Preferred citation style

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Distributional assumptions in the representation of random taste heterogeneity

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Mixed Logit

- No a priori constraints on choice of f(β|Ω)

$$P_{ni} = \int_{\beta} \left(\frac{e^{f(\beta, X_{ni})}}{\sum_{j=1}^{I} e^{f(\beta, X_{nj})}} f(\beta|\Omega) \right) d\beta$$

- No closed-form solution for choice-probabilities
  - ➔ Need simulation in estimation and application

Issues for the a-priori distribution

- Generally no a priori knowledge about true distribution
- Limited set of distributions available in standard estimation packages
- Most researchers/practitioners use *Normal*
- Limited success with using alternative distributions

Current choices

- Normal
- Log-normal
- Triangular
- Sb

Effect of poor distributional assumptions

- Unbounded:
  - risk of wrong conclusions wrt existence of counter-intuitively signed coefficients and related trade-offs (e.g. VTTS)
  - risk especially great with symmetrical distributions
- Strictly bounded:
  - can prevent data or model specification issues to manifest themselves

Effect of poor distributional assumptions

- Symmetry
  - Over/underestimation of the tails of the distributions
  - Bias in the mean recovery with skewed underlying distributions
- Consideration of mass points

Framework

- Recovery of true distribution
  - Normal
  - Truncated Normal
  - Censored Normal
  - Normal with Mass
  - Lognormal
  - S_B

- Four different settings per distribution (24 exp)

- 8,000 independent draws per experiment (ExpertFit used for analysis)

