Meeting functional requirements for real-time railway traffic management with mathematical models

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Literature reviews


Modelling needs assumptions…

A farmer has some chickens who don't lay any eggs. The farmer calls a physicist to help. The physicist does some calculation and says "I have a solution but it only works for spherical chickens in a vacuum!".
Contents

- Introduction and motivation
- Functional requirements
- Mathematical models
- Meeting functional requirements with mathematical models
- Conclusion and outlook
# Functional requirements from real-time traffic management

<table>
<thead>
<tr>
<th>Infrastructure</th>
<th>Rolling Stock</th>
<th>Operations</th>
</tr>
</thead>
</table>

Functional requirements from real-time traffic management

Infrastructure  Rolling Stock  Operations
Example: Railway network of the Railway operations laboratory (EBL) at the ETH Zurich

Source: archive of the institute for transport planning and systems (IVT)
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Source: archive of the institute for transport planning and systems (IVT)
Example: Railway network of the Railway operations laboratory (EBL) at the ETH Zurich

- Macroscopic representation of the entire network
Example: Railway network of the Railway operations laboratory (EBL) at the ETH Zurich

Source: archive of the institute for transport planning and systems (IVT)
Example: Railway network of the Railway operations laboratory (EBL) at the ETH Zurich

- Microscopic representation of station Ypslikon

Source: archive of the institute for transport planning and systems (IVT)
Functional requirements from real-time traffic management

Infrastructure
- Macroscopic
- Microscopic

Rolling Stock

Operations
Functional requirements from real-time traffic management

Infrastructure
- Macroscopic
- Mesoscopic
- Microscopic

Rolling Stock

Operations
Functional requirements from real-time traffic management

Infrastructure

- Macroscopic
- Mesoscopic
- Microscopic

Rolling Stock

Operations

Functional requirement (Radtke 2014)
"the microscopic infrastructure is not only suitable but even mandatory for exact running time calculation, timetable construction, possession planning and railway operational simulation, conflict detection and resolution."

Functional requirements from real-time traffic management

Infrastructure
- Macroscopic
- Mesoscopic
- Microscopic

Rolling Stock

Operations

Functional requirement (Radtke 2014)
"the microscopic infrastructure is not only suitable but even mandatory for exact running time calculation, timetable construction, possession planning and railway operational simulation, conflict detection and resolution."

Functional requirements from real-time traffic management
Example: Speed limits on consecutive sections
Example: Speed limits on consecutive sections

Maximal speed

(speed limit)

(speed)

section

1  2  3
Example: Speed limits on consecutive sections
Example: Speed limits on consecutive sections

Maximal speed

Realizable speed

Speed limits on consecutive sections

1 2 3

section
## Functional requirements from real-time traffic management

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</thead>
<tbody>
<tr>
<td></td>
<td>Maximal speed</td>
<td>Realizable speed</td>
</tr>
</tbody>
</table>
Functional requirements from real-time traffic management

- Infrastructure
- Rolling Stock
  - Maximal speed
  - Realizable speed
  - Length
- Operations
Functional requirements from real-time traffic management

Infrastructure  Rolling Stock  Operations

- Maximal speed
- Realizable speed
- Length
## Functional requirements from real-time traffic management

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Functional requirements from real-time traffic management

- Infrastructure
- Rolling Stock
- Operations
  - Monitoring
Functional requirements from real-time traffic management

- Monitoring
- Punctuality
Functional requirements from real-time traffic management

- Monitoring
- Punctuality
- Infrastructure
Functional requirements from real-time traffic management

Operations
- Monitoring
- Punctuality
- Infrastructure
- Rolling Stock
Functional requirements from real-time traffic management

Infrastructure  Rolling Stock  Operations

- Monitoring
- Punctuality
- Infrastructure
- Rolling Stock
- Staff

<table>
<thead>
<tr>
<th>Travel with</th>
<th>Dep.</th>
<th>Prognosis</th>
<th>To</th>
<th>Platform</th>
<th>Station</th>
<th>Occupancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>IC 982</td>
<td>17:32</td>
<td>ca. +7 min</td>
<td>Basel SBB Oll 17:32</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ICN 1532</td>
<td>17:40</td>
<td>ca. +3 min</td>
<td>Lausanne Oll 17:40</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 20 0673</td>
<td>17:42</td>
<td></td>
<td>Turgi Oll 17:42</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Functional requirements from real-time traffic management

Infrastructure

Rolling Stock

Operations

- Monitoring
- Punctuality
- Infrastructure
- Rolling Stock
- Staff
Functional requirements from real-time traffic management

