

Thresholds in choice behaviour and the size of travel time savings

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Abstract

Travel time savings are usually the most substantial economic benefit of transport infrastructure projects. However, there are concerns whether small time savings are of the same monetary unit value as larger savings. One reason for a discounted unit value for small time savings might be thresholds in individual choice behaviour. Different approaches for modelling these thresholds will be demonstrated using synthetic and stated choice data. We show that the consideration of thresholds can be important, even if the discounted unit value approach for travel time savings is rejected for transport scheme appraisal. If an existing threshold is ignored, the value of travel time savings will be biased. The presented procedure might also be useful to model thresholds in other contexts of choice behaviour.

Keywords: discrete choice model, logit model, value of travel time savings, threshold

1. Introduction

One of the main outcomes of transport infrastructure improvements are travel time reductions. Their evaluation plays therefore a major role in infrastructure planning and assessment. For instance, travel time reductions may change individual route or mode choices and consequently they may affect the flow of traffic. Usually benefit-cost analyses are performed in order to judge whether a project is beneficial or not. In cases where numerous proposed projects compete for limited financial resources, they are also applied in order to find out which project generates the highest net social benefit. In such analyses, travel time savings represent usually the highest share of the economic benefits of transport infrastructure construction projects. Welch and Williams (1997), for example, report a range of 70 to 90 per cent of total benefits.

There has been a long and ongoing debate on how to treat small travel time savings, since thresholds in the individual choice behaviour might be present for small travel time savings. The discussion usually focuses on the monetarization of travel time savings.¹ In several studies, it has been found that small travel time savings are valued by travellers less than larger ones (e.g. Fosgerau 2007; Hultkrantz and Mortazavi 2001; Gunn 2001; Mackie et al. 2003). One of the main arguments in favour of a reduced monetary value for small travel time savings is that people could not make effective use of them. However, counterarguments support, that, in the long run, this may not be the case (e.g. Mackie et al. 2001; Fowkes 1999). In addition, the fact that travellers might not identify small time differences (because they are below their cognition threshold), does not mean that they do not gain the associated benefits (Mackie et al. 2001). Following these arguments, small travel time savings should be equally valued as large ones in cost benefit analyses. However, for modelling purposes, the presence of thresholds in individual choice behaviour might still be important. As we show in this paper, estimated asymptotic values of travel time savings can deviate substantially from that of a model ignoring these thresholds. Furthermore, the consideration of thresholds may be important for predicting choice behaviour.

The explicit consideration of thresholds with respect to travel time savings is a rather underrepresented topic in the literature on travel choice modelling. Basically, such approaches focus either on the utility or the attribute level. The former are called indifference thresholds and have been modelled for example by Krishnan (1977), Lioukas (1984) and Cantillo (2010). We, however, will concentrate on attribute thresholds. Work in this area has been done for example by Cantillo et al. (2006) or

¹ Another important issue is the potential error in measurement of small travel time savings within transport models. However, this is not subject to this paper.

Li and Hultkranz (2004). Empirical results support the existence of indifference as well as attribute thresholds.

A somewhat newer approach has been put forward by Hjorth and Fosgerau (2012). They have tested if prospect theory can be an explanation for the observed lower valuation of small travel time savings. They applied a sophisticated power function transformation to time and cost differences to model the main propositions of the prospect theories' value function, which are increased sensitivity for small differences and loss aversion. At first glance, the approach to explain a lower value of time for small time savings with an increased sensitivity for small attribute differences seems to be counterintuitive. However, this can be explained by a relatively stronger transformation of cost in comparison to time differences in their study.

This paper is focused solely on thresholds with respect to travel time differences. Two new functions will be presented that allow to model smooth thresholds. These can easily be applied in any estimation tool for discrete choice analysis which can handle non-linear utility functions. We demonstrate the usefulness of these functions with synthetic data and apply them to stated choice data.

The structure of the paper is as follows. In section 2 and 3, the modelling approach is described and tested using synthetic data. Section 4 presents the calculation of the value of travel time savings and discusses the topic of project scheme appraisal and under time thresholds. In section 5, the modelling approach is applied to stated choice data. Section 6 concludes.

2. Modelling Approach

We model the choice between two options which are characterised by the same attributes (e.g. travel cost or travel time). Route choice between a cheap but slow and a fast but expensive alternative is a typical case. The modelling approach focuses on detection of possible deviations in the sensitivity to attribute differences between both alternatives, if these differences are small. The aim is to test whether travellers exhibit different sensitivities between large and small travel time differences. In this case the rate of substitution between travel cost and travel time will be different between small and large changes in travel time (assuming constant cost sensitivity).