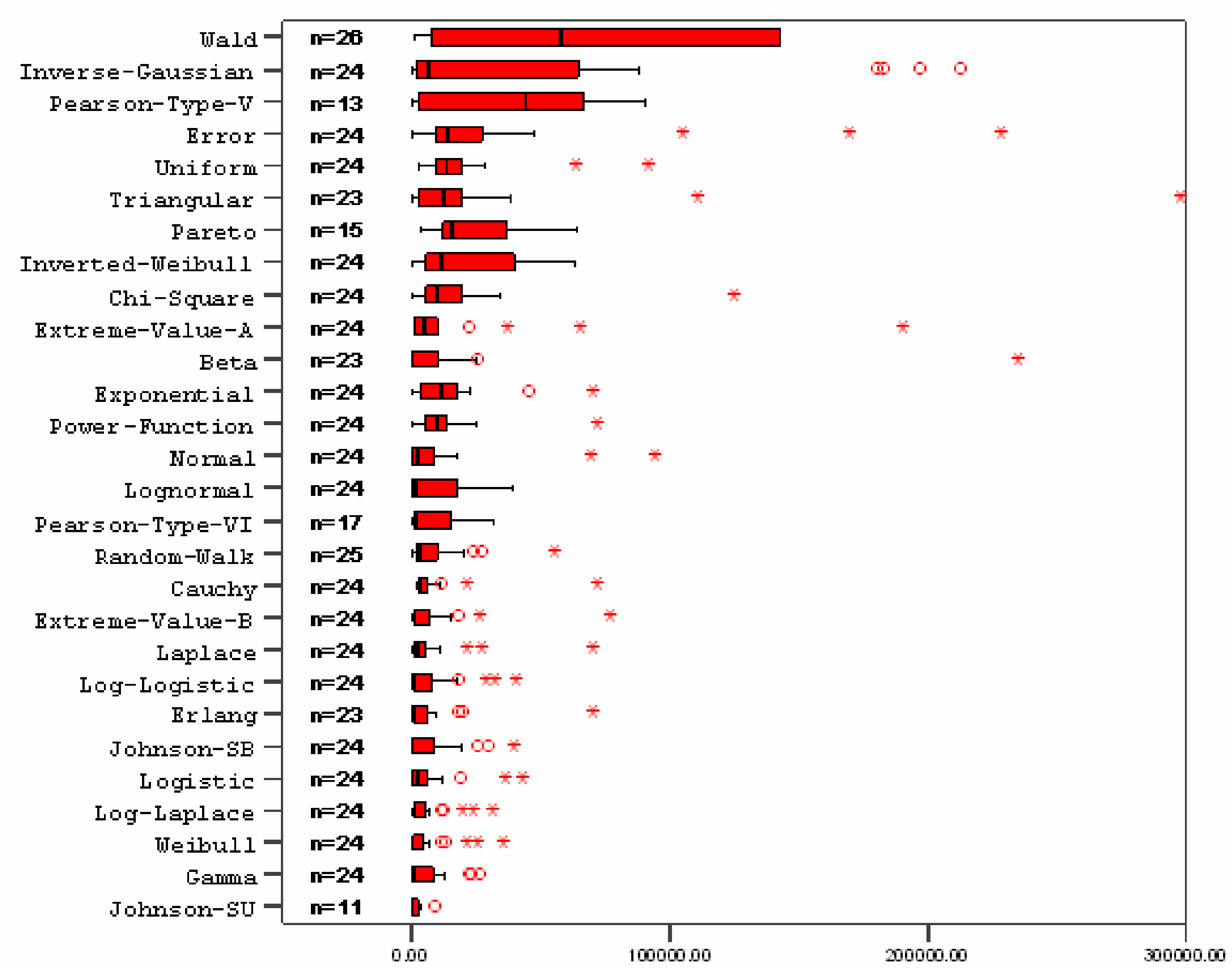
Distributions

- Unbounded (*Normal*, S_u, Cauchy, Error, Exp. Power, extreme value A&B, Laplace, Student‘s t, Logistic)
- Bounded on one side (*Log normal*, Chi2, Erlang, F, Gamma, Inverse Gaussion & Weibull, Log-Laplace, Log-logistic, Pareta, Pearson V&VI, Random walk, Wald, Weibull)
- Bounded on both sides (*Triangular, Uniform,* S_b, Beta, Power function)

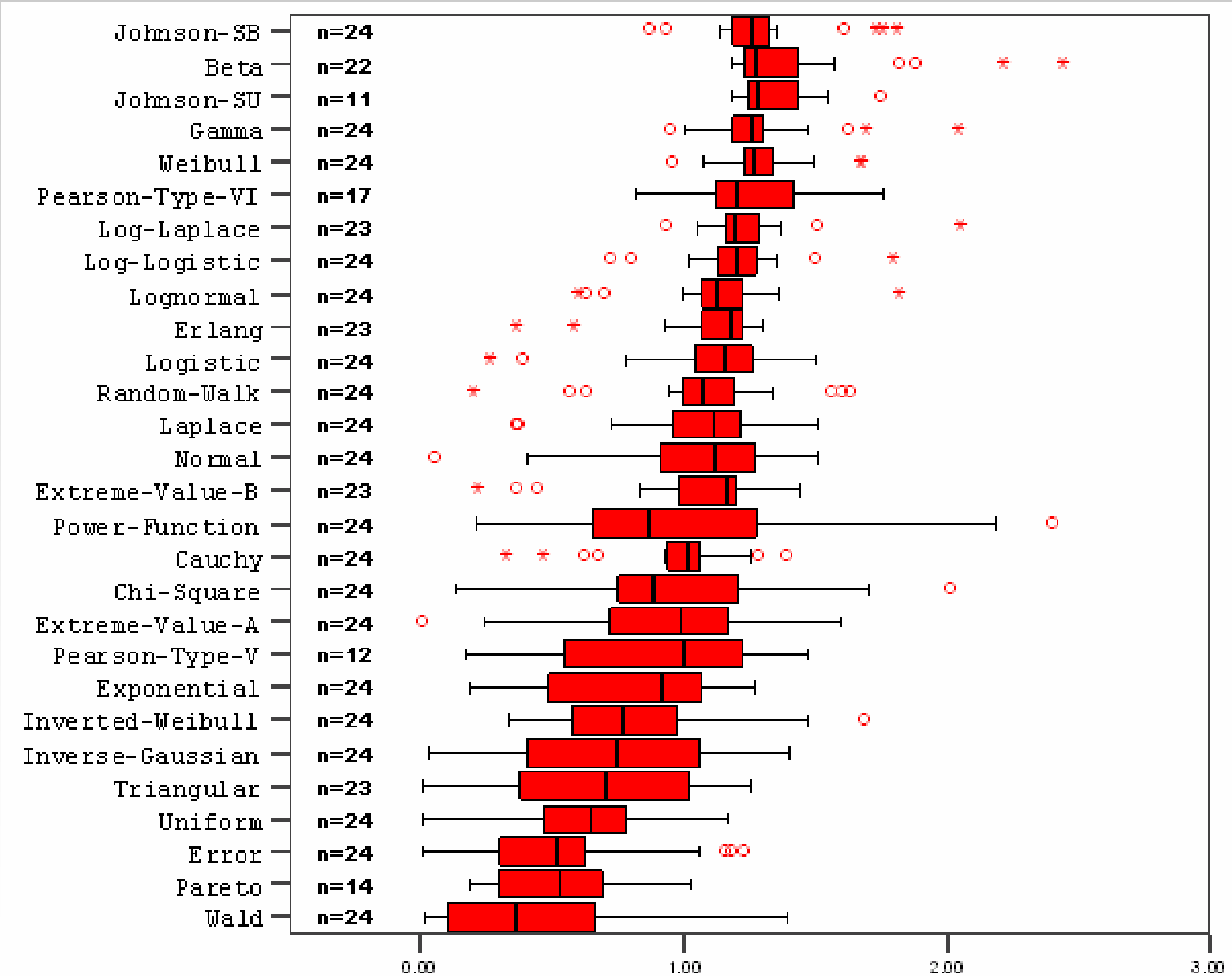
Comparison Criteria

- Equal chi-squared test
- Recovery of true mean & variance
- Likelihood function of observed values
- Recovery of lower & upper octiles
- Difference in density functions, mean & max
- Weighted sum of differences across lower & upper octiles, quartiles and median

