
Assignment and route choice: Similarities and differences

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What do we do today?

Demand

\[ q(t,i,j,m) \quad k(t,r,m) \sim q(t,r,m) \]

Distribution of demand across the network

What is the task at hand?

Distribution of demand between two locations onto the possible and sensible routes:

- Identification of all sensible routes
- Calculation of generalised costs as a function of route demands
- Selection of the approach used to allocate demand between routes
What are the differences?

User equilibrium is normative, i.e. the solution of a mathematical programme:

User equilibrium:
- \( k'_{ijr} = k'(q'_{ijr}) \), for all routes \( r \) between \( i \) and \( j \) with \( q'_{ijr} > 0 \); for all \( i, j \)

System optimum:
- \( k'_{rm} = k'(q_{irm}) + q_{irm} \cdot \left[ \frac{\partial k'(q_{irm})}{\partial q_{irm}} + \frac{\partial q'_{irm}}{\partial q_{irm}} \right] \), for all \( r \) between all \( i,j \) with \( q'_{irm} > 0 \)

Route choice is descriptive, i.e. models real behaviour

Structure of equilibrium algorithms

1) Find the current cheapest paths
2) Distribute demand among all paths found so far according to the criteria chosen
3) Are there any substantial differences to the previous solution?
   3a) No, equilibrium reached
   3b) Yes, update set of paths found, recalculate link costs \([k'(s) = f(q'(s))]\) and go to 1)
Please note:

- The set of routes is empty at the start of the calculations
- All valid routes have been found at the end

- We have a solution for which we can say
  - No user can unilaterally improve his situations
  - Average costs are minimal

Example: IVT - Switzerland 1999

Size:
- 3066 zones
- 7949 nodes and 20620 links

Basis:
- Network data of ARE and SBB
- OD matrices of ARE and SBB

1999 version not fully calibrated
Example: UE IVT – Switzerland

Example: Difference All-or-nothing and UE
Why route choice?

- Types of traffic without strong volume - travel time feedbacks (cycling, (partially) public transport)
- Modelling differences in user preferences (tolls, elements of the total travel times, safety)
- Integration with other choices (departure time, mode choice)

How do we achieve consistent solutions?

- The share of each route $r$ between zones $i$ and $j$ is proportional to the probability that is optimal for the users with regards to the total utility of the route, i.e. including non-measurable elements of the utility:

$$q'_{rijm} = q'_{ijm} \times P(r), \text{ for all } r, i \text{ and } j$$

- $P(r) = f(k'_{rijm}(q_{rijm}))$ is calculated with a suitable model
- Travel times of routes between the same $i$ and $j$ need not be the same
Example of an allocation rule

Multinomial logit - model:

\[ P(i) = \frac{\exp[\beta V(i)]}{\sum_{\forall r} \exp[\beta V(r)]} \]

with:

\[ \beta = 1 \]

\[ V(r) = \sum_{i} \alpha_i X_{ir} \]

\[ X_{ir} : \text{Value of attribute } i \text{ of route } r \]

\[ \alpha_i : \text{Parameter of attribute } i \]

Example

Route 1

Route 2
Example

Route 1

Exp. Logit

R1 50%
R2 50%

Route 2

Example

Route 1

Exp. Logit

R1 50% 50%
R2 50% 50%
Example

All routes have the same generalised costs!

Exp. Logit

R1a 28% 33%
R1b 28% 33%
R2 44% 33%
Example

All routes have the same generalised costs!

<table>
<thead>
<tr>
<th></th>
<th>Exp.</th>
<th>Logit</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1a</td>
<td>33%</td>
<td>33%</td>
</tr>
<tr>
<td>R1b</td>
<td>33%</td>
<td>33%</td>
</tr>
<tr>
<td>R2</td>
<td>33%</td>
<td>33%</td>
</tr>
</tbody>
</table>

Route 1a

Route 1b

Route 2

Why the error?

*Independence of Irrelevant Alternatives*:

\[
P(\text{i})/P(\text{j}) = \frac{\exp(V(\text{i}))}{\exp(V(\text{j}))} = \text{constant}!
\]

\[
P(1)/P(2) = 1 \rightarrow P(1) = P(2) = 50%
\]

Introducing the new route

\[
P(1a)/P(1b) = 1 \rightarrow P(1a) = P(1b) = P(2) = 33%
\]

The simple model ignores the similarities of the routes!
What can we do?

Correction of the MNL:

- C-Logit (Cascetta)
- Path-Size - Logit (Bierlaire and Ben-Akiva)
- Correction of temporal overlap (Friedrich and Wekeck)

More general logit-type models:

- Cross-nested logit
- Probit

What attributes should we account for?

<table>
<thead>
<tr>
<th></th>
<th>PT</th>
<th>mIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Access and egrees times</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>In-vehicle time</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With congestion</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without congestion</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of transfers</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Transfer time</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Headway</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Reliability</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Comfort (Vehicle type, ride quality)</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Variable costs</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Tolls, supplements</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
How to find the routes?