It is assumed that trip makers always choose the option with the highest utility, which is decomposed into a deterministic (V) and a stochastic (ε) part. The stochastic component is assumed to be Gumbel-iid, and, therefore, the difference of the two stochastic components is logistically distributed. In the following, we consider just the utility difference between the two alternatives, because this is what matters for the choice decision. The utility difference is a function of the attribute differences. To model potentially different sensitivities depending on the size of time differences, an attribute

transformation function is applied. The parameter α_r of the transformation function has to be estimated along with the remaining coefficients of the model.

We assume that the utility difference ΔU is separable in time ΔT (in minutes) and cost ΔC (in CHF) components according to

$$\Delta U(\Delta T, \Delta C) = \Delta V(\Delta T, \Delta C) + \Delta \varepsilon = \beta_T * f_r(\Delta T, \alpha_r) + \beta_C \Delta C + \Delta \varepsilon \quad (1)$$

and that the time component is non-linear according to the following three specifications of the transformation function $f_r(\Delta T, \alpha_r)$ (cf. Figure 1).²

$$f_{HTF}(\Delta T, \alpha_{HTF}) = \begin{cases} 0 & \text{abs}(\Delta T) < \alpha_{HTF} \\ \text{sign}(\Delta T) * (\text{abs}(\Delta T) - \alpha_{HTF}) & \text{abs}(\Delta T) \geq \alpha_{HTF} \end{cases} \quad (2)$$

$$f_{STF1}(\Delta T, \alpha_{STF1}) = \Delta T - \alpha_{STF1} \tanh\left(\frac{\Delta T}{\alpha_{STF1}}\right) \quad (3)$$

$$f_{STF2}(\Delta T, \alpha_{STF2}) = \Delta T \left(1 - 1 / \sqrt{\left(\frac{\Delta T}{\alpha_{STF2}}\right)^2 + 1} \right) \quad (4)$$

Eq. (2) is a hard threshold function (HTF). It is a piecewise linear function, where the slope within the threshold area is zero. For functions (3) and (4), called soft threshold functions (STF), the slope is continuously increasing from zero to the limit of one. Figure 1 depicts the different transformation functions for $\alpha_r = 5$.

² For procedures to estimate such models see for example Train (2009).

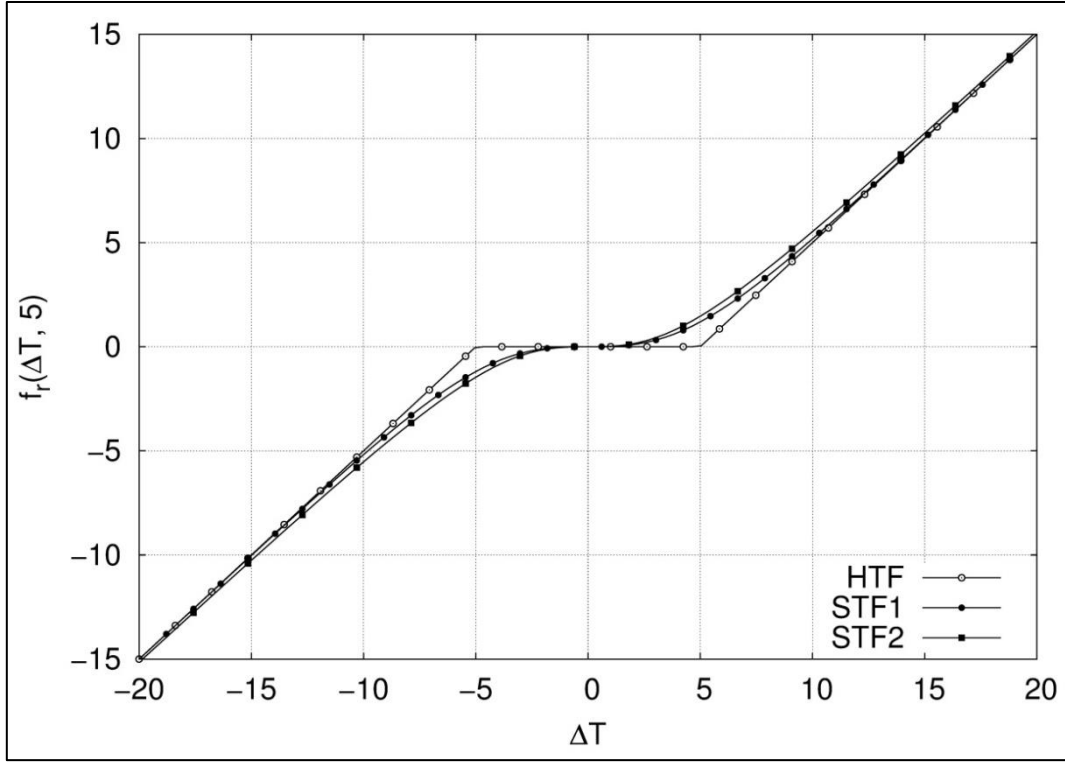


Figure 1: Transformation functions

The HTF models the extreme case of no sensitivity at all within the threshold. This is the common understanding of a threshold. However, in the following, the term threshold will be used to describe the area with significantly reduced sensitivity of the STF as well. Both STF are approximations to the HTF. The limit of the slopes of the STF for large positive and negative time differences is one and hence the asymptotes of the STF correspond to the HTF (if $\alpha_{HTF} = \alpha_{STF1} = \alpha_{STF2}$).