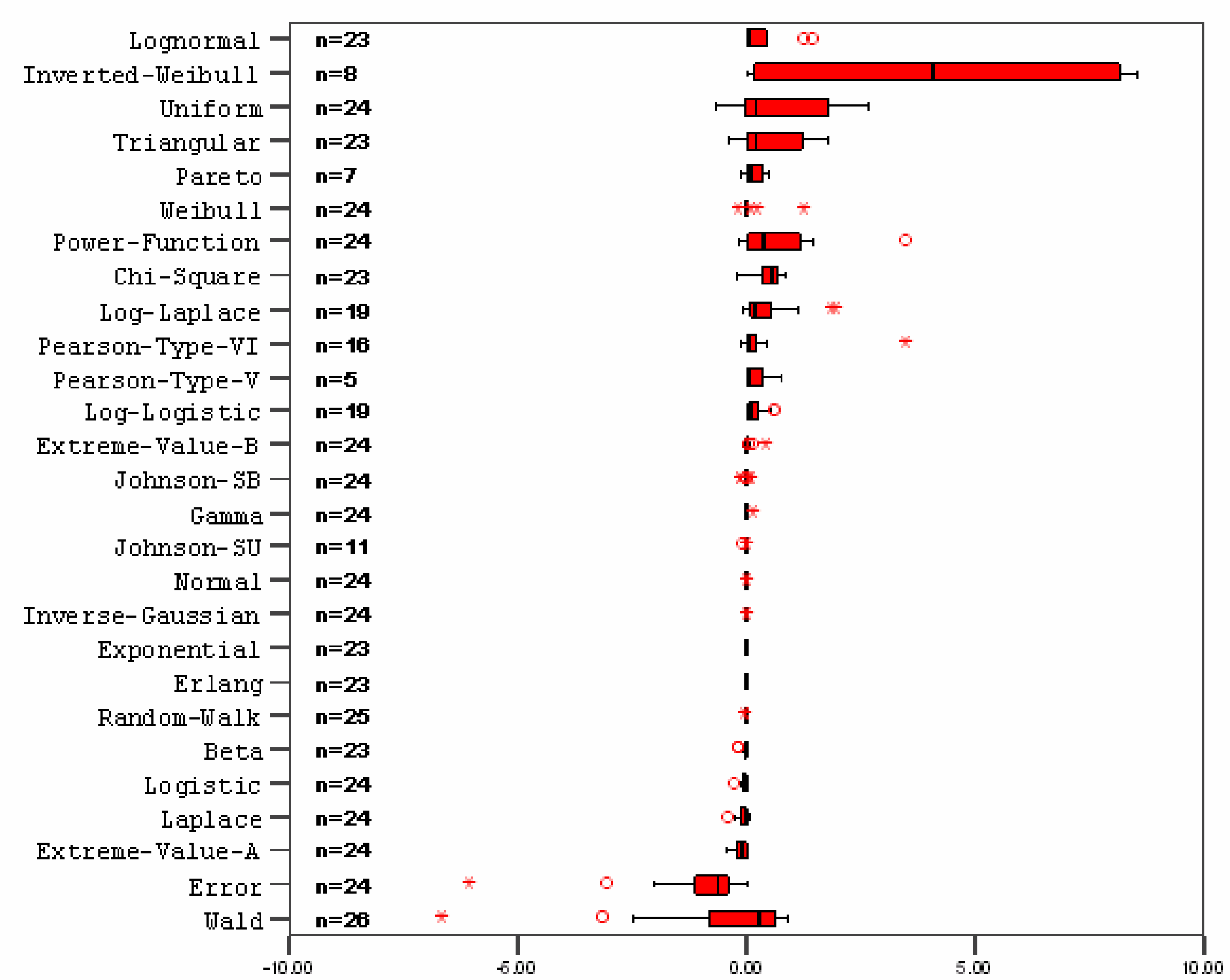
Equal probability chi-square test



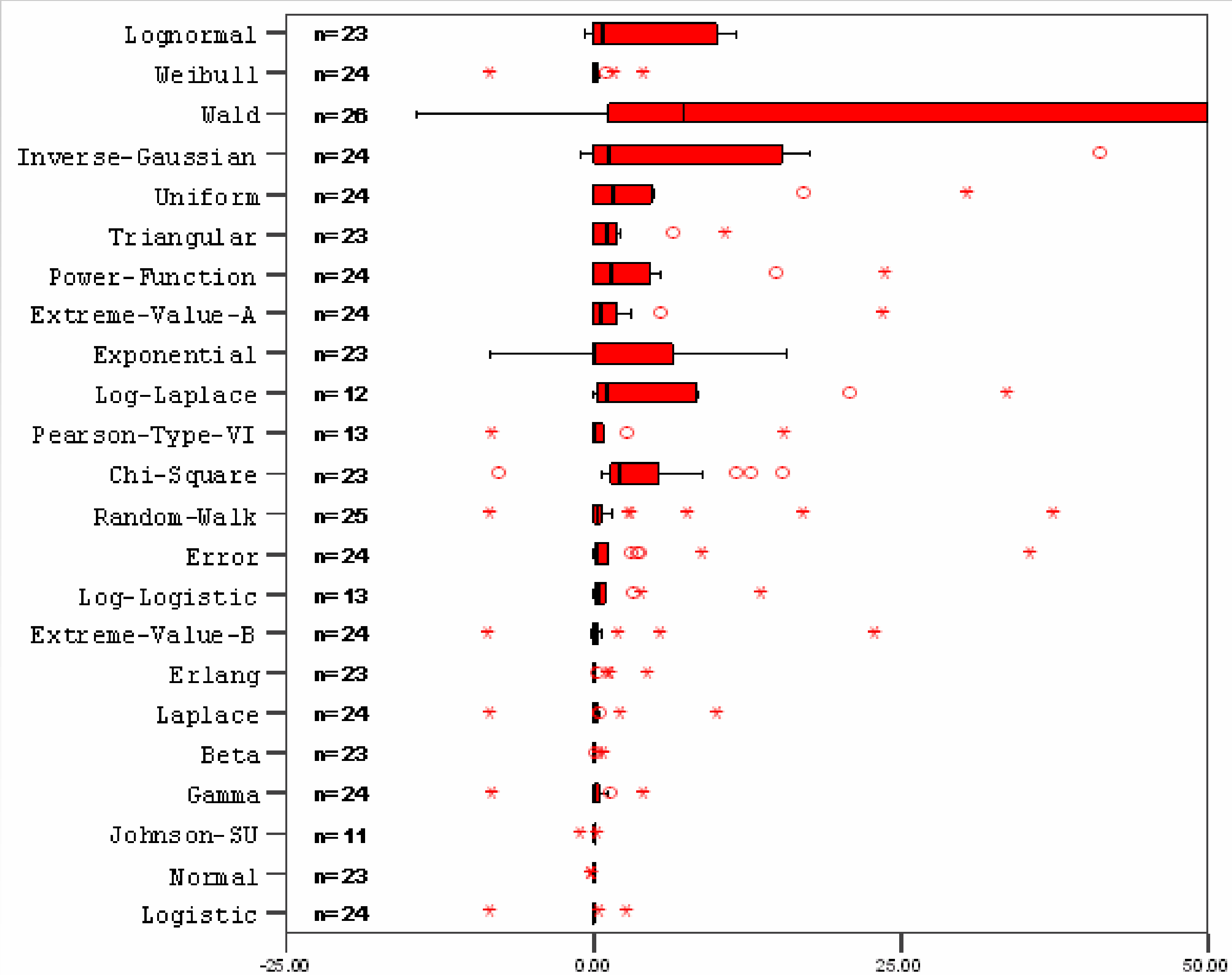
Relative likelihood per observation



