Meeting functional requirements for real-time railway traffic management with mathematical models

Ambra Toletti – Marco Laumanns – Peter Grossenbacher – Ulrich Weidmann

Conference on Advanced Systems in Public Transport **CASPT 2015**

Literature reviews

- **Cordeau JF, Toth P, Vigo D (1998)** A survey of optimization models for train routing and scheduling. Transportation Science 32(4):380–404
- **Törnquist J (2006)** Computer-based decision support for railway traffic scheduling and dispatching: A review of models and algorithms. In: Kroon LG, Möhring RH (eds) 5th Workshop on Algorithmic Methods and Models for Optimization of Railways (ATMOS'05), Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, Dagstuhl, Germany, OpenAccess Series in Informatics (OASIcs), vol 2, DOI 10.4230/OASIcs.ATMOS.2005.659
- **Lusby RM, Larsen J, Ehrgott M, Ryan D (2011)** Railway track allocation: models and methods. OR Spectrum 33(4):843–883, DOI 10.1007/s00291-009-0189-0
- **Corman F, Meng L (2013)** A review of online dynamic models and algorithms for railway traffic control. In: Intelligent Rail Transportation (ICIRT), 2013 IEEE International Conference on, pp 128–133, DOI 10.1109/ICIRT.2013.6696281
- **Cacchiani V, Huisman D, Kidd M, Kroon L, Toth P, Veelenturf L, Wagenaar J (2014)** An overview of recovery models and algorithms for real-time railway rescheduling. Transportation Research Part B: Methodological 63(0):15 – 37, DOI 10.1016/j.trb.2014.01.009

Modelling needs assumptions…

A farmer has some chickens who don't lay any eggs. The farmer calls a physicist to help. The physicist does some calculation and says "I have a solution but it only works for spherical chickens in a vacuum!".

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Infrastructure

Rolling Stock

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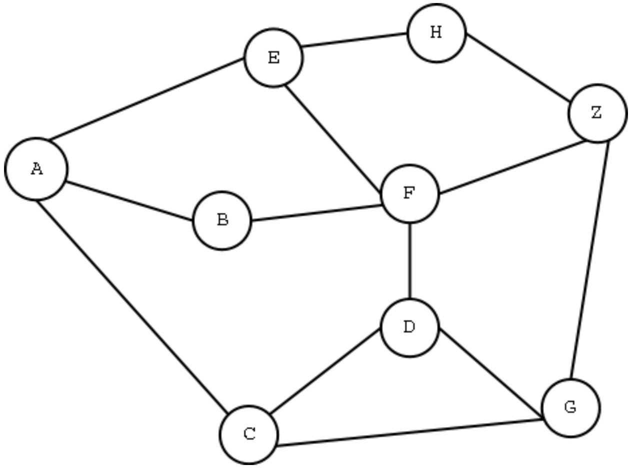
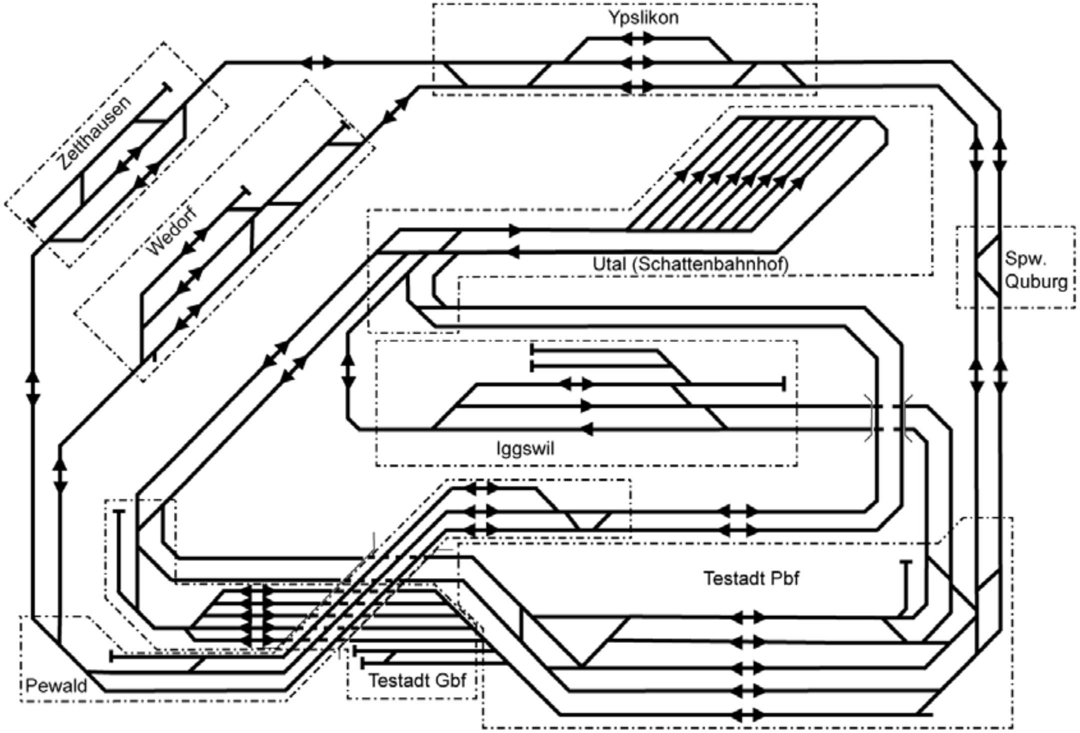
Functional requirements from real-time traffic management

Infrastructure Rolling Stock Operations



Example: Railway network of the Railway operations laboratory (EBL) at the ETH Zurich

Source: archive of the institute for transport planning and systems (IVT)



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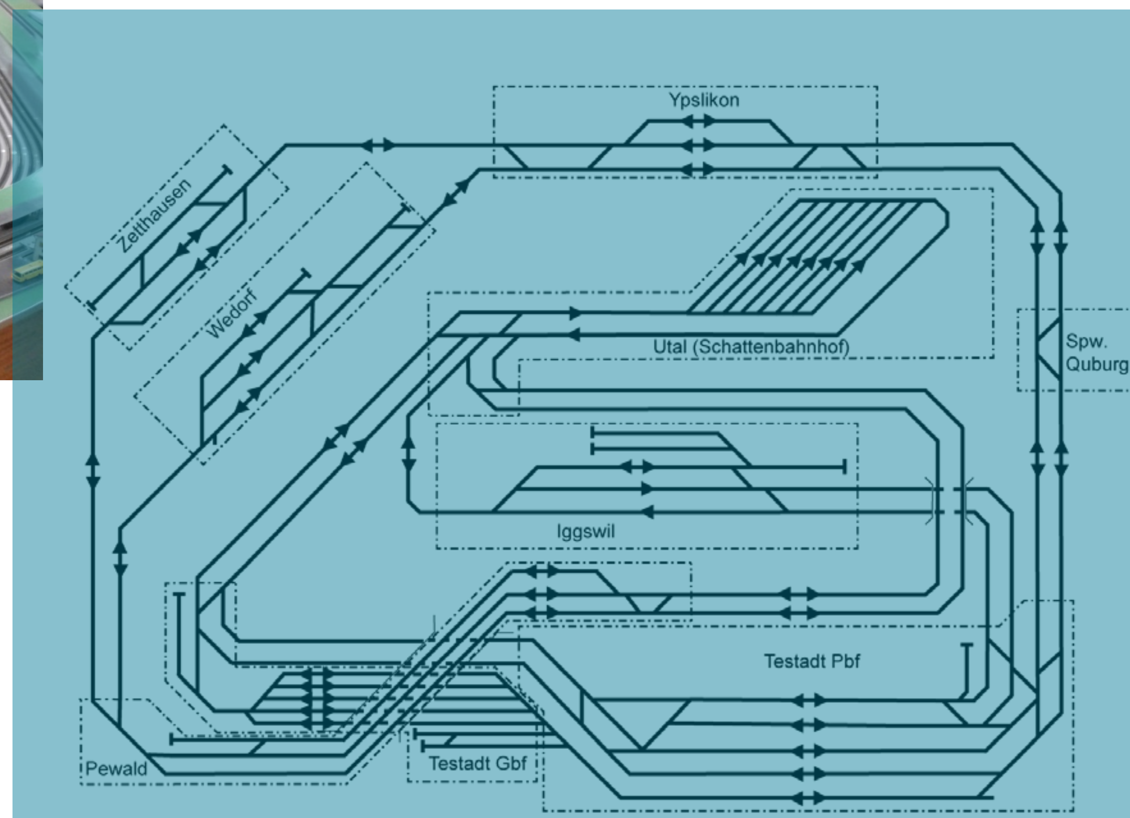