Another option to model reduced sensitivity for small time differences is to employ a power function transformation.³

$$f_{Power}(\Delta T, \alpha_{Power}) = sign(\Delta T) * abs(\Delta T)^{\alpha_{Power}} \quad (5)$$

Usually, this kind of transformation is used to model increased sensitivity around a reference point as predicted by prospect theory.⁴ An exponent greater than one means a reduced sensitivity for small differences. However, in contrast to the STF, the power function exhibits no limit for the slope when

³ Commonly, power function transformations are somewhat more sophisticated allowing for different sensitivities for gains and losses (e.g. Hjorth and Fosgerau 2012). However, for the kind of data used here, this is not necessary.

⁴ Both STF can simply be adjusted to incorporate increased sensitivity for small attribute differences by including a further parameter. In (3) α_{STF1} in front of the hyperbolic tangent and in (4) the numerator one have to be replaced by a separate parameter. This is useful for testing on an increased cost sensitivity which has a similar effect on the value of time as a time threshold. However, in this paper we concentrate on time thresholds.

differences tend to infinity. This might be problematic for detecting thresholds as we will show with synthetic data below.

3. Application to synthetic data

To test the different specifications regarding their goodness of fit, a synthetic database has been set up. For 5000 database records time and cost differences as well as a logistically distributed error component have been generated. Cost and time differences have been assumed to be independent and uniformly distributed in the range of [-10, 10 CHF] and [-25, 25 min], respectively. Dominant choice sets (with only positive or negative time and cost differences) have been excluded. The utility differences have been calculated according to the hard threshold function. Option one has been selected for utility differences greater than zero. Based on this data, estimations have been carried out with all four presented transformations.⁵ Table 1 summarizes the parameters used in the data generation process and the estimated values. Plots of ΔV against ΔT for all functions can be found in Figure 2.

	Synthetic	Linear	HTF	STF1	STF2	Power
Cost	-0.600	-0.630 * (0.12)	-0.596 * (0.83)	-0.598 * (0.92)	-0.598 * (0.92)	-0.602 * (0.92)
Time	-0.100	-0.080 * (0.00)	-0.106 * (0.47)	-0.113 * (0.35)	-0.119 * (0.26)	-0.013 + (0.00)
Alpha	5.000	---	5.410 * (0.69)	6.34 * (0.45)	7.48 * (0.29)	1.600 * [0.00]
VTTS^a	10.00	7.62	10.67	11.33	11.94	---
Null-LL				-3465.736		
Final-LL	---	-1787.714	-1779.042	-1779.105	-1779.051	-1779.064

*, #, + Significant on 1%, 5%, and 10% level, respectively.
 (.) p-value for null hypotheses that parameter is equal to its target value (synthetic column).
 [.] p-value for null hypotheses that parameter is equal to one.
^a Asymptotic value of travel time savings in CHF per hour. See section 4.

Table 1: Estimation results for synthetic data

The HTF and the two STF apparently fit really well and reproduce the target values. Not surprisingly, the threshold width of the HTF is closest to its original since this function is determining the data generation process. The threshold parameters of the STF show a somewhat greater difference to the predefined one. However, since the STF are smooth approximations to the HTF this deviation is not surprising and, moreover, not significant. Despite the good fit of the power function the estimated time coefficient is significantly different from its target value. This is a consequence of the infinite slope of the power function for large attribute levels. The power is significantly larger than one indicating a reduced sensitivity for small time differences. However, to avoid a steeply increasing slope,

⁵ All estimations have been carried out with Python Biogeme.

the time sensitivity coefficient needs to be small. A purely linear specification has also been estimated to examine the error when ignoring the threshold. Although the p-value has fallen dramatically the cost coefficient is still not significantly different from its target. However, the time coefficient has not been reproduced correctly. In general, we observe that many observations are necessary to detect an existing threshold. For 5000 observations the Log-Likelihood difference between the linear and the threshold models is just about 9 units,⁶ which means an average improvement of roughly 1 Log-Likelihood unit per 500 observations in this situation.

