spatial and temporal network.

Adequate rolling stock (RS) capacity especially for the rush periods; many passengers cannot be transported according to the usual service standards or an excess of material is used in any period.
Which is new in our approach?

-The Rolling Stock (RS) problem studies the assignment of a given train fleet in a dense urban network to satisfy timetable and demand.

-The RTS_RS in metropolitan context has different characteristics to Interurban Railway System: High density networks, where distances between nodes are relatively short and the frequencies have high values.

“Considering a given timetable and demand, in a context where composition changes are considered, the goal is to determine each car type and units number to compose the trains that must be assigned to satisfy them, in the context of the urban dense metropolitan rapid transit networks”.
Rolling Stock for underground and suburban trains

- In this paper a model has been defined to find the optimal assignment of train compositions: aggregation and disaggregation of different car types in deposit stations to form convoys.

- The RS also will consider the optimization of empty trains and the optimal allocations of the convoys in the deposits, all considering the character and capacities of this type of networks.
SPACE-TIME NETWORK: Given a directed space-time graph $G(S,A)$, where $S$ is the station’s set and $T$ the period’s set. $A$ is the arc’s set. Each arc $a$ is defined by $(s,t,s’,t’)$, where $s$ and $s’$ are the nodes origin and destination, $t$ is the depart time, and $t’$ is the arrival time. This is, $t’=t+t_a$, where $t_a$ is the arc train time to move from $s$ to $s’$. It is assumed that this time is constant. This means that is denoted by “$a$” the arc $(s,s’,t)$, or $(a’,t)$, where $a’$ is the arc in the physical network.

INPUT: Train characteristics, demand, scheduling,

OUTPUT: Train composition moving, train inventory in deposit, empty trains, maniouvre trains.
RS Sets

\[ T(t) : \text{period’ set.} \]

\[ S(s) : \text{station’ set.} \]

\[ A(a) : \text{arc’ set.} \]

\[ L(l) : \text{service’ set.} \]

\[ M(m) : \text{train composition type’ set.} \]

\[ Sc(s) : \text{deposit’ set.} \]
RSM is defined by an objective function, where the train operative costs, the train investment costs and some penalization costs are minimized

\[
\begin{align*}
\text{Min} & \quad \sum_{m \in M} \sum_{l \in L} c_{m,l} x_{m,l} + \sum_{m \in M} \sum_{l \in L} c_{m,l} x_{m,l} \\
& + \sum_{m \in M} ic_{m} yn_{m} + \text{penalizations}
\end{align*}
\]
Piecewise Demand Penalization

In the flexible demand constraints the passenger capacity in arc “a” may be inferior to the demand but some extra demand “h1” and “h2” may be considered paying a penalization.
Flexible Demand constraints

In the flexible demand constraints the demand may be superior to the arc passenger capacity, but some extra demand “h1” and “h2” may be considered exceeding the capacity levels:

\[ \sum_{m \in M} q_1 x_{m,l} \delta_a^l \geq g_{a,l} - h_{1a,l} - h_{2a,l}, \forall a \in A, \forall l \in L \]

The excess of demand is between q1 and q2:

\[ h_{1a,l} \leq \sum_{m \in M} (q_2 - q_1) x_{m,l} \delta_a^l, \forall a \in A, \forall l \in L \]

The excess of demand is between q2 and q3:

\[ h_{2a,l} \leq \sum_{m \in M} (q_3 - q_2) x_{m,l} \delta_a^l, \forall a \in A, \forall l \in L \]
Flexible Demand penalization

\[ p_1 \sum_{a \in A} \sum_{l \in L} h_{1,a,l} + p_2 \sum_{a \in A} \sum_{l \in L} h_{2,a,l} \]
Car section and length station constraints

The car capacity in the sections:

$$\sum_{m \in M, \ l \in L} N \nu_m x_{m,l} \delta_a^l \leq N_a, \ \forall a \in A$$