- Infrastructure
- Rolling Stock
- Operations
  - Monitoring
  - Punctuality
  - Infrastructure
  - Rolling Stock
  - Staff
  - Intervention

Functional requirement (Corman and Meng 2013)

Actions considered by rescheduling:

- re-timing an event (e.g. the arrival at or the departure from a station);
- re-ordering trains on a shared infrastructure;
- local re-routing (e.g. platform change);
- global re-routing;
- re-servicing. → Breaking connections, cancelling trains, skipping or adding stops

Mathematical models

Continuous time
- Event Scheduling Problem (ESP);
- Alternative Graph (AG);
- Flexible Path (FP).

Discrete time
- Arc Packing Problem (APP) and its weak version (APP’);
- Path Packing Problem (PPP);
- Arc Configuration Problem (ACP);
- Path Configuration Problem (PCP);
- Resource Tree Conflict Graph (RTCG) and Tree Conflict Graph (TCG);
- Resource Conflict Graph (RCG);
- REFormulated Simultaneous train Rerouting and Rescheduling (REF-SRR).
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Mathematical models

Continuous time

- Discrete events: $v^z_S$
- Times: $t^z_S \geq 0$
- Fixed routes
  - Train run:

$$cType: t^z_{S_2} - t^z_{S_1} \geq f^z(s_1, s_2)$$

Discrete time
Continuous time

- Discrete events: $v_S^z$
- Times: $t_S^z \geq 0$
- Fixed routes
  - Train run:
    \[ cType : t_S^{z_2} - t_S^{z_1} \geq f^{z}(s_1, s_2) \]

Discrete time

- $cRun$: minimum running time
- $cRun$: maximum running time
- $cDwell$: minimum dwell time
- $cDwell$: maximum dwell time
- $cPass$: earliest time*
- $cPass$: latest time
- $cOverall/cOverall$: minimum/maximum running time from the departure from the first station $v$ to the arrival at destination

* Sometimes referred to as passing constraints
**Mathematical models**

**Continuous time**
- Discrete events: $v_S^z$
- Times: $t_S^z \geq 0$
- Fixed routes
  - Train run:
    \[
    cType : t_{S_2}^z - t_{S_1}^z \geq f_{(s_1,s_2)}^z
    \]
  - Interactions
    \[
    cType : t_{S_2}^w - t_{S_1}^z \geq f_{(s_1,s_2)}^{z,w}
    \]
    \[
    cHead : (t_{S_3}^w - t_{S_2}^z \geq f_{(s_1,s_2,s_3,s_4)}^{z,w}) \lor (t_{S_2}^z - t_{S_4}^w \geq f_{(s_1,s_2,s_3,s_4)}^{w,z})
    \]

**Discrete time**
- $cConn$: minimum connection time
- $cConn$: maximum connection time
- $cDep/cDep$: minimum/maximum

**usually defined for ESP only**
Mathematical models

- ESP

- AG
Mathematical models

Continuous time

- Discrete events: $v^z_S$
- Times: $t^z_S \geq 0$
- With routing: $x^z_S \in \{0,1\}$

Discrete time

\[
\begin{aligned}
\begin{cases}
  x^z_O_z = 1 \\
  x^z_D_z = 1
\end{cases}
\forall z \\
\sum_{i \in \delta_-(J)} x^z_i = \sum_{v \in \delta_+(J)} x^z_i \quad \forall J, \forall z
\end{aligned}
\]
Mathematical models

Continuous time

- Discrete events: \( v_S^Z \)
- Times: \( t_S^Z \geq 0 \)
- With routing: \( x_S^Z \in \{0,1\} \)

\[
\begin{align*}
    x_{O_S}^Z & = 1 & \forall Z \\
    x_{D_S}^Z & = 1 & \forall Z \\
    \sum_{i \in \delta^-(J)} x_i^Z & = \sum_{v \in \delta^+(J)} x_v^Z & \forall J, \forall Z
\end{align*}
\]

Discrete time

- Train run:
  \[
  c\text{Type} : t_s^Z - t_s^{Z_1} + M(1 - x_i^Z) \geq f_{(s_1,s_2)}^Z
  \]