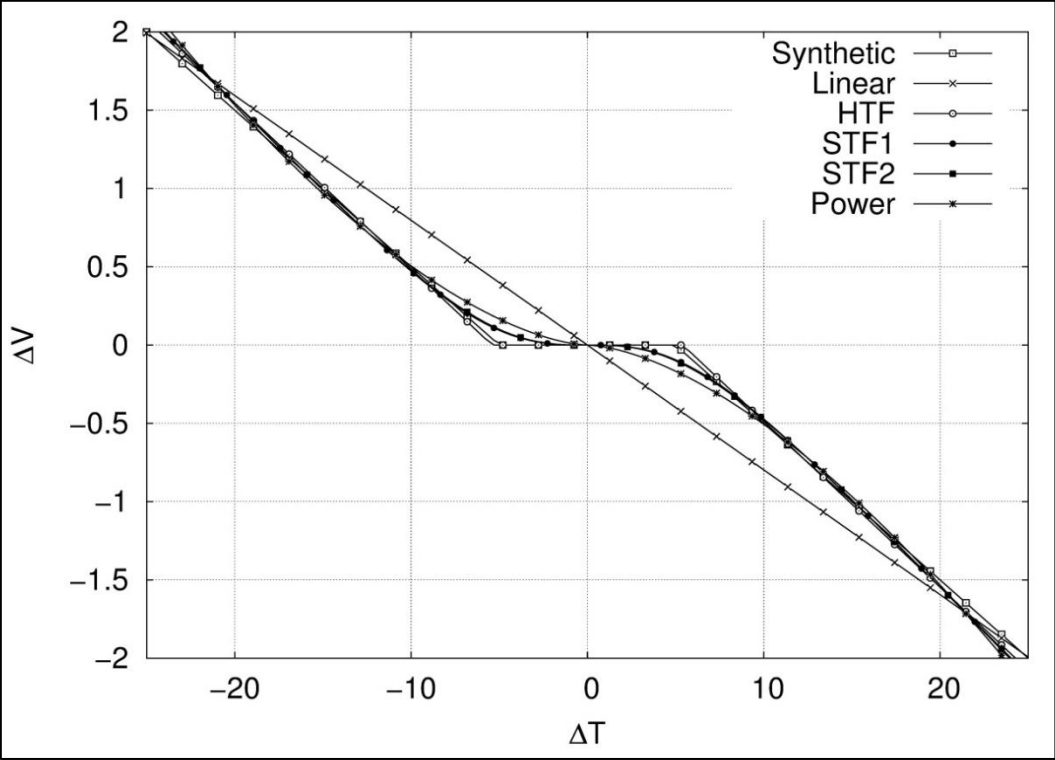


Figure 2: Utility functions - synthetic data

4. The value of travel time savings

The individual value of travel time is calculated as the compensatory variation per unit of travel time. The compensatory variation is the maximum amount of money a person is willing to pay for a time saving. This payment keeps the person on the same utility level as in the situation without the time saving. In this context the compensatory variation for a specific time saving is defined as the corresponding cost increase, which in turn is equivalent to an income reduction. Hence, the value of travel time savings (VTTs) is defined by (6). The resulting formulas for the various transformations are given by (7) - (11). It is assumed that costs are measured in Swiss franc (CHF) and time in minutes.

⁶ Nonetheless, the threshold models are significantly better than the linear model.

The individual value of travel time savings (VTTS) for finite amounts of time differences is expressed in CHF per hour.

$$VTTS = -\frac{\Delta C}{\Delta T} \Big|_{\Delta V=0} * 60 \quad (6)$$

$$VTTS_{HTF} = \begin{cases} 0 & \text{abs}(\Delta T) < \alpha_{HTF} \\ \beta_T / \beta_C * \left(1 - \frac{\alpha_{HTF}}{\text{abs}(\Delta T)}\right) * 60 & \text{abs}(\Delta T) \geq \alpha_{HTF} \end{cases} \quad (7)$$

$$VTTS_{Linear} = \frac{\beta_T}{\beta_C} * 60 \quad (8)$$

$$VTTS_{Power} = \frac{\beta_T}{\beta_C} * \text{sign}(\Delta T) * \text{abs}(\Delta T)^{\alpha_{Power}} * \Delta T^{-1} * 60 \quad (9)$$

$$VTTS_{STF1} = \frac{\beta_T}{\beta_C} * \left(1 - \alpha_{STF1} \tanh\left(\frac{\Delta T}{\alpha_{STF1}}\right) * \Delta T^{-1}\right) * 60 \quad (10)$$

$$VTTS_{STF2} = \frac{\beta_T}{\beta_C} * \left(1 - 1 / \sqrt{\left(\frac{\Delta T}{\alpha_{STF2}}\right)^2 + 1}\right) * 60 \quad (11)$$

Figure 3 depicts the corresponding VTTS for the synthetic data. As expected, the models considering thresholds exhibit a lower VTTS for smaller time changes and higher VTTS for larger time changes in comparison to the linear model.