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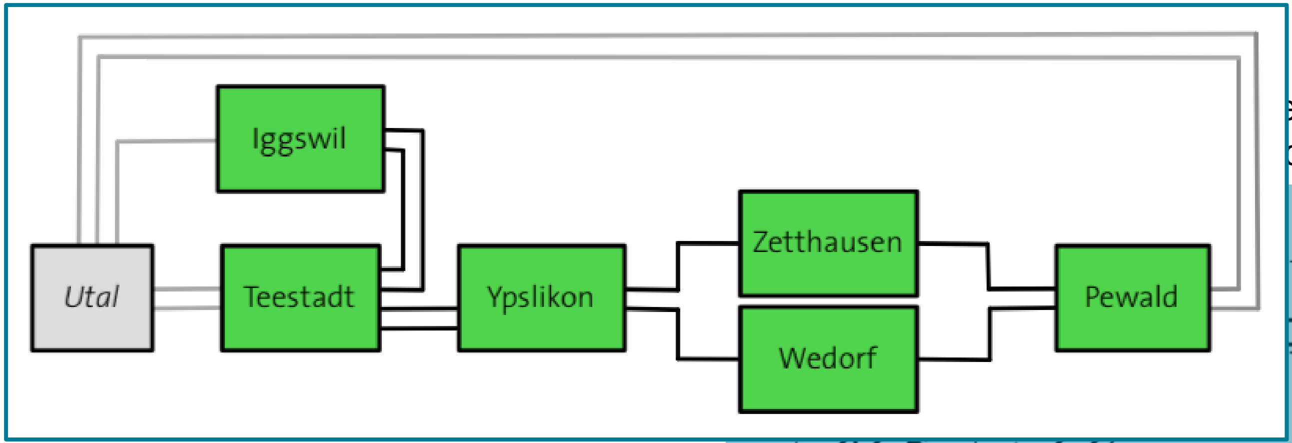

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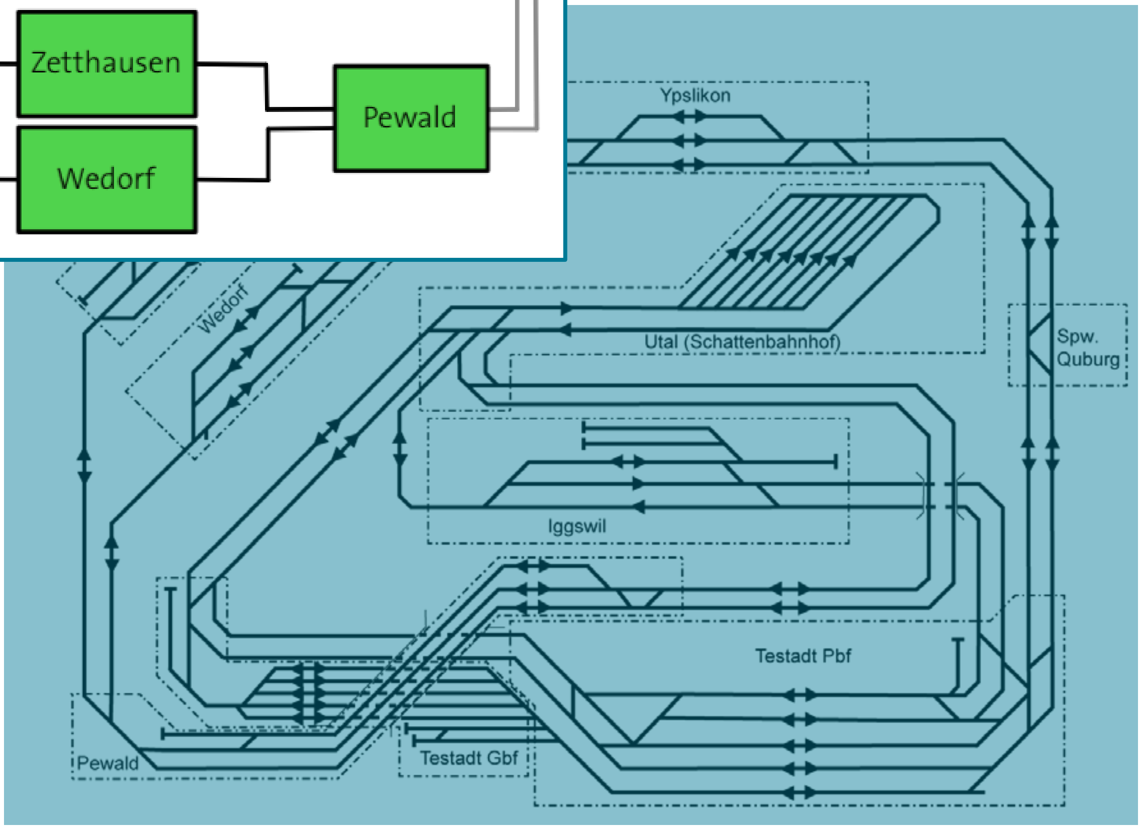

- Macroscopic representation of the entire network

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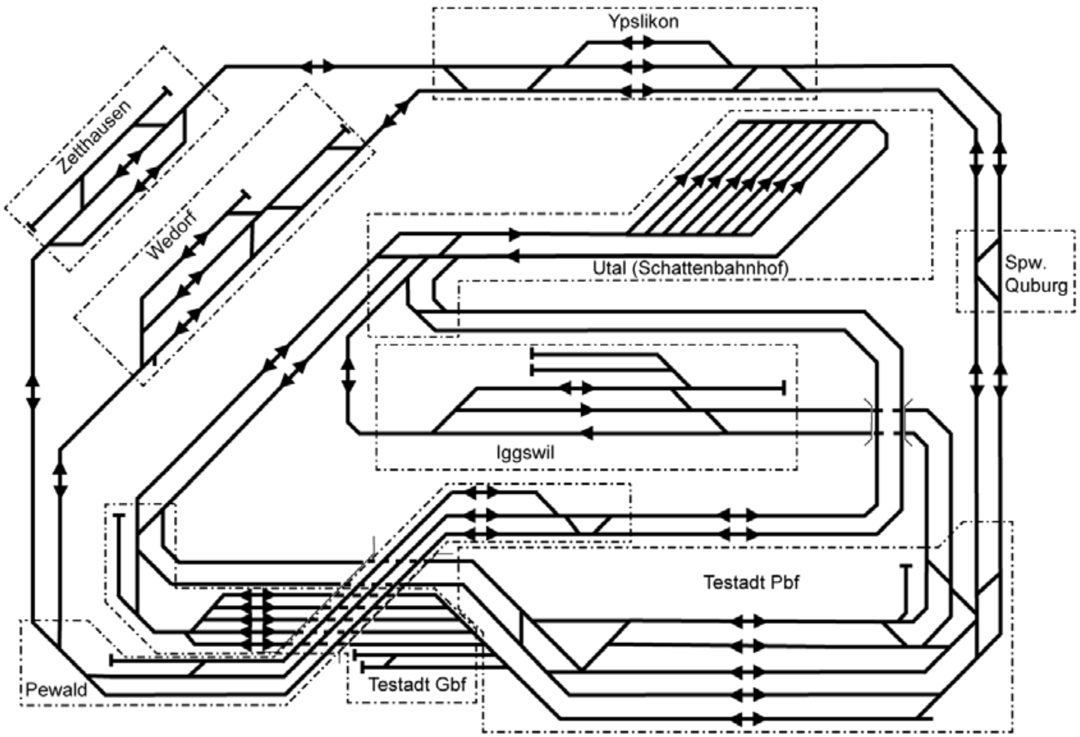

