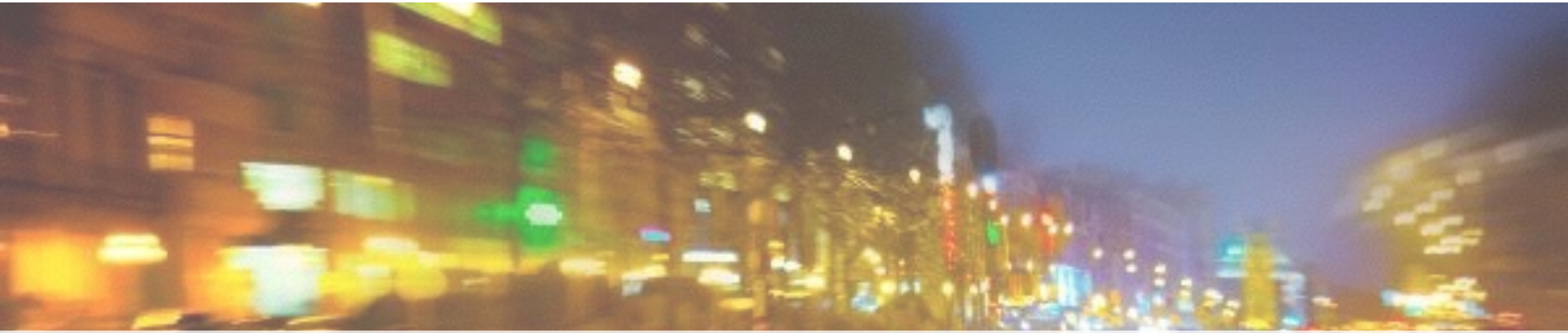

W–SPSA in practice: Approximation of weight matrices and calibration of traffic simulation models

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Outline

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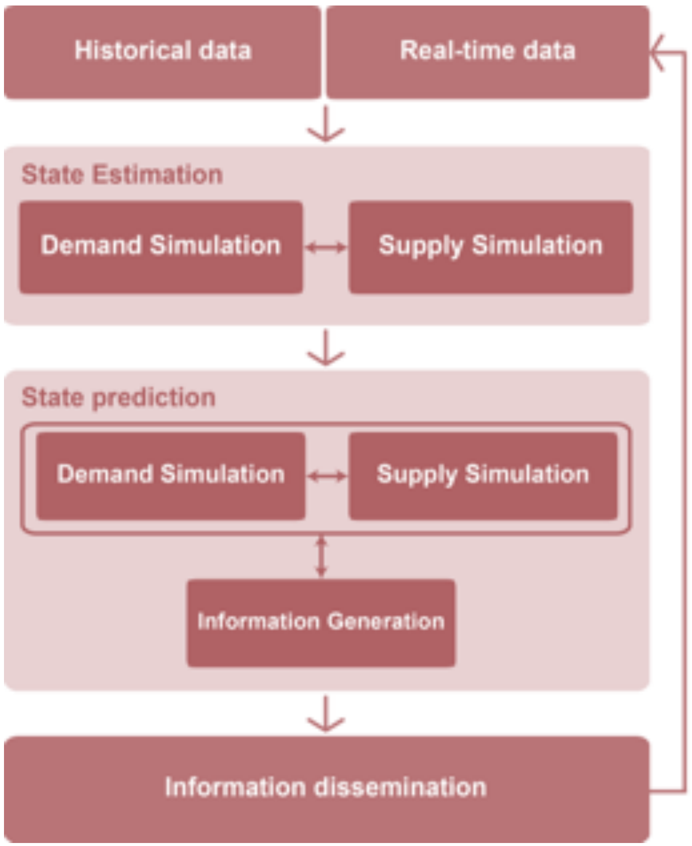


Antoniou, C., C. L. Azevedo, L. Lu, F. Pereira and M. Ben-Akiva (2015). W-SPSA in practice: Approximation of weight matrices and calibration of traffic simulation models. Transportation Research Part C: Emerging Technologies, Vol. 59, October, pp. 129-146.

Introduction

- The **challenge**: the calibration of all supply and demand parameters of traffic simulation tools in order to reflect the reality.
- The **problem:** number of parameters & unknown search space
- The **constraints:** computational costs & lack of analytical models

⇒ **Simultaneous Perturbation Stochastic Approximation (SPSA)**

Balakrishna (2006), Ma et al., (2007), Lee & Ozbay (2008), Vaze et al. (2009), Huang et al. (2010), Paz et al. (2012), etc.