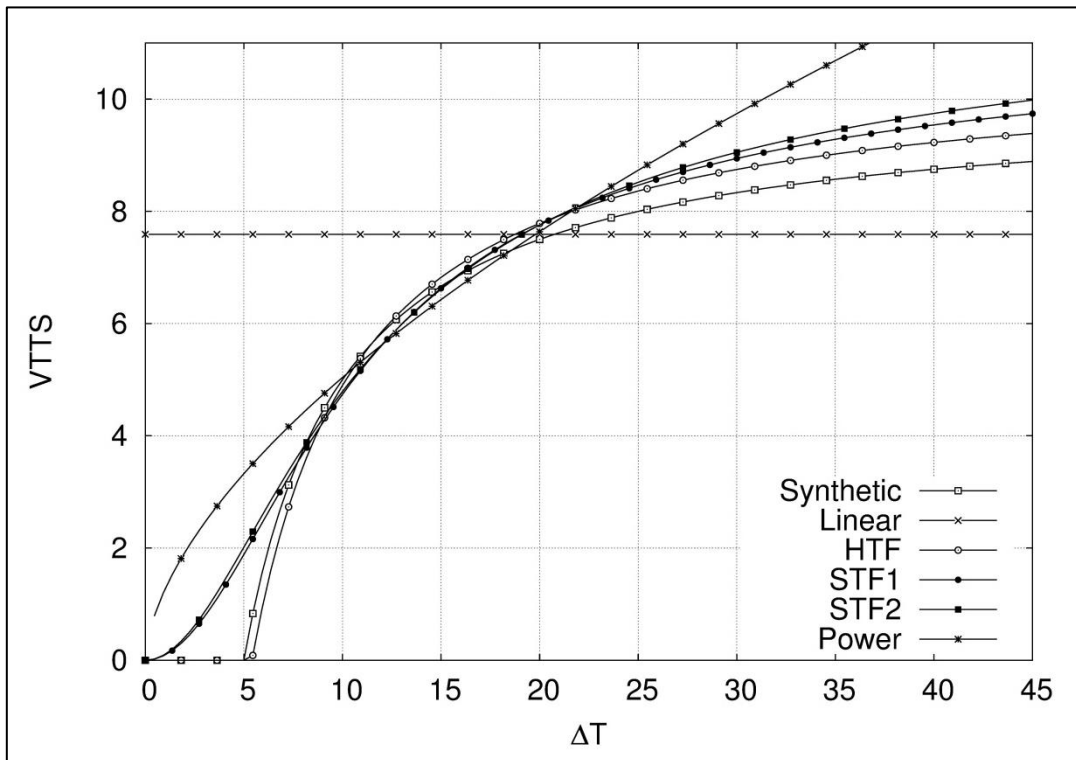


Figure 3: VTTS - synthetic data

Furthermore, an asymptotic VTTS could be calculated for the two STF and the HTF. This is the VTTS for $\Delta T \rightarrow \pm\infty$, which is in this case simply the ratio of the time and cost coefficient. The ratios of the time and cost coefficients for the different specifications are reported in Table 1. An asymptotic VTTS however does not exist for the power function. Calculating the VTTS as the ratio of the time and cost coefficient would result in an incorrect value of 1.30 CHF/hour. Thus, the power function specification appears to be problematic for estimating the correct value of time, if thresholds exist.⁷ It is also easy to see that the linear specification deviates strongest from the synthetic data. Thus, the problems of the linear and power function in estimating the correct time coefficient affect the VTTS calculations as well.

With regard to transport project scheme appraisal, two major arguments against a lower valuation of travel time savings (as shown above) can be found in the literature. At first, on an individual level, it might be questionable whether thresholds really exist, even if they can be found in stated choice data (e.g. Fowkes 1999). Individuals might show these thresholds in choice experiments, but this would be more or less an artificial result caused by the survey method. A possible explanation for this is that people do not consider the opportunity of rescheduling their activities in the short run to make use of a small time saving (Mackie et al. 2001). Thus, the central question is, whether transport users' valuation differs with the size of the saving *under real-world conditions*. Here, an analysis based on revealed preference data might help. However, this comes along with the problem of misperception, that is, trip makers simply might not perceive small time differences and consequently do not consider them in their choice decision. Thus, valuation and perception thresholds may be confounded when considering real choices. The perception effect might be relevant for modelling and forecasting route choices in reality but the matter of interest for evaluating time savings in the framework of a benefit-cost analysis seems to be, whether transport users care about them, not if they perceive them. The latter seems to be just a problem of incomplete information. With stated choice data, the perception effect can be ruled out.