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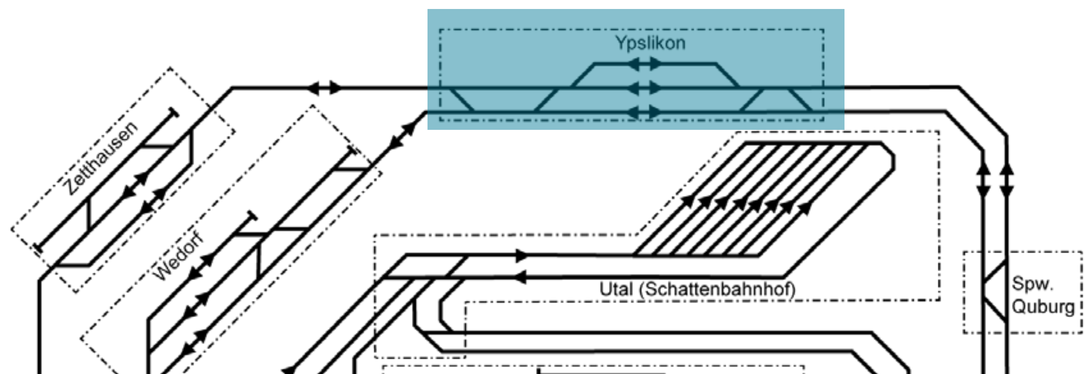
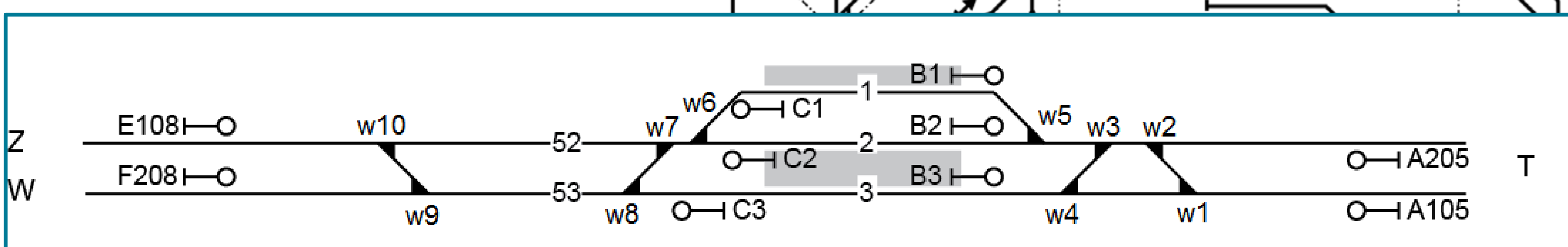


- Microscopic representation of station Ypslikon



Functional requirements from real-time traffic management

Infrastructure Rolling Stock Operations

- Macroscopic
- Microscopic

Functional requirements from real-time traffic management

Infrastructure | Rolling Stock | Operations

- Macroscopic
- Mesoscopic
- Microscopic

Functional requirements from real-time traffic management

Infrastructure

- Macroscopic
- Mesoscopic
- Microscopic

Rolling Stock

Operations

Functional requirement (Radtke 2014)

”the microscopic infrastructure is not only suitable but even mandatory for exact running time calculation, timetable construction, possession planning and railway operational simulation, conflict detection and resolution.”

Radtke A (2014) Infrastructure modelling. In: Hansen IA, Pachl J (eds) Railway Timetabling and Operations, 2nd edn, Eurail press, Hamburg, Germany, chap 3, pp 47–63.

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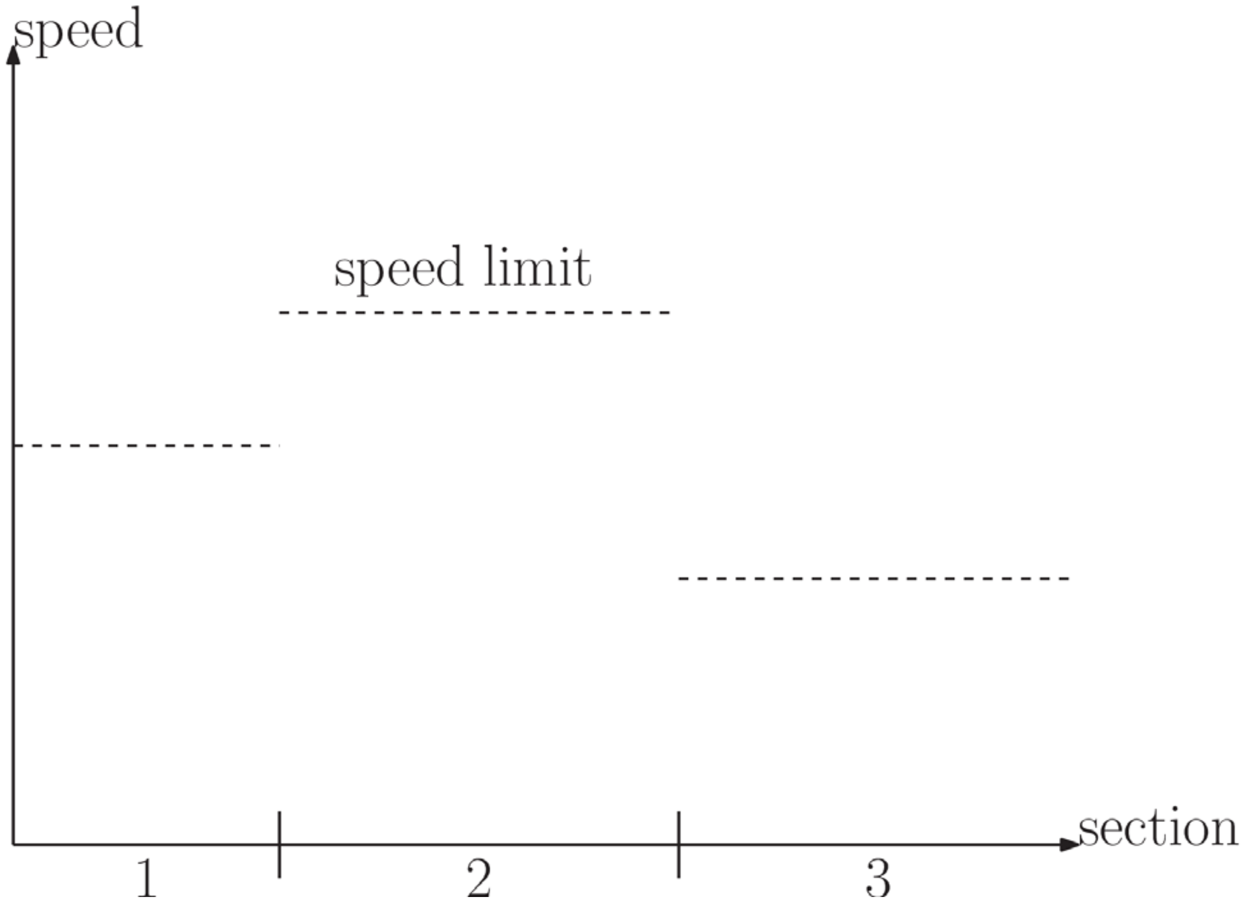
Functional requirements from real-time traffic management