The length train capacity in stations:

$$\sum_{m \in M} l_m x_{m,l} \delta_{s,t}^l \leq l_s, \ \forall s \in S, \ \forall l \in L, \ \forall t \in T$$
Node train conservation constraints and maneuvering penalization

trains arriving: $e_{x_{m,a}}$ and $x_{m,l}$

$y_{t(s,t-1,m)}$

$y_{t1(s,t,m)}$

$y_{t(s,t,m)}$

$y_{t2(s,t,m)}$

trains leaving: $e_{x_{m,a}}$ and $x_{m,l}$
Node conservation constraints and maneuvering penalization

In the deposit balance constraints are considered: the convoys waiting in station, the train manoeuvring the station, the empty and commercial convoys:

\[ yt_{s,t-1,m} + yt_{1s,t,m} + \sum_{a \in A} ex_{m,a} \bar{\delta}^{a}_{s,t} + \sum_{l \in L} x_{m,l} \bar{\delta}^{l}_{s,t} = \]

\[ yt_{s,t,m} + yt_{2s,t,m}, \forall s \in Sc, \forall t \in T, \forall m \in M \]

maneuvering penalization: \[ p3 \sum_{s \in S} \sum_{m \in M} \sum_{t \in T} yt_{1s,t,m} \]
Fleet capacity constraints

It ensures the cars used do not exceed the available ones. The constraint counts the commercial and empty moving trains, the staying trains and the maneuvering trains.

\[
\sum_{l \in L} x_{m,l} \delta_t^l + \sum_{a \in A} ex_{m,a} \delta_t^a + \sum_{s \in S} y_{t_s,t,m} + \sum_{s \in S} y_{t_1 s,t,m} \leq N_m + y n_m ; \forall m \in M, \forall t \in T
\]
Depart services and inventory cars constraints

Timetable is satisfied:

\[ \sum_{m \in M} x_{m,l} = z_{s,t}^l, \forall s \in Sc, \forall t \in T, \forall l \in L \]

The train distribution is equal at each station at the initial and last periods of the planning period:

\[ y_{t_s,t_i,m} = y_{t_s,t_f,m}, \forall s \in Sc, \forall m \in M \]
Study Case:
The Line C5 of Renfe-Madrid
Case of study: Renfe Cercanías Madrid

- The RS model tests have been done for the line C5 of the network. This line has four different deposits where the trains flow is studied.

- In C5 line there only is one car type (m1) and the trains can go in simple (one convoy) or double (two convoys) composition.

- We use a demand and a timetable given by Renfe. This demand is not symmetric and can vary much from an arc and the following one. That is why we introduce critic arcs in order to make smaller the model.
Car types

m1: Simple or double?
Train Composition

Carriage

Simple Convoy

Double convoy
Line C5

Line C5 of Suburban Madrid network

Deposit considered for the Line C5
<table>
<thead>
<tr>
<th>Car Type</th>
<th>Capacity</th>
<th>Seats</th>
<th>Standing</th>
<th>Density (Pax/M2)</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1</td>
<td>q1</td>
<td></td>
<td>261</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>m1</td>
<td>q2</td>
<td>240</td>
<td>348</td>
<td>4</td>
<td>80</td>
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<tr>
<td>m1</td>
<td>q3</td>
<td></td>
<td>870</td>
<td>10</td>
<td></td>
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</table>
### Train composition