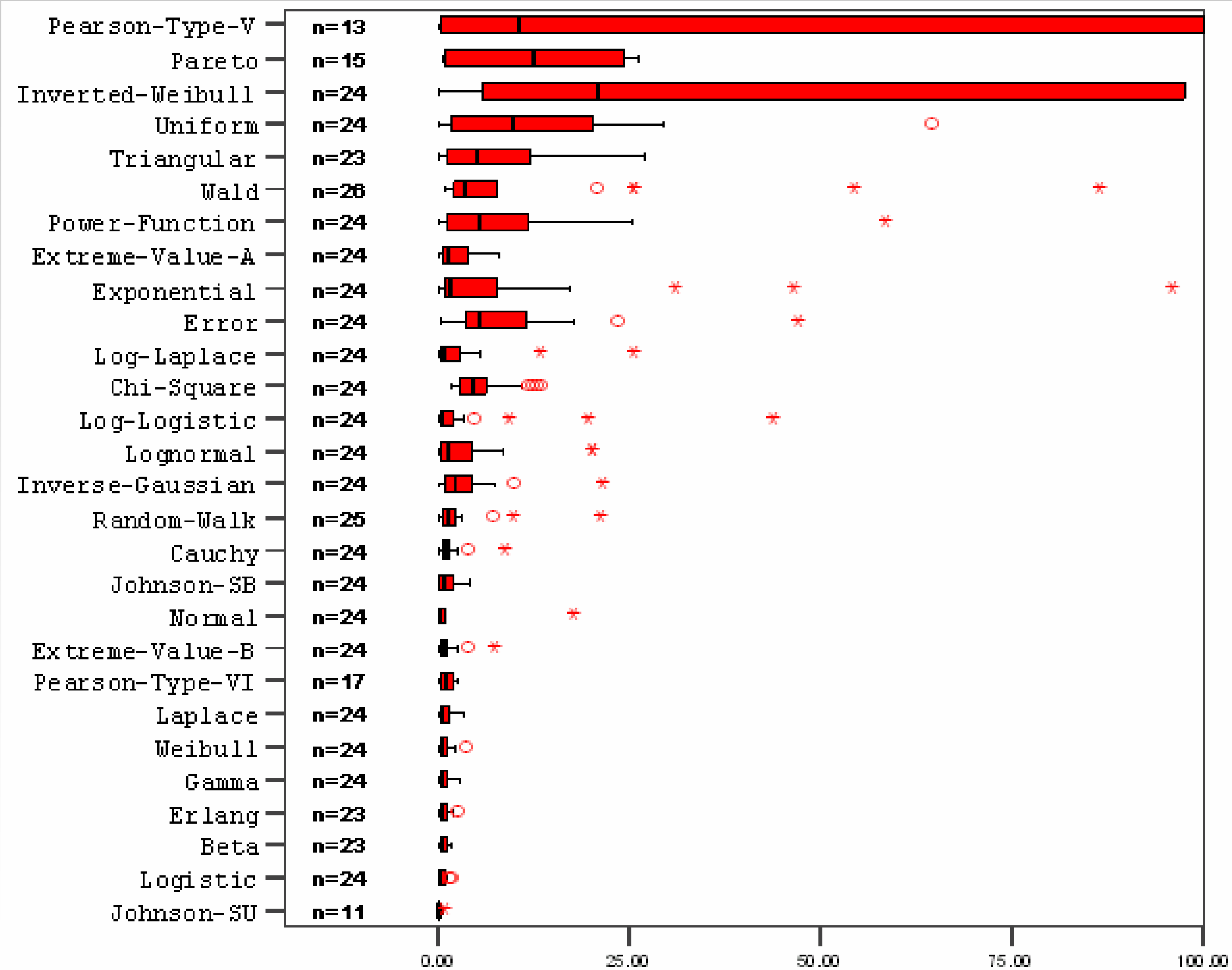
Recovery of mean



Recovery of variance



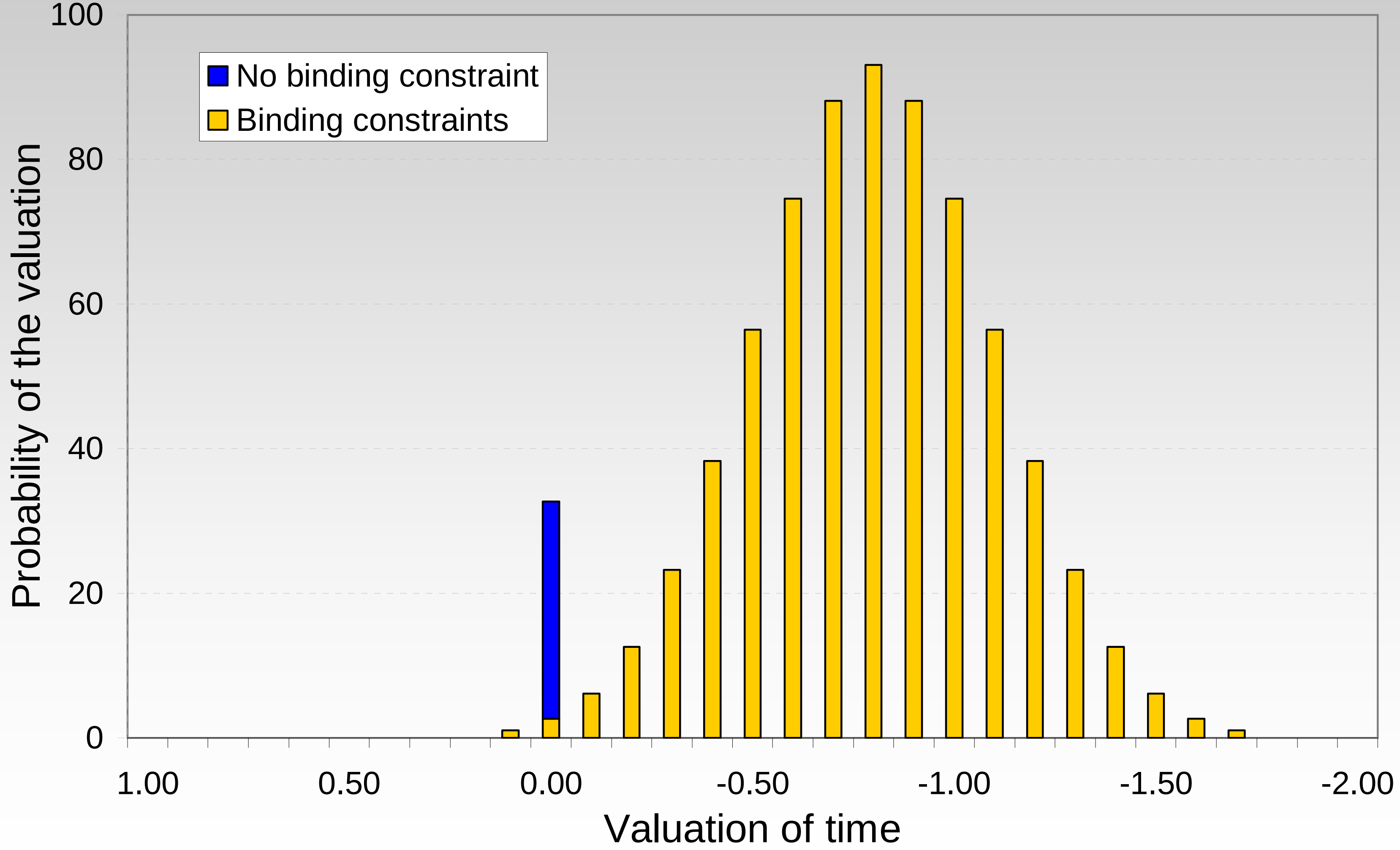