At second, time savings might add up across different (transport) projects (Fowkes 1999; Mackie et al. 2001). Therefore all transport users exceed (after several projects) the threshold. Even if people do not add up explicitly in mind, it can be argued that it is just important that all time differences are actually *received* by the individuals. However, for combining time savings from different activities, people probably have to reschedule their activities. We think, in circumstances, where individual timetables underlie certain restrictions a threshold can still be of relevance. Beside this, we see a

⁷ This is not necessarily the case if people do really exhibit a sensitivity of infinity for large time changes, which, however, seems to be unrealistic to us.

further counterargument which refers to the cost of thinking. People might avoid the cognitive costs for rescheduling their activities, when a time saving is sufficiently small.

Closely related to this, it has been argued (Fowkes 1999), that with discounting small time savings the breaking up of a large project into smaller parts would result in different benefits of travel time savings. The inclusion of thresholds would therefore favour large-scale projects and penalize small-scale ones, since the threshold will be exceeded only at large scaled projects. However, one should not ignore that a large scale project might have a higher total effect (e. g. on the modal split) than several smaller ones with the same accumulated time savings, if thresholds are present and people continuously update their reference points. Hence, thresholds should not be ignored in the modelling process.

We want to emphasise that, regardless if one agrees with above arguments or not, the inclusion of thresholds in the modelling approach seems to be necessary. Otherwise, as we have shown, the estimated VTTS can be substantially biased downwards. In this sense, the transformation function corrects for the threshold, which might have been just caused by the survey method. Thus, even if the discounted unit value approach for transport scheme appraisal is rejected, the asymptotic VTTS based on a threshold model should be used.

5. Application to stated choice data

The data we use originate from route choice experiments for commuting trips by train in Switzerland. In these experiments, respondents had to choose between two routes which were characterised by the attributes travel time, travel cost, headway (H) and the number of changes (K). The data contains around 1600 observations of roughly 180 respondents. The average travel time in the data is 30 minutes and the mean of the cost variable is 12 CHF. The range of time differences reaches from one minute to around 45 minutes with 20 per cent of the observations less than or equal to two minutes.⁸ A binary logit model with the following deterministic utility function has been estimated.⁹

$$\Delta V(\Delta T, \Delta C) = \beta_T * f_r(\Delta T, \alpha_r) + \beta_C \Delta C * \left(\frac{I}{\bar{I}}\right)^{\lambda_I} * \left(\frac{T}{\bar{T}}\right)^{\lambda_T} + \beta_H \Delta H + \beta_K \Delta K \quad (12)$$

The above formulation of the utility additionally considers elasticities of the cost parameter with respect to Income, λ_I , and the average travel time of the two offered alternatives, λ_T . Both variables are normalised to their average. The values of λ_I and λ_T have to be estimated. In contrast to Ax-

⁸ For a more detailed description of the complete database and the survey design see Axhausen et al. (2006). Based on preliminary tests we restricted our analysis to rail route choices and therefore, we used just a part of the whole database.

⁹ This function is based on the model of Axhausen et al. (2006).

hausen et al. (2006) and others, we did not estimate the elasticity for distance because people usually consider travel time rather than distance when deciding on a trip by train. This has been confirmed by preliminary tests, where we could observe a clear improvement of the model fit when using an elasticity of time instead of distance. The estimation results for the different transformations mentioned above are summarized in Table 2.

	Linear	HTF	STF1	STF2
Cost	-0.305 *	-0.274 *	-0.285 *	-0.286 *
Time	-0.127 *	-0.159 *	-0.151 *	-0.152 *
Alpha	---	2.760 *	2.170 #	2.310 #
Headway	-0.050 *	-0.051 *	-0.051 *	-0.051 *
Changes	-1.420 *	-1.430 *	-1.430 *	-1.430 *
Elasticity Income	-0.252 *	-0.251 *	-0.250 *	-0.249 *
Elasticity Time	-0.489 *	-0.347 *	-0.387 *	-0.391 *
Scale^a	0.797 [*]	0.787 [*]	0.790 [*]	0.790 [*]
VTTS^b	24.98	34.82	31.79	31.89
Null-LL			-1110.42	
Final-LL	-687.064	-684.184	-684.651	-684.798
LL-ratio test against Linear	---	0.02	0.03	0.03
*, #, + Significant on 1%, 5%, and 10% level, respectively. [.] Significance level for null hypotheses that parameter is equal to one. ^a Controls for error scale differences. ^b Asymptotic value of travel time savings in CHF per hour.				

Table 2: Estimation results for stated choice data

The scale variable allows for different magnitudes of the error components of different user groups, in our case car drivers and rail users, that both participated in the experiments.¹⁰ The scale parameter of the rail users has been set to unity. An estimated scale of less than one for the group of car drivers indicates that they exhibit a higher error variance than rail users.¹¹ Hence, the variance of the unobserved factors is greater for car than for rail users.