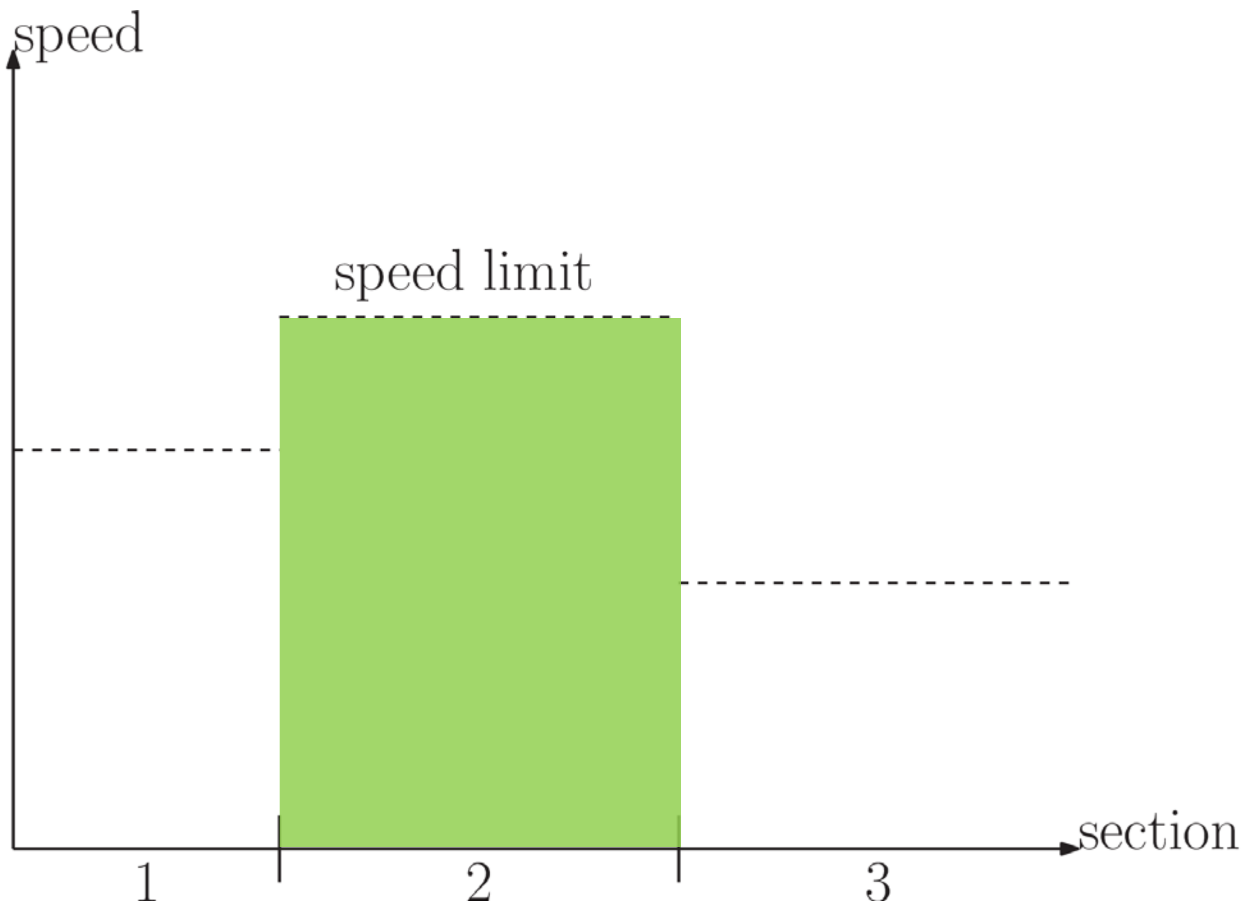
Infrastructure Rolling Stock Operations

Example: Speed limits on consecutive sections



Example: Speed limits on consecutive sections

Maximal speed



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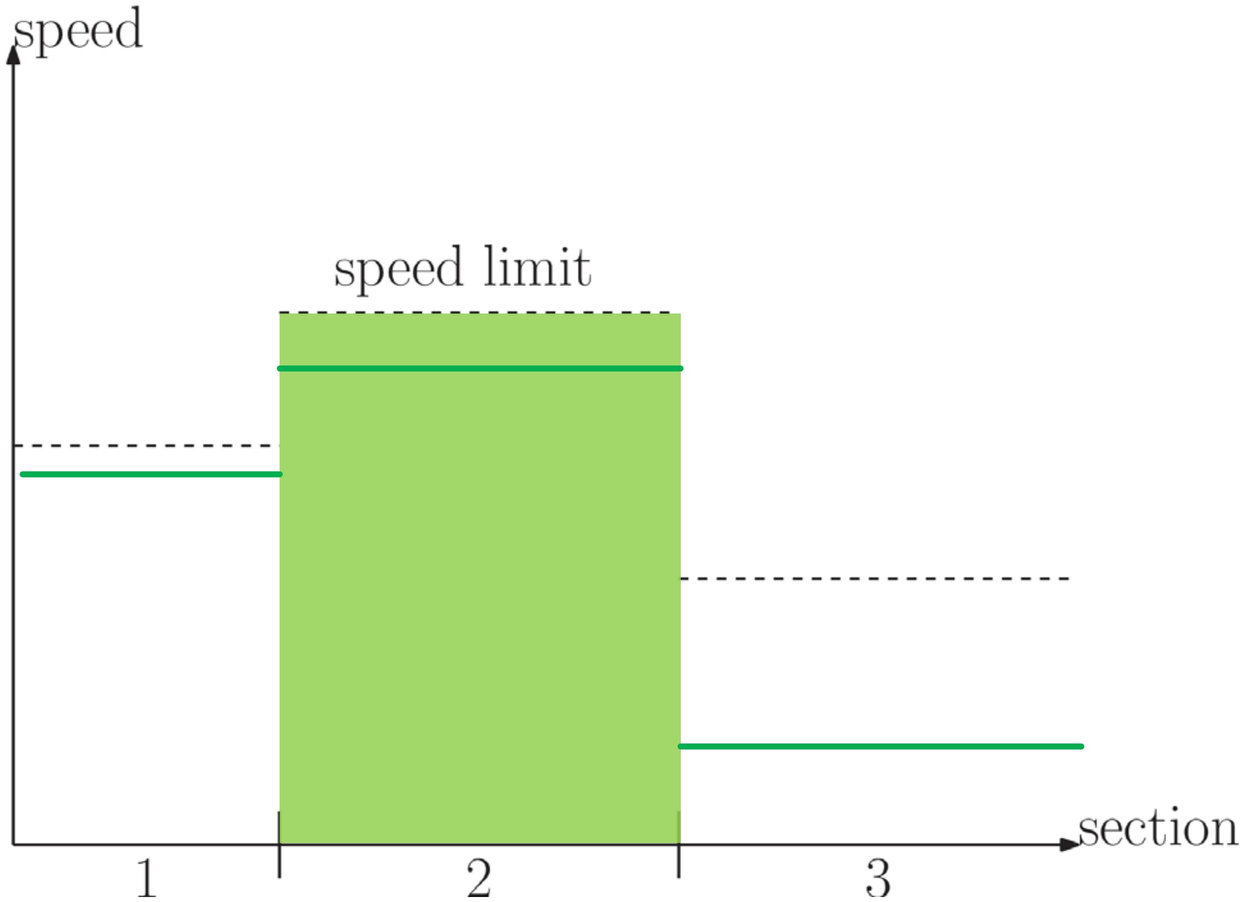
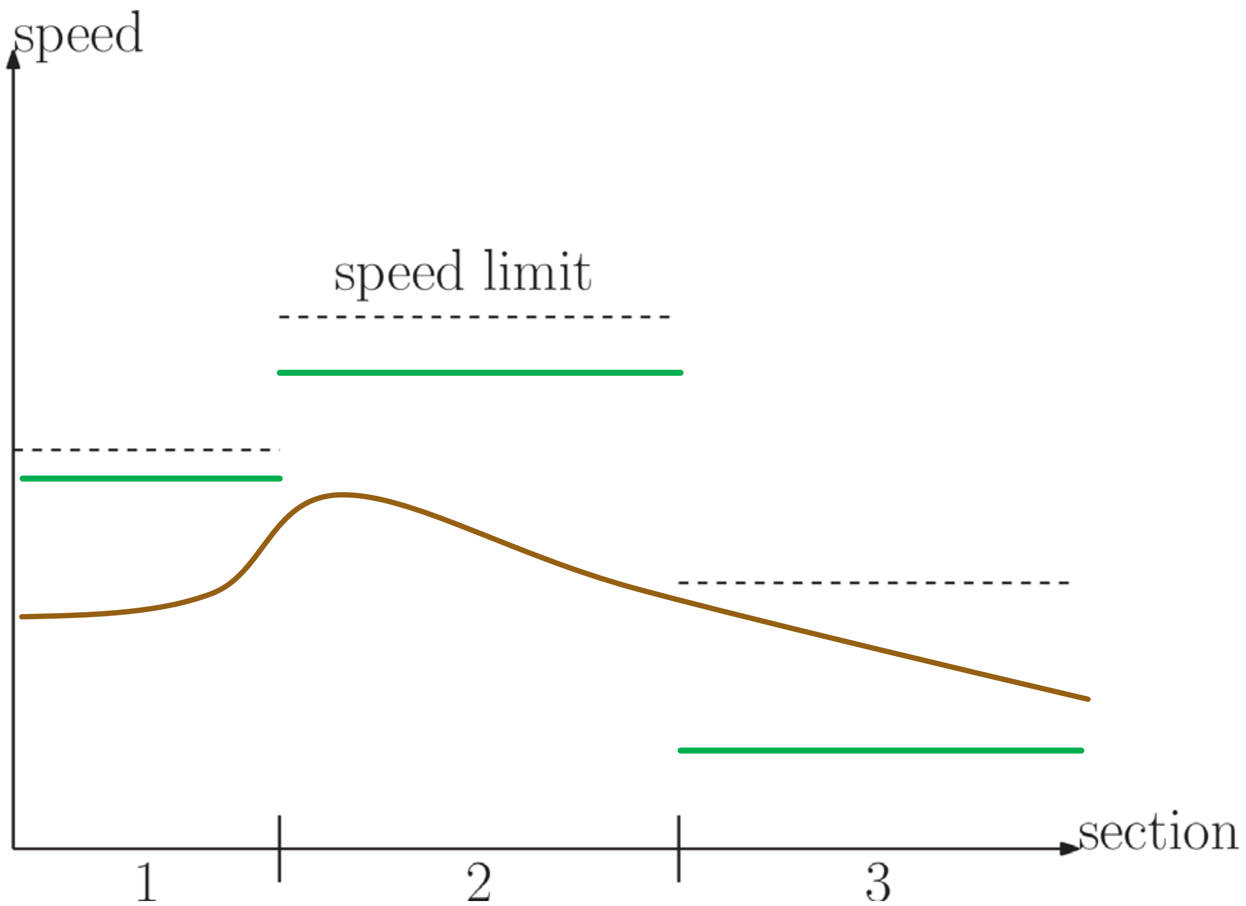