Weighted difference, 5 points



Conclusions

- Group of well performing distributions (Su, Sb, Beta, Gamma, Weibull)
- Normal not too bad, but a priori difficulties
- Software availability
- Still, what to do .....

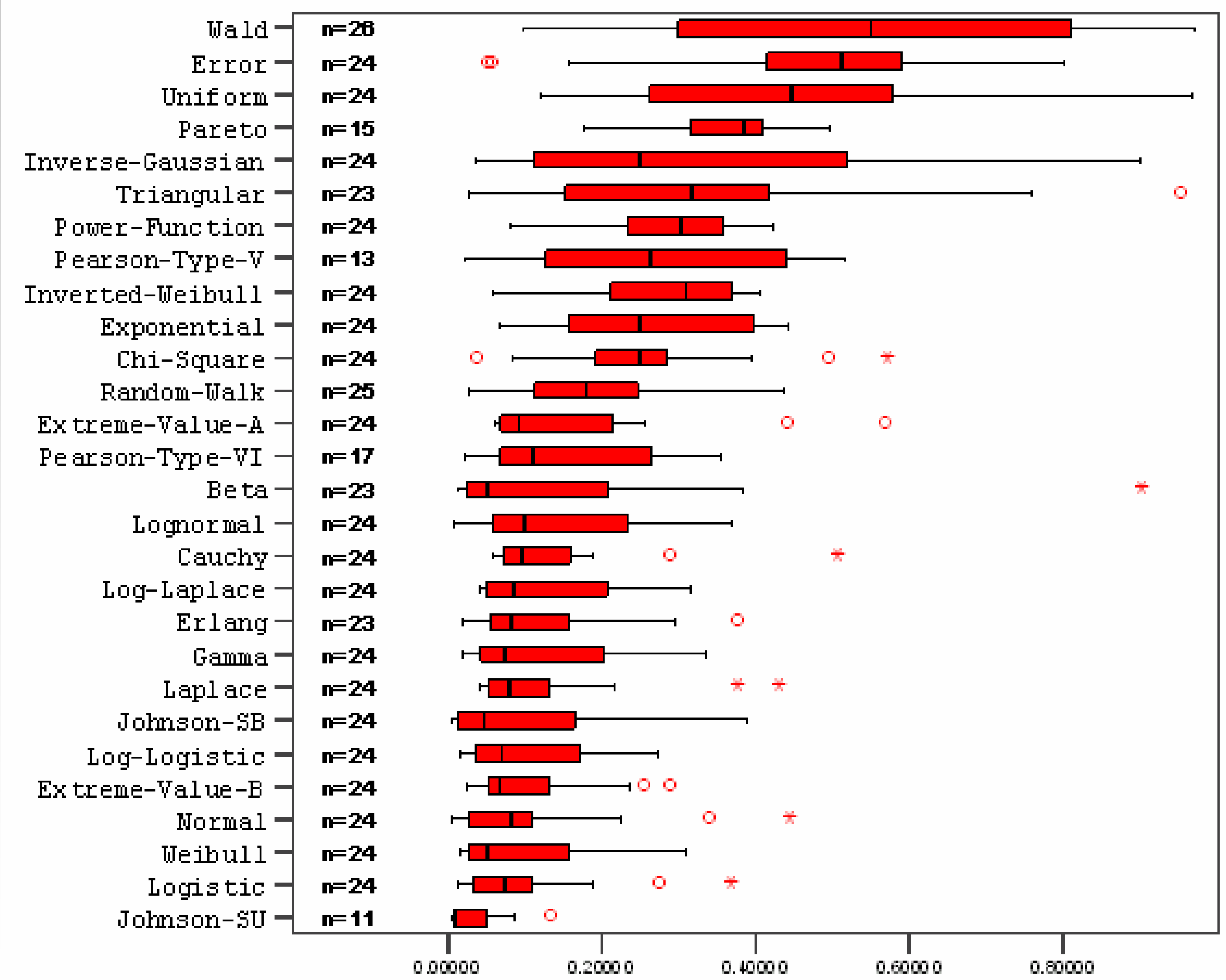
In this case ?