<table>
<thead>
<tr>
<th>Train</th>
<th>Origin Station</th>
<th>Departure Time</th>
<th>Arrival Station</th>
<th>Arrival Time</th>
<th>Composition</th>
<th>Occupation index (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19522</td>
<td>MO</td>
<td>7:02</td>
<td>HU</td>
<td>8:01</td>
<td>1</td>
<td>48.78</td>
</tr>
<tr>
<td>27320</td>
<td>MO</td>
<td>7:05</td>
<td>FU</td>
<td>8:00</td>
<td>1</td>
<td>43.27</td>
</tr>
<tr>
<td>27327</td>
<td>FU</td>
<td>7:14</td>
<td>MO</td>
<td>8:09</td>
<td>2</td>
<td>48.70</td>
</tr>
<tr>
<td>19526</td>
<td>MO</td>
<td>7:22</td>
<td>HU</td>
<td>8:21</td>
<td>2</td>
<td>25.09</td>
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<tr>
<td>27333</td>
<td>FU</td>
<td>7:28</td>
<td>MO</td>
<td>8:23</td>
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<td>31.71</td>
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<tr>
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<td>MO</td>
<td>7:32</td>
<td>HU</td>
<td>8:31</td>
<td>1</td>
<td>44.30</td>
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<td>MO</td>
<td>7:39</td>
<td>FU</td>
<td>8:34</td>
<td>2</td>
<td>40.47</td>
</tr>
<tr>
<td>19530</td>
<td>MO</td>
<td>7:42</td>
<td>HU</td>
<td>8:41</td>
<td>2</td>
<td>27.62</td>
</tr>
<tr>
<td>27339</td>
<td>FU</td>
<td>7:44</td>
<td>MO</td>
<td>8:39</td>
<td>2</td>
<td>34.15</td>
</tr>
<tr>
<td>19525</td>
<td>HU</td>
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<td>MO</td>
<td>8:45</td>
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<td>35.52</td>
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<tr>
<td>19532</td>
<td>MO</td>
<td>7:52</td>
<td>HU</td>
<td>8:51</td>
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<tr>
<td>19527</td>
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<tr>
<td>27347</td>
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<td>MO</td>
<td>8:59</td>
<td>2</td>
<td>48.09</td>
</tr>
<tr>
<td>27348</td>
<td>MO</td>
<td>8:15</td>
<td>FU</td>
<td>9:10</td>
<td>2</td>
<td>27.97</td>
</tr>
</tbody>
</table>
Empty trains and car composition between 7 to 8 am.
### Rolling Stock results versus penalizations

<table>
<thead>
<tr>
<th>p1-p2 Convoy</th>
<th>Commercial costs (€)</th>
<th>Empty train costs (€)</th>
<th>Savings (%)</th>
<th>Passenger excess (%)</th>
<th>Occupation index (%)</th>
<th>CPU TIME (in seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,5-4</td>
<td>66</td>
<td>76918,8</td>
<td>997,64</td>
<td>30,43</td>
<td>0,46</td>
<td>44,24</td>
</tr>
<tr>
<td>1-5</td>
<td>65</td>
<td>78530,16</td>
<td>1121,44</td>
<td>28,88</td>
<td>0,28</td>
<td>43,16</td>
</tr>
<tr>
<td>2-5,5</td>
<td>64</td>
<td>80085,36</td>
<td>1426,56</td>
<td>27,21</td>
<td>0,18</td>
<td>42,21</td>
</tr>
<tr>
<td>Actual</td>
<td>72</td>
<td>109765,2</td>
<td>2232,04</td>
<td>-</td>
<td>-</td>
<td>28,3</td>
</tr>
</tbody>
</table>
Conclusions

- The Rolling Stock model is a first approach in the new subject of the urban Rapid Transit network.

- The results do not only include the commercial movement: the empty movements, the adequate allocation of the material in the deposits and the optimal combining and splitting of the convoys to form the trains are also included.

- Other aspect considered in the model has been the inclusion in the planning period of all the rush hours in a daily period.

- The results obtained in the network tests have been satisfactory, not only because of the improved quality of the obtained allocations, but also by the reduction in the throughput time of the planning process that will be enabled by the application of the model.
Further researchs

- Influence of the car delay (respect to the planning timetable) in the Rolling Stock: Robust Rolling Stock in the Rapid Transit networks.

- The Rolling Stock solution in terms of the car successions.
Thanks for your attention

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