Example: Speed limits on consecutive sections

Maximal speed

Realizable speed



Functional requirements from real-time traffic management

Infrastructure

Rolling Stock

- Maximal speed
- Realizable speed

Operations

Functional requirements from real-time traffic management

Infrastructure

Rolling Stock

- Maximal speed
- Realizable speed
- Length

Operations

Institut für Verkehrsplanung und Transportsysteme
Institute for Transport Planning and Systems

Functional requirements from real-time traffic management

Infrastructure

Rolling Stock

- Maximal speed
- Realizable speed
- Length

Operations

Functional requirements from real-time traffic management

Infrastructure Rolling Stock Operations

Functional requirements from real-time traffic management

Infrastructure

Rolling Stock

Operations

- Monitoring

Functional requirements from real-time traffic management

Infrastructure　　Rolling Stock　　Operations

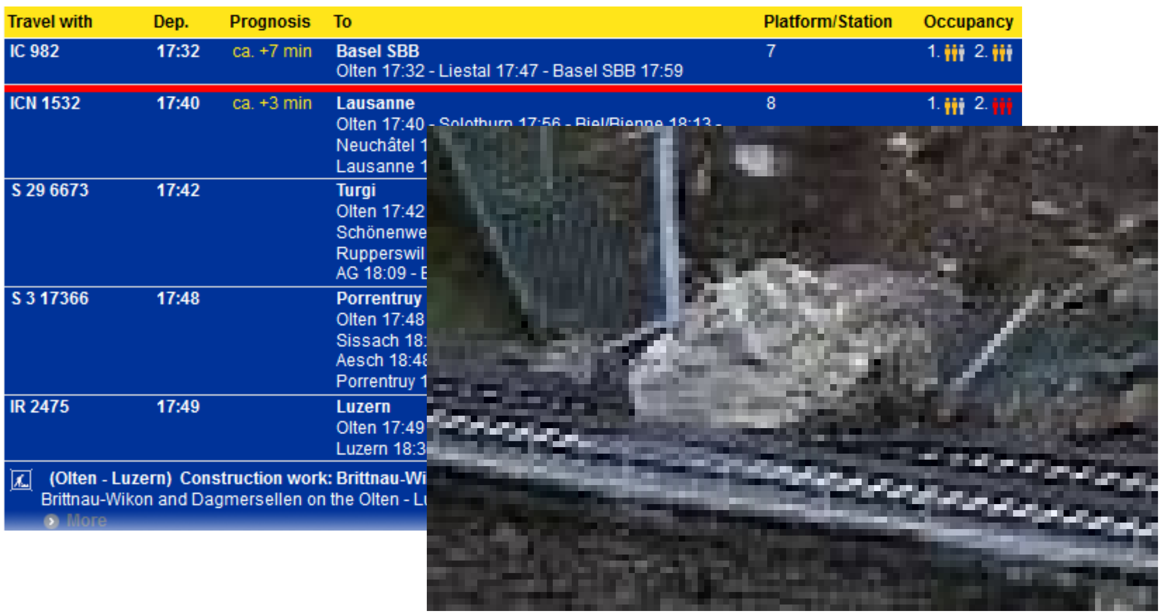
Travel with	Dep.	Prognosis	To	Platform/Station	Occupancy
IC 982	17:32	ca. +7 min	**Basel SBB** Olten 17:32 - Liestal 17:47 - Basel SBB 17:59	7	1. 2.
ICN 1532	17:40	ca. +3 min	**Lausanne** Olten 17:40 - Solothurn 17:56 - Biel/Bienne 18:13 - Neuchâtel 18:32 - Yverdon-les-Bains 18:51 - Lausanne 19:15	8	1. 2.
S 29 6673	17:42		**Turgi** Olten 17:42 - Dulliken 17:45 - Däniken 17:48 - Schönenwerd SO 17:50 • Aarau 17:56 - Rupperswil 18:01 - Wildegg 18:06 - Holderbank AG 18:09 - Brugg AG 18:17 - Turgi 18:26	9	1. 2.
S 3 17366	17:48		**Porrentruy** Olten 17:48 - Tecknau 17:54 - Gelterkinden 17:59 - Sissach 18:02 • Liestal 18:10 - Basel SBB 18:26 - Aesch 18:48 - Laufen 19:02 - Delémont 19:17 - Porrentruy 19:52	11	1. 2.
IR 2475	17:49		**Luzern** Olten 17:49 - Zofingen 17:56 - Sursee 18:10 - Luzern 18:30	12	1. 2.

(Olten - Luzern) Construction work: Brittnau-Wikon - Dagmersellen (SBB CFF FFS) IR Olten - Luzern Between Brittnau-Wikon and Dagmersellen on the Olten - Luzern line, only limited train services are operating. IR Olten - Luzern
More

- Monitoring
  - Punctuality

Functional requirements from real-time traffic management

Infrastructure Rolling Stock **Operations**



- Monitoring
  - Punctuality
  - Infrastructure

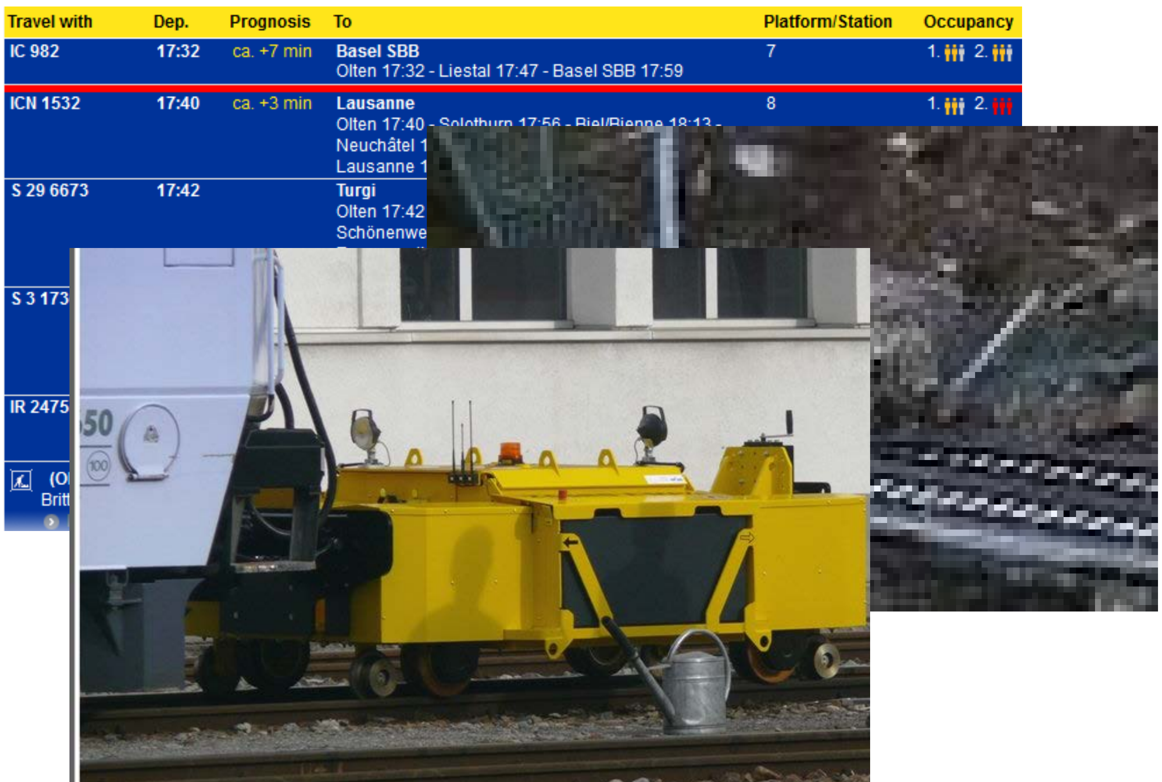
Functional requirements from real-time traffic management