General Problem Formulation

observed time-dependent measurements

simulated time-dependent measurements

a-priori time-dependent parameters

a-priori static parameters

$$\underset{\boldsymbol{\beta}, \boldsymbol{x}}{minimize}\; z(\boldsymbol{M}^O, \boldsymbol{M}^S, \boldsymbol{x}, \boldsymbol{\beta}, \boldsymbol{x}^a, \boldsymbol{\beta}^a)$$

static parameters

time-dependent parameters

fixed parameters

s.t.: $\boldsymbol{M}_h^S = \boldsymbol{f}(\boldsymbol{x}_1, \dots, \boldsymbol{x}_h; \boldsymbol{\beta}; \boldsymbol{G}_1, \dots, \boldsymbol{G}_h)$

$$\boldsymbol{l}_{x_h} \leq \boldsymbol{x}_h \leq \boldsymbol{u}_{x_h} \quad \boldsymbol{l}_{\beta} \leq \boldsymbol{\beta} \leq \boldsymbol{u}_{\beta}$$

General Problem Formulation (2)

Under GLS:

variance–covariance matrix

$\boldsymbol{M}_h^O - \boldsymbol{M}_h^S$

$$\underset{\beta, x}{minimize} \sum_{h=1}^{H} \left[\boldsymbol{\epsilon}_{\boldsymbol{M}_h}^T \boldsymbol{\Omega}_{\boldsymbol{M}_h}^{-1} \boldsymbol{\epsilon}_{\boldsymbol{M}_h} + \boldsymbol{\epsilon}_{\boldsymbol{x}_h}^T \boldsymbol{\Omega}_{\boldsymbol{x}_h}^{-1} \boldsymbol{\epsilon}_{\boldsymbol{x}_h} \right] + \boldsymbol{\epsilon}_{\boldsymbol{\beta}}^T \boldsymbol{\Omega}_{\boldsymbol{\beta}}^{-1} \boldsymbol{\epsilon}_{\boldsymbol{\beta}}$$

Generalized formulation:

$$\underset{\boldsymbol{\theta}}{minimize}\, z(\boldsymbol{\theta}) \Rightarrow \underset{\boldsymbol{\theta}}{minimize}\, z\left(\boldsymbol{\epsilon}_{\boldsymbol{M}}^T \boldsymbol{\Omega}_{\boldsymbol{M}}^{-1} \boldsymbol{\epsilon}_{\boldsymbol{M}} + \boldsymbol{\epsilon}_{\boldsymbol{\theta}}^T \boldsymbol{\Omega}_{\boldsymbol{\theta}}^{-1} \boldsymbol{\epsilon}_{\boldsymbol{\theta}} \right)$$

$$\boldsymbol{\epsilon}_{\boldsymbol{M}} = F_M(\boldsymbol{\theta}, \boldsymbol{M}^O, G) \quad \boldsymbol{\epsilon}_{\boldsymbol{\theta}} = F_\theta(\boldsymbol{\theta}, \boldsymbol{\theta}^a, G) \quad \boldsymbol{l}_\theta \le \boldsymbol{\theta} \le \boldsymbol{u}_\theta$$

SPSA

- SA are iterative stochastic optimization algorithms when the objective function has no known analytical form and can only be estimated with noisy observations.

$$\hat{\boldsymbol{\theta}}_{k+1} = \hat{\boldsymbol{\theta}}_k - a_k \hat{g}_k(\hat{\boldsymbol{\theta}}_k)$$

$$a_k = \frac{a}{(A + k + a)^{\alpha}}$$

$$\hat{g}_{ki}(\hat{\boldsymbol{\theta}}_k) = \frac{z(\hat{\boldsymbol{\theta}}_k + c_k \boldsymbol{\Delta}_k) - z(\hat{\boldsymbol{\theta}}_k - c_k \boldsymbol{\Delta}_k)}{2 c_k \Delta_{ki}}$$

Spall (1992, 1998)

Implementation of SPSA

1. Set the current step $k = 0$, $\widehat{\boldsymbol{\theta}}_k = \widehat{\boldsymbol{\theta}}_0$; decide the values of the algorithm parameters.
2. Evaluate the initial objective function value z_0.
3. Update $k = k + 1$. Calculate a_k and c_k.
4. Generate the independent random perturbation vector Δ_k.
5. Evaluate the objective function values at two points: $\widehat{\boldsymbol{\theta}}_k + c_k\Delta_k$ and $\widehat{\boldsymbol{\theta}}_k - c_k\Delta_k$. Parameter boundaries are imposed before the objective function evaluation.
6. Approximate the gradient vector $\hat{g}_{ki}(\widehat{\boldsymbol{\theta}}_{\boldsymbol{k}})$.
7. Calculate $\widehat{\boldsymbol{\theta}}_{k+1}$.
8. If converged, stop the process. If not, return to step 3.

SPSA Problems

- Let's assume

$$\mathbf{\Omega}_{M}^{-1} = \mathbf{I} \quad \mathbf{\Omega}_{\theta}^{-1} = \mathbf