The linear model is always a special case of the threshold models. Likelihood-ratio tests show that the HTF and the STF are significantly better than the linear model on a 2 and 3 percent significance level, respectively. Interestingly, as with the synthetic data, we observe a difference in the Log-Likelihood value of around 1 per 500 observations. With a test developed by Horowitz (1983) to compare non-nested models, HTF and STF can be tested against each other. The null hypothesis that the model with the lower Log-Likelihood is the true model cannot be rejected for the data used here,

¹⁰ See Train (2009) section 2.5.2 and 3.2 for an in depth discussion of the error scale.

¹¹ This is in line with the findings of König et al. (2004).

meaning the STF formulations are not significantly worse than the HTF model.¹² In addition, the power function formulation has been proved significantly worse than the other three models and has therefore been omitted from the analysis. Furthermore, the power coefficient was not significantly different from one. This is a further indication for the problem of the power function to detect thresholds correctly.

Across all three threshold specifications, significant threshold parameters have been estimated. They indicate a threshold between 2 and 3 minutes. All other coefficients are significant as well and within the range of expectations. Figure 4 depicts the utility difference against the time difference and reveals great similarities between the two STF.¹³

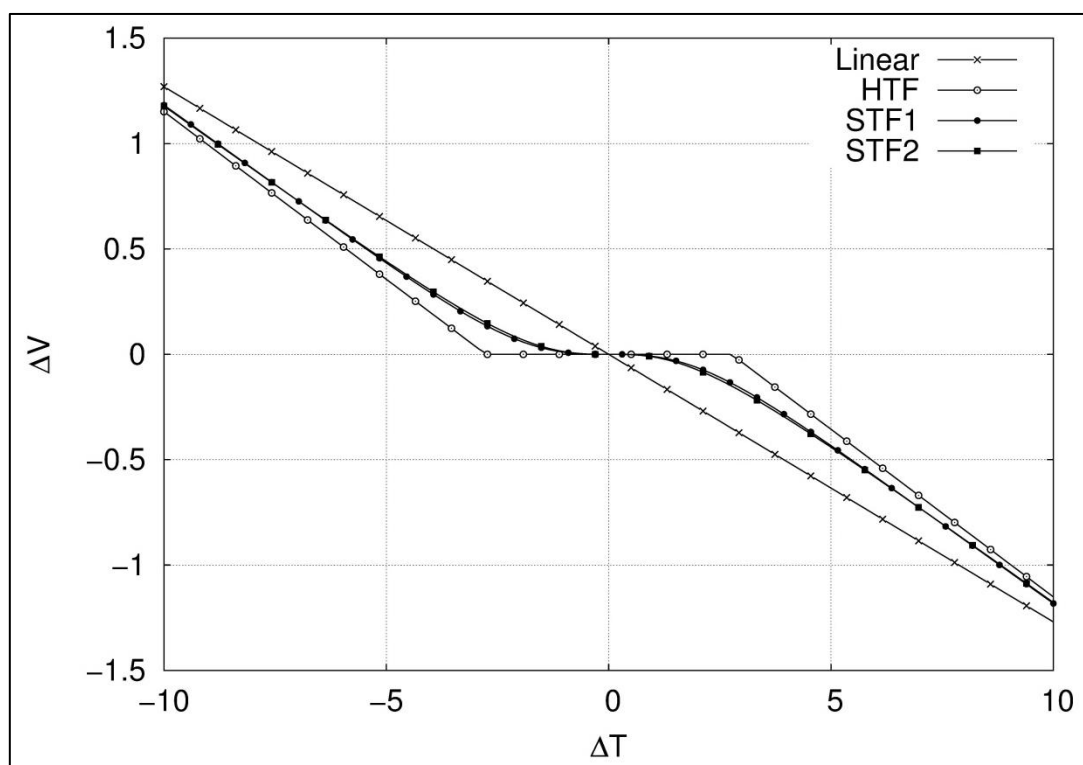


Figure 4: Utility functions - stated choice data

Finally, in Figure 5 the resulting VTTS are plotted for mean income and travel time. As with the synthetic data, the threshold formulations show a lower VTTS for smaller time changes and higher VTTS for larger time changes in comparison to the linear model. The asymptotic values are reported in Table 2. Again, as in the case of the synthetic data, we can observe, that the inclusion of thresholds leads to substantially higher asymptotic VTTS values.

¹² We based our calculations on formula 52 in Horowitz (1983, p. 336). Note that a different formulation can be found in the literature, e.g. in Ben-Akiva and Lerman (2007, p. 172). We however, regard the original formula of Horowitz as the correct one.

¹³ All other explanatory variables have been set to zero to reduce dimensionality.