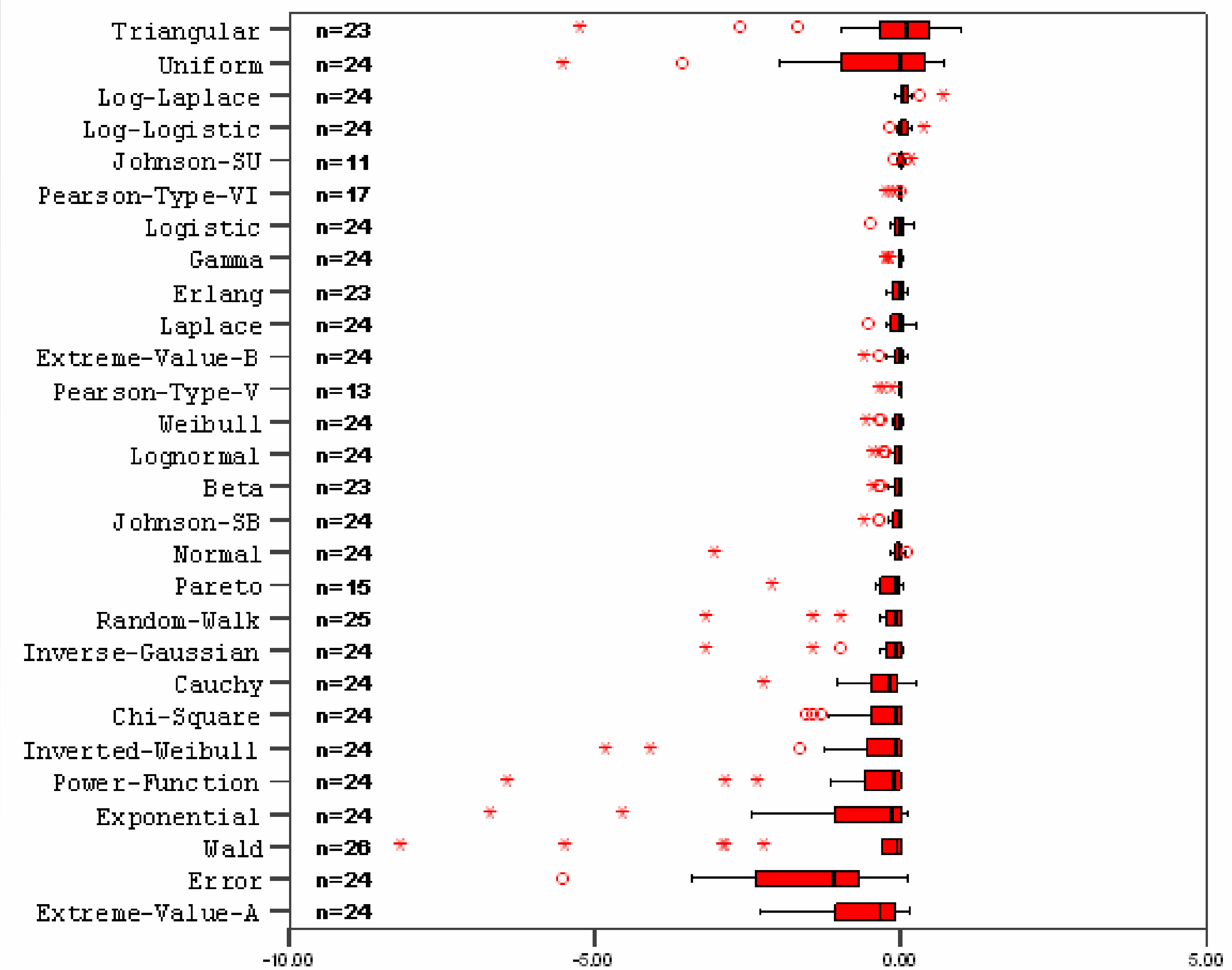
No binding constraint
Binding constraints
100
80
60
40
20
0
Probability of the valuation
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
Valuation of time