Infrastructure

Rolling Stock

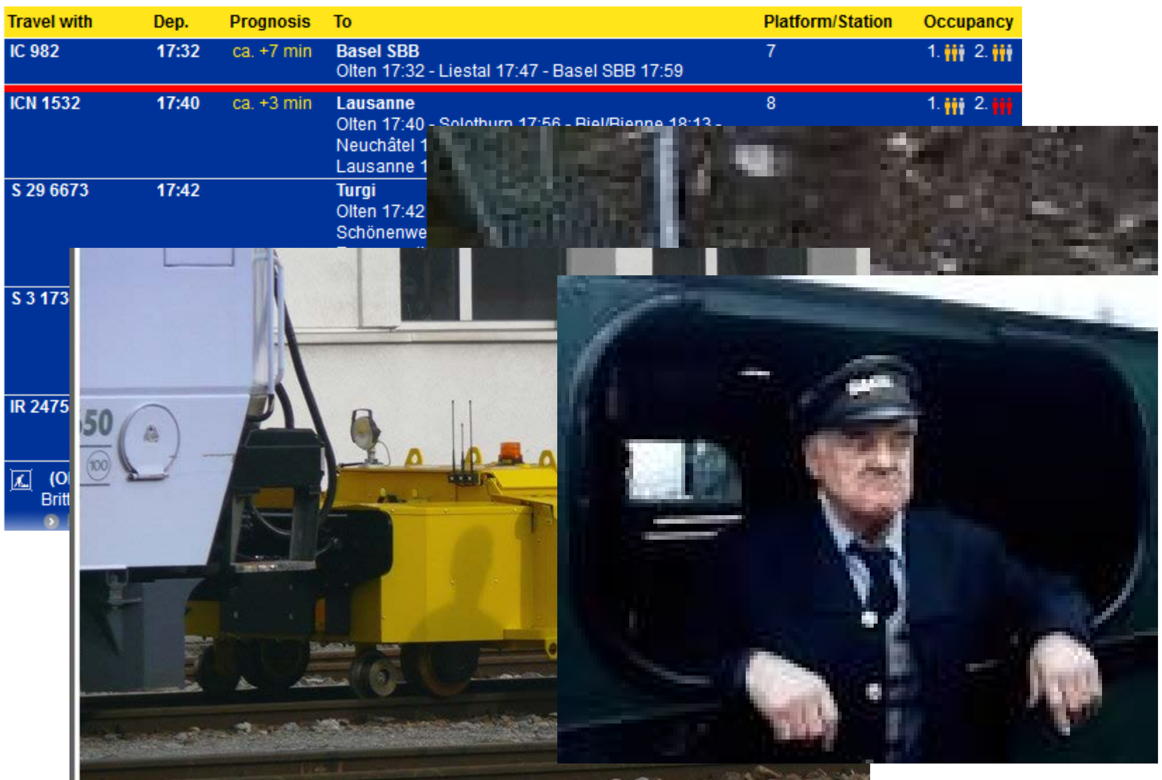
Operations

- Monitoring
  - Punctuality
  - Infrastructure
  - Rolling Stock



Functional requirements from real-time traffic management

Infrastructure Rolling Stock **Operations**

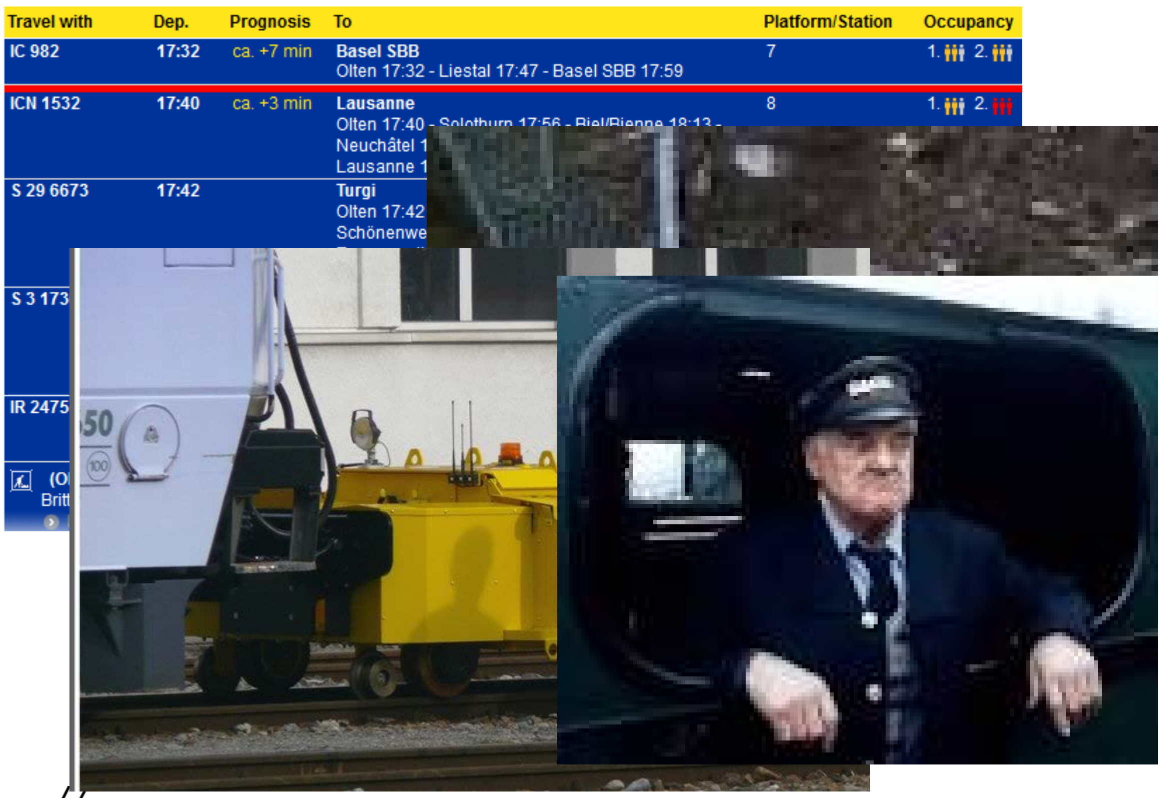


- Monitoring
  - Punctuality
  - Infrastructure
  - Rolling Stock
  - Staff

Functional requirements from real-time traffic management

Infrastructure | Rolling Stock | Operations

- Monitoring
  - Punctuality
  - Infrastructure
  - Rolling Stock
  - Staff



Functional requirements from real-time traffic management

Infrastructure

Rolling Stock

Operations

- Monitoring
  - Punctuality
  - Infrastructure
  - Rolling Stock
  - Staff
- Intervention

Functional requirement (Corman and Meng 2013)

Actions considered by rescheduling:

- re-timing an event (e.g. the arrival at or the departure from a station);
- re-ordering trains on a shared infrastructure;
- local re-routing (e.g. platform change);
- global re-routing;
- re-servicing. ←Breaking connections, cancelling trains, skipping or adding stops

Corman F, Meng L (2013) A review of online dynamic models and algorithms for railway traffic control. In: Intelligent Rail Transportation (ICIRT), 2013 IEEE International Conference on, pp 128–133, DOI 10.1109/ICIRT.2013.6696281

Mathematical models

Continuous time

- Event Scheduling Problem (ESP);
- Alternative Graph (AG);
- Flexible Path (FP).

Discrete time

- Arc Packing Problem (APP) and its weak version (APP’);
- Path Packing Problem (PPP);
- Arc Configuration Problem (ACP);
- Path Configuration Problem (PCP);
- Resource Tree Conflict Graph (RTCG) and Tree Conflict Graph (TCG);
- Resource Conflict Graph (RCG);
- REFormulated Simultaneous train Rerouting and Rescheduling (REF-SRR).