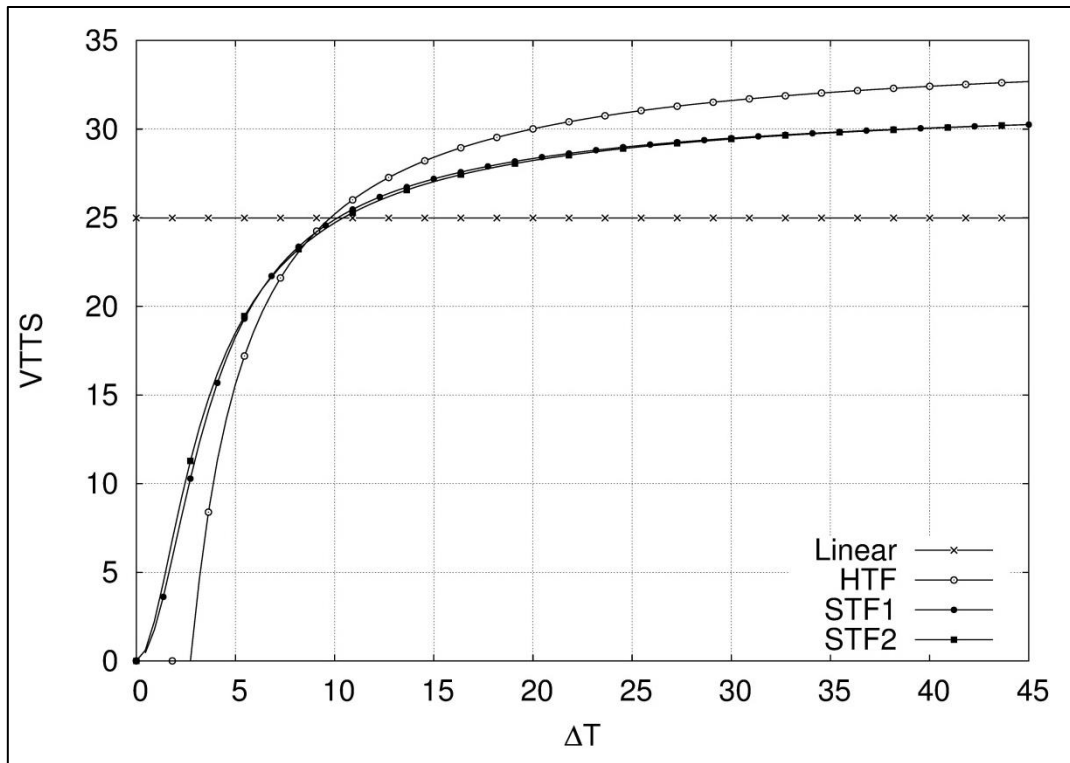


Figure 5: VTTs - stated choice data

6. Conclusion and discussion

In the above analysis, we have tested for thresholds in individual choice behaviour. For this purpose, we have proposed different functions including a piecewise linear function with a hard threshold, two smooth functions with soft thresholds, and a power function. To the best of our knowledge, the two soft threshold functions have not been used elsewhere in the literature. We applied these functions to synthetic and stated choice data. Estimating the generated data showed that many observations are necessary to detect even hard thresholds. Interestingly, for both, the synthetic and the real data, we observed a difference in the Log-Likelihood value of around 1 per 500 observations between the linear and the threshold models. As stated choice data we employed the rail route choice experiments from the Swiss value of travel time study (Axhausen et al. 2006). The results indicate a time threshold between two and three minutes. However, we are not able to detect, if it is a hard or soft one. Further, tests with the synthetic data have shown that all functions except the power transformation worked reasonably well in reproducing the known values. It has to be emphasised that the performance of the different transformations depends on the underlying data generation process. Nevertheless, we think that it is more plausible that trip makers' sensitivities converge to a limit instead of rising to infinity.

The detection of thresholds affects the inferred value of time. According to the estimates, small travel time savings should be valued at a lower rate than larger ones. Moreover, the large ones should be

valued even higher than currently. Arguments against such a treatment in project scheme appraisal have been raised in literature before. However, as we have shown in this paper, thresholds should not be ignored, even if they are just an artificial result caused by the survey method. Otherwise, the estimated VTTS may be substantially biased downwards. Thus, even if the discounted unit value approach for transport scheme appraisal is rejected, the asymptotic VTTS based on a threshold model should be used. Furthermore, regardless the monetisation issues discussed in this paper, thresholds might be important for predicting choice behaviour.

Clearly, there remain numerous unresolved questions regarding the value of time for project evaluation. Finally, we emphasise, that the estimation procedure and transformation functions presented in this paper may be useful to model more general choice situations beyond mode or route choice, as well as to model smooth indifference thresholds in the sense of Cantillo et al. (2010).

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