Appendices

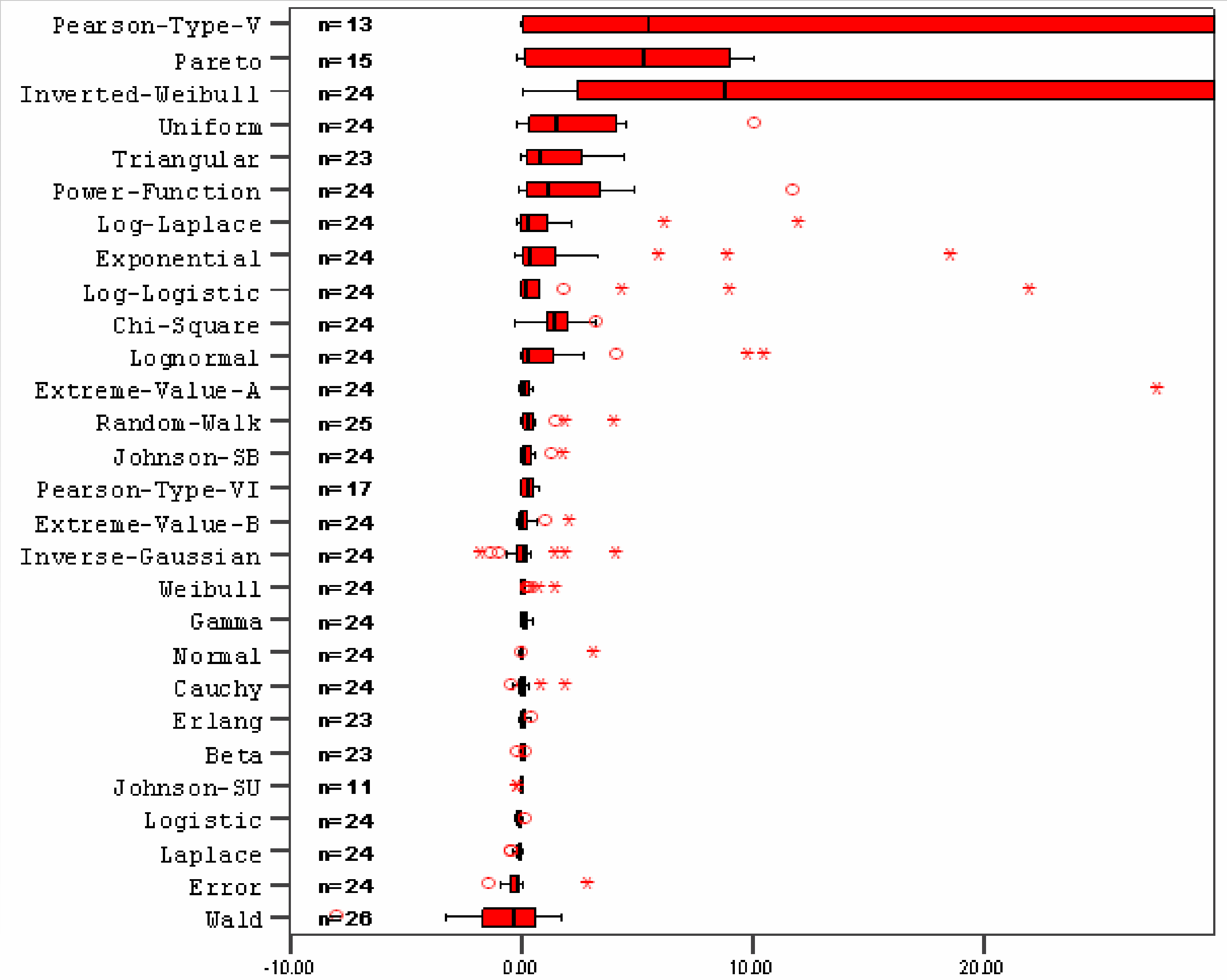
Density function maximum difference



Difference in lower octile



Difference in upper octile



Density function mean difference