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Mathematical models

Continuous time

- Discrete events: v_S^z
- Times: $t_S^z \geq 0$
- Fixed routes
  - Train run:

$$cType : t_{S_2}^z - t_{S_1}^z \geq f_{(S_1,S_2)}^z$$

Discrete time

Mathematical models

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Discrete time

- $\underline{cRun}$: minimum running time
- $\overline{cRun}$: maximum running time
- $\underline{cDwell}$: minimum dwell time
- $\overline{cDwell}$: maximum dwell time
- $\underline{cPass}$: earliest time*
- $\overline{cPass}$: latest time
- $\underline{cOverall}$/ $\overline{cOverall}$: minimum/maximum running time from the departure from the first station v to the arrival at destination

* Sometimes referred to as passing constraints

Mathematical models

Continuous time

- Discrete events: v_S^z
- Times: $t_S^z \geq 0$
- Fixed routes
  - Train run:

$$cType : t_{S_2}^z - t_{S_1}^z \geq f_{(S_1,S_2)}^z$$

  - Interactions

$$cType : t_{S_2}^w - t_{S_1}^z \geq f_{(S_1,S_2)}^{z,w}$$

- $\underline{cConn}$: minimum connection time
- $\overline{cConn}$: maximum connection time
- $cDep$/ $\overline{cDep}$: minimum/maximum

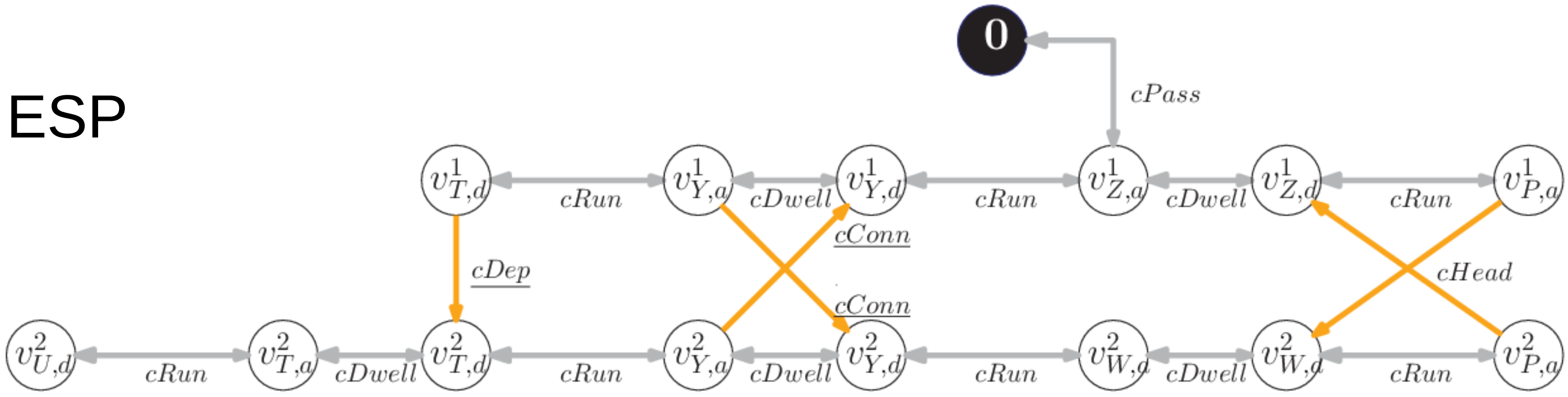
$$cHead : (t_{S_3}^w - t_{S_2}^z \geq f_{(S_1,S_2,S_3,S_4)}^{z,w}) \vee (t_{S_1}^z - t_{S_4}^w \geq f_{(S_1,S_2,S_3,S_4)}^{w,z})$$

Discrete time

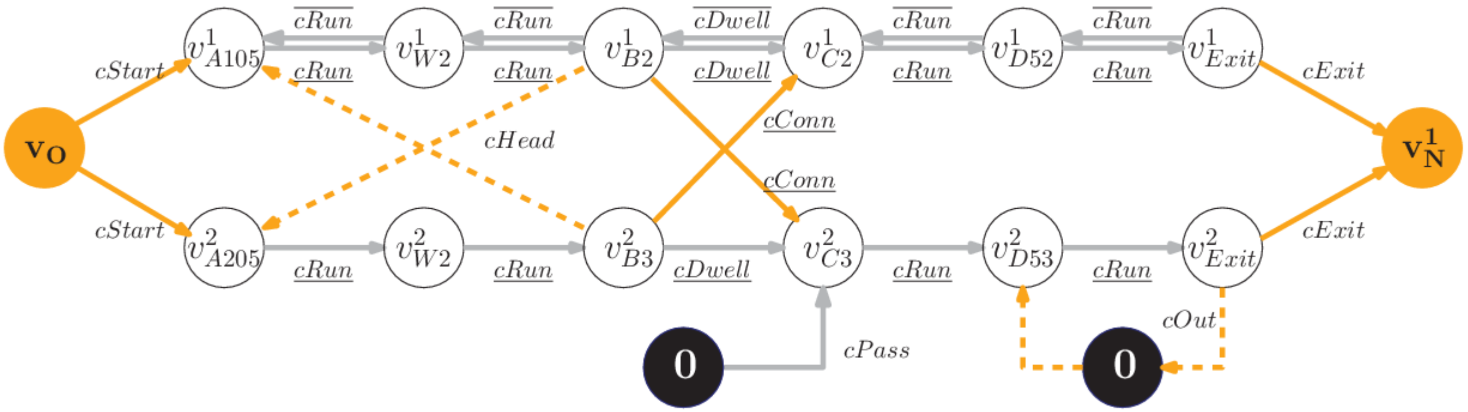
** usually defined for ESP only

Mathematical models

- ESP



- AG



Mathematical models

Continuous time

- Discrete events: v_S^z
- Times: $t_S^z \geq 0$
- With routing: $x_S^z \in \{0,1\}$

$$\begin{cases} x_{O_z}^z = 1 \\ x_{D_z}^z = 1 \end{cases} \quad \forall z$$

$$\sum_{i \in \delta_-(J)} x_i^z = \sum_{v \in \delta_+(J)} x_i^z \quad \forall J, \forall z$$

Discrete time

Mathematical models

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Discrete time

-Train run:

$$cType: t_{S_2}^z - t_{S_1}^z + M(1 - x_i^z) \geq f_{(S_1,S_2)}^z$$

-Interactions

$$cType: t_{S_2}^w - t_{S_1}^z + M(1 - x_i^z) + M(1 - x_j^w) \geq f_{(S_1,S_2)}^{z,w}$$

$$cHead: t_{S_3}^w - t_{S_2}^z + Mh_i^{z,w} + M(1 - x_j^z) + M(1 - x_k^w) \geq f_{(S_1,S_2,S_3,S_4)}^{z,w}$$
$$t_{S_1}^z - t_{S_4}^w + M(1 - h_i^{z,w}) + M(1 - x_j^z) + M(1 - x_k^w) \geq f_{(S_1,S_2,S_3,S_4)}^{w,z}$$

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Mathematical models

Continuous time

Discrete time

- Action: a
- Decision: $x_a^z \in \{0,1\}$
  - Train run: Uniqueness and continuity

$$\sum_{a \in \delta_+(s_z)} x_a^z \leq 1 \quad \forall z$$

$$\sum_{a \in \delta_+(v)} x_a^z - \sum_{a \in \delta_-(v)} x_a^z = 0 \quad \forall v \notin \{s_z, t_z\}, \forall z$$

- Decision for a sequence of actions: $x_p^z \in \{0,1\}$
  - Train run: uniqueness:

$$\sum_{p} x_p^z \leq 1 \quad \forall z$$

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  - Train run: uniqueness:

$$\sum_p x_p^z \leq 1 \quad \forall z$$

Discrete time

- *aStart*: departures from the first node;
- *aEnd*: arrivals at the last station;
- *aRun*: runs;
- *aDwell*: dwells in stations;
- *aInfeasibility*: infeasibility