- Interactions
  \[
  c\text{Type} : t_s^w - t_s^{Z_1} + M(1 - x_i^Z) + M(1 - x_j^w) \geq f_{(s_1,s_2)}^{Z,w}
  \]
  \[
  c\text{Head} : t_s^w - t_s^{Z_2} + Mh_i^{Z,w} + M(1 - x_j^w) + M(1 - x_k^w) \geq f_{(s_1,s_2,s_3,s_4)}^{Z,w}
  \]
  \[
  t_S^Z - t_S^{Z_1} + M(1 - h_i^{Z,w}) + M(1 - x_j^Z) + M(1 - x_k^Z) \geq f_{(s_1,s_2,s_3,s_4)}^{Z,w}
  \]
Mathematical models

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Mathematical models

Continuous time

- Action: \( a \)
- Decision: \( x_{a}^{z} \in \{0,1\} \)
  - Train run: Uniqueness and continuity
    \[
    \sum_{a \in \delta_{\pm}(s_{z})} x_{a}^{z} \leq 1 \quad \forall z
    \]
    \[
    \sum_{a \in \delta_{\pm}(v)} x_{a}^{z} - \sum_{a \in \delta_{-}(v)} x_{a}^{z} = 0 \quad \forall v \notin \{s_{z},t_{z}\}, \forall z
    \]

- Decision for a sequence of actions: \( x_{p}^{z} \in \{0,1\} \)
  - Train run: uniqueness:
    \[
    \sum_{p} x_{p}^{z} \leq 1 \quad \forall z
    \]

Discrete time

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Mathematical models

Continuous time

- Action: \( a \)
- Decision: \( x^z_a \in \{0,1\} \)
  - Train run: Uniqueness and continuity
    \[
    \sum_{a \in \delta_+ (s_z)} x^z_a \leq 1 \quad \forall z
    \]
    \[
    \sum_{a \in \delta_+ (v)} x^z_a - \sum_{a \in \delta_- (v)} x^z_a = 0 \quad \forall v \notin \{s_z, t_z\}, \forall z
    \]
- Decision for a sequence of actions: \( x^Z_p \in \{0,1\} \)
  - Train run: uniqueness:
    \[
    \sum_p x^Z_p \leq 1 \quad \forall z
    \]

Discrete time

- \( aStart \): departures from the first node;
- \( aEnd \): arrivals at the last station;
- \( aRun \): runs;
- \( aDwell \): dwells in stations;
- \( aInfeasibility \): infeasibility
Mathematical models

Continuous time

- Action: \( a \)
- Decision: \( x^z_a \in \{0,1\} \)
  - Conflicts
    \[
    \sum_{(z,a) \in C} x^z_a \leq 1 \quad \forall C \in \mathcal{C}_r, r
    \]
    \[
    x^z_a + x^w_b \leq 1
    \]

Discrete time

- Decision for a sequence of actions: \( x^z_p \in \{0,1\} \)
  - Conflicts:
    \[
    \sum_{(z,p) \cap C \neq \emptyset} x^z_p \leq 1 \quad \forall C \in \mathcal{C}_r, r
    \]
Mathematical models

- PPP
Meeting functional requirements with mathematical models
### Table 1: Models and functional requirements

× means that the model satisfies the functional requirement; (×) means that the requirement is satisfied off-line (i.e. for timetabling); ○ means that there is a model extension which satisfies the requirement.

<table>
<thead>
<tr>
<th></th>
<th>continuous time</th>
<th>discrete time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ESP</td>
<td>AG</td>
</tr>
<tr>
<td>infrastructure</td>
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<td></td>
</tr>
<tr>
<td>macroscopic</td>
<td>×</td>
<td>×</td>
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<tr>
<td>microscopic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rolling stock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>max. speed</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>real. speed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>operations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>timetable closed</td>
<td>(×)</td>
<td>×</td>
</tr>
<tr>
<td>tracks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>re-timing</td>
<td>(×)</td>
<td>×</td>
</tr>
<tr>
<td>re-ordering</td>
<td></td>
<td></td>
</tr>
<tr>
<td>re-routing</td>
<td>(×)</td>
<td>×</td>
</tr>
<tr>
<td>connections</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cancel train</td>
<td>(×)</td>
<td>×</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>pairwise conflicts</th>
<th>on tracks</th>
<th>on paths</th>
</tr>
</thead>
<tbody>
<tr>
<td>conflicts</td>
<td></td>
<td></td>
<td>conflict cliques</td>
</tr>
</tbody>
</table>
Conclusion and outlook

- Models that satisfy all the functional requirements identified exist.
- Some models are very similar to each others.

Future work:
- Develop a model for rescheduling starting from the current existing models, which already satisfy the functional requirements.
- Take advantage of the similarity with scheduling models that have fast solving techniques.
Thank you for your kind attention!

Questions?