- Incremental heuristics, e.g. set of all cheapest paths across all iterations

- A priori using rules:
  - k-cheapest paths (with and without overlaps)
  - set of path matching various cost criteria (distance, travel time, number of nodes, shares of certain types of roads, scenery etc.)
  - Selection based on random travel times

Problems with route sets: Share of used routes found by

<table>
<thead>
<tr>
<th>Approach</th>
<th>Required overlap/match</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>Shortest path by distance</td>
<td>20%</td>
</tr>
<tr>
<td>Time shortest path</td>
<td>34%</td>
</tr>
<tr>
<td>16 multicriteria searches</td>
<td>72%</td>
</tr>
<tr>
<td>K-cheapest paths</td>
<td>57%</td>
</tr>
<tr>
<td>48 „random“ shortest paths</td>
<td>50%</td>
</tr>
<tr>
<td>All of the above</td>
<td>84%</td>
</tr>
</tbody>
</table>

Ben-Akiva (2002)
Structure of the route-choice approaches

All the relevant routes should be known at the start

At convergence:

- Route shares are consistent with user perceptions
- The set of routes has been checked and, if required, been expanded
- A Stochastic User Equilibrium (SUE) has been achieved

Summary

<table>
<thead>
<tr>
<th>Perception of costs</th>
<th>Criterion</th>
<th>Consistent solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without error (Objective)</td>
<td>User costs</td>
<td>User equilibrium (UE)</td>
</tr>
<tr>
<td></td>
<td>Social costs</td>
<td>System optimum (SO)</td>
</tr>
<tr>
<td>With error (Subjective)</td>
<td>User costs</td>
<td>Stochastic user equilibrium (SUE) for given set of routes and choice rule</td>
</tr>
</tbody>
</table>
What next?

- Hypernetwork of public transport and private transport
- Hypernetwork with departure time choice
- Improvement of "corrected" logit approaches
- Development of models for the selection of the route choice set
- Improvement estimation of the choice model parameters (better accounting for similarities between choices)

Literature

Utility of route $r$: $U(r) = \beta \cdot CF(r) + \sum_{i} \alpha_i X_{ir}$

The "commonality factor (CF)" is defined as:

$$CF(r) = \sum_{j} \left( \frac{l(rj)}{[l(rj) / l(j)]^{1/2}} \right)^\mu$$

with
- $l(rj)$ Joint length of routes $r$ and $j$
- $l(j)$ Length of route $j$
- $\beta$, $\mu$ Parameters
Pathsize - Logit

Utility of route $r$:  $U(r) = \ln(S(r)) + \sum_{i \notin S} \alpha_i X_{ir}$

The “path size factor (PSF)” is defined as:

$PSF = \sum_{a \in S(r)} g(a) \cdot \frac{l(a)}{l(r)}$

$1/g(a) = \sum_{j \in R} \delta(a) \cdot \frac{l'(j)}{l(j)}$

with

$s(a)$  Length of link $a$

$S(r)$  Set of the links of route $r$

$R$  Set of routes

$l(j)$  Length of route $j$

$l'(j)$  Length of the cheapest route in $R$

$\delta(a) = 1$, if link $a$ is part of route $j$; otherwise $= 0$

Valuation of the generalised cost attributes

<table>
<thead>
<tr>
<th></th>
<th>Commuters</th>
<th>Shopping</th>
<th>Leisure / vacation</th>
<th>Business</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOT in-vehicle time [CHF/h]</td>
<td>11.9</td>
<td>20.1</td>
<td>15.8</td>
<td>52.4</td>
</tr>
<tr>
<td>VOT headway [CHF/h]</td>
<td>3.5</td>
<td>4.1</td>
<td>3.6</td>
<td>1.0</td>
</tr>
<tr>
<td>VOT transfer time [CHF/h]</td>
<td>7.7</td>
<td>25.0</td>
<td>6.5</td>
<td>43.9</td>
</tr>
<tr>
<td>Transfer [CHF/transfer]</td>
<td>1.5</td>
<td>2.0</td>
<td>5.9</td>
<td>4.5</td>
</tr>
<tr>
<td>IR-doubledecker [CHF]*</td>
<td>1.2</td>
<td>4.1</td>
<td>3.6</td>
<td>2.7</td>
</tr>
<tr>
<td>IC/EC [CHF]*</td>
<td>1.2</td>
<td>2.9</td>
<td>4.2</td>
<td>7.9</td>
</tr>
<tr>
<td>ICN [CHF]*</td>
<td>1.9</td>
<td>3.9</td>
<td>4.6</td>
<td>2.8</td>
</tr>
<tr>
<td>Number of transfer / in-vehicle time [min. in-vehicle time / transfer]</td>
<td>7.7</td>
<td>5.9</td>
<td>22.6</td>
<td>5.2</td>
</tr>
<tr>
<td>Transfer time / in-vehicle time</td>
<td>0.7</td>
<td>1.2</td>
<td>0.4</td>
<td>0.8</td>
</tr>
<tr>
<td>Headway / in-vehicle time</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Vitec and Axhausen (2002)