Mathematical models

Continuous time

Discrete time

- Action: a
- Decision: $x_a^z \in \{0,1\}$
  - Conflicts

$$\sum_{(z,a)\in C} x_a^z \leq 1 \quad \forall C \in \mathscr{C}_r, r$$

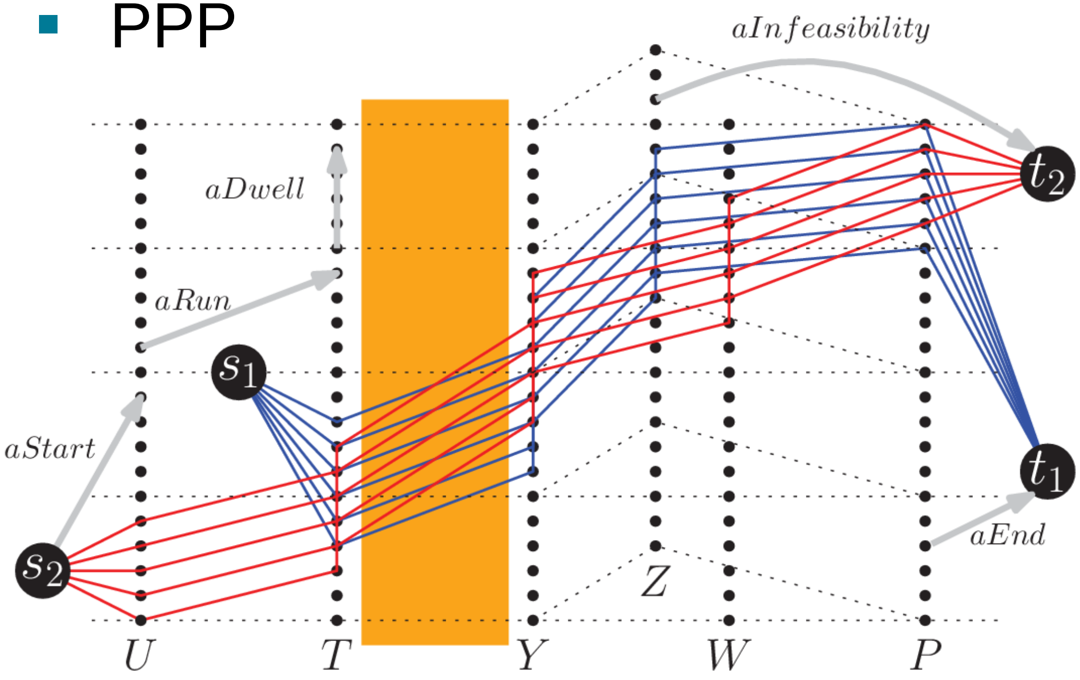
$$x_a^z + x_b^w \leq 1$$

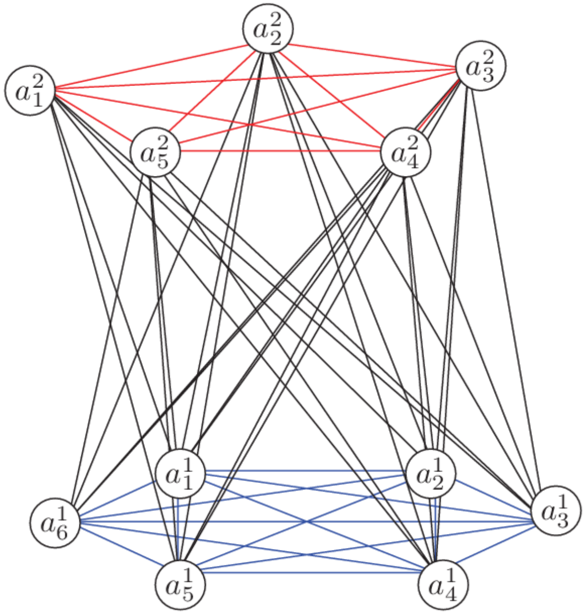
- Decision for a sequence of actions: $x_p^z \in \{0,1\}$
  - Conflicts:

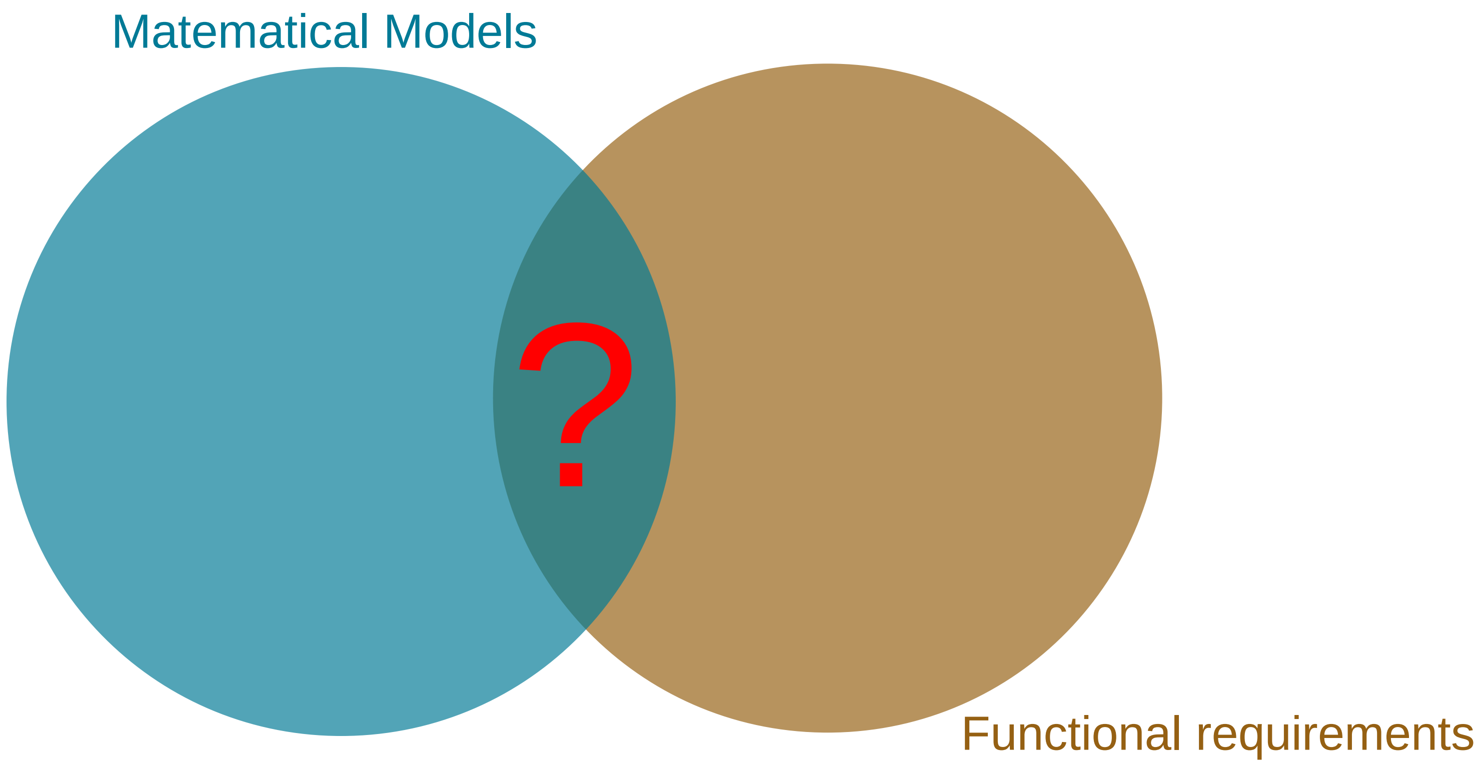
$$\sum_{(z,p)\cap C\neq\emptyset} x_p^z \leq 1 \quad \forall C \in \mathscr{C}_r, r$$

Mathematical models

- PPP







Meeting functional requirements with mathematical models

Table 1 Models and functional requirements: × means that the model satisfies the functional requirement; (×) means that the requirement is satisfied off-line (i.e. for timetabling); ∘ means that there is a model extension which satisfies the requirement

		continuous time			discrete time						
		ESP	AG	FP	APP'	TCG	APP/ ACP	RTCG	REF–SRR	PPP/ PCP	RCG
infrastructure	macroscopic	×			×		×			×	
	microscopic		×	×		×		×	×		×
rolling stock	max. speed	×	×	×	×	×	×	×	×	×	×
	real. speed		∘				∘			×	×
operations	timetable		×	×					×		×
	closed tracks		×	×					×		×
	re-timing	(×)	×	×	(×)	(×)	(×)	(×)	×	(×)	×
	re-ordering	(×)	×	×	(×)	(×)	(×)	(×)	×	(×)	×
	re-routing		∘	×	(×)	(×)	(×)	(×)	×	(×)	×
	connections		∘								×
	cancel train		∘	×	(×)	(×)	(×)	(×)	×	(×)	×
		pairwise conflicts					on tracks			on paths	
							conflict cliques				

Conclusion and outlook

- Models that satisfy all the functional requirements identified exist.
- Some models are very similar to each others.

- Future work:
  - Develop a model for rescheduling starting from the current existing models, which already satisfy the functional requirements.
  - Take advantage of the similarity with scheduling models that have fast solving techniques.

Thank you for your kind attention!

Questions?