Distributional assumptions in the representation of random taste heterogeneity

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- No a priori constraints on choice of $f(\beta|\Omega)$

$$P_{ni} = \int_{\beta} \left( \frac{\sum_{j=1}^{l} e^{f(\beta, X_{nj})}}{\sum_{j=1}^{l} e^{f(\beta, X_{ni})}} \right) f(\beta|\Omega) \, d\beta$$

- No closed-form solution for choice probabilities
  - Need simulation in estimation and application
Issues for the a-priori distribution

- Generally no a priori knowledge about true distribution
- Limited set of distributions available in standard estimation packages
- Most researchers/practitioners use Normal
- Limited success with using alternative distributions
Current choices

- Normal
- Log normal
- Triangular
- Sb
Effect of poor distributional assumptions

- Unbounded:
  - risk of wrong conclusions wrt existence of counter-intuitively signed coefficients and related trade-offs (e.g. VTTS)
  - risk especially great with symmetrical distributions

- Strictly bounded:
  - can prevent data or model specification issues to manifest themselves
Effect of poor distributional assumptions

- Symmetry
  - Over/underestimation of the tails of the distributions
  - Bias in the mean recovery with skewed underlying distributions

- Consideration of mass points
Framework

• Recovery of true distribution
  – Normal
  – Truncated Normal
  – Censored Normal
  – Normal with Mass
  – Lognormal
  – $S_B$

• Four different settings per distribution (24 exp)

• 8,000 independent draws per experiment (ExpertFit used for analysis)
Distributions

• Unbounded (*Normal*, $S_u$, Cauchy, Error, Exp. Power, extreme value A&B, Laplace, Student‘s t, Logistic)

• Bounded on one side (*Log normal*, Chi2, Erlang, F, Gamma, Inverse Gaussion & Weibull, Log Laplace, Log logistic, Pareta, Pearson V&VI, Random walk, Wald, Weibull)

• Bounded on both sides (*Triangular*, *Uniform*, $S_b$, Beta, Power function)
Comparison Criteria

- Equal chi-squared test
- Recovery of true mean & variance
- Likelihood function of observed values
- Recovery of lower & upper octiles
- Difference in density functions, mean & max
- Weighted sum of differences across lower & upper octiles, quartiles and median
Equal probability chi-square test
Relative likelihood per observation
Recovery of mean

- Lognormal: n=23
- Inverted-Weibull: n=8
- Uniform: n=24
- Triangular: n=23
- Pareto: n=7
- Weibull: n=24
- Power-Function: n=24
- Chi-Square: n=23
- Log-Laplace: n=19
- Pearson-Type-VI: n=16
- Pearson-Type-V: n=5
- Log-Logistic: n=19
- Extreme-Value-B: n=24
- Johnson-SB: n=24
- Gamma: n=11
- Johnson-SU: n=24
- Normal: n=24
- Inverse-Gaussian: n=24
- Exponential: n=23
- Erlang: n=23
- Random-Walk: n=25
- Beta: n=23
- Logistic: n=24
- Laplace: n=24
- Extreme-Value-A: n=24
- Error: n=24
- Wald: n=26
Recovery of variance
Weighted difference, 5 points
Conclusions

• Group of well performing distributions (Su, Sb, Beta, Gamma, Weibull)

• Normal not too bad, but a \textit{a priori} difficulties

• Software availability

• Still, what to do .....
In this case?

![Graph showing valuation of time and probability distribution with two categories: No binding constraint and Binding constraints. The x-axis represents valuation of time ranging from -2.00 to 1.00, and the y-axis represents the probability of the valuation ranging from 0 to 100. The graph depicts a distribution with a peak around the value 0, with a higher probability for no binding constraint compared to binding constraints.]
Appendices
Density function maximum difference

Wald  n=28
Error  n=24
Uniform  n=24
Pareto  n=15
Inverse-Gaussian  n=24
Triangular  n=23
Power-Function  n=24
Pearson-Type-V  n=13
Inverted-Weibull  n=24
Exponential  n=24
Chi-Square  n=24
Random-Walk  n=25
Extreme-Value-A  n=24
Pearson-Type-VI  n=17
Beta  n=23
Lognormal  n=24
Cauchy  n=24
Log-Laplace  n=24
Erlang  n=23
Gamma  n=24
Laplace  n=24
Johnson-SB  n=24
Log-Logistic  n=24
Extreme-Value-B  n=24
Normal  n=24
Weibull  n=24
Logistic  n=24
Johnson-SU  n=11
Difference in lower octile

- Triangular: n=23
- Uniform: n=24
- Log-Laplace: n=24
- Log-Logistic: n=24
- Johnson-SU: n=11
- Pearson-Type-VI: n=17
- Logistic: n=24
- Gamma: n=24
- Erlang: n=23
- Laplace: n=24
- Extreme-Value-B: n=24
- Pearson-Type-V: n=13
- Weibull: n=24
- Lognormal: n=24
- Beta: n=23
- Johnson-SB: n=24
- Normal: n=24
- Pareto: n=24
- Random-Walk: n=25
- Inverse-Gaussian: n=24
- Cauchy: n=24
- Chi-Square: n=24
- Inverted-Weibull: n=24
- Power-Function: n=24
- Exponential: n=24
- Wald: n=26
- Error: n=24
- Extreme-Value-A: n=24
Difference in upper octile
Density function mean difference

![Graph showing various density functions and their differences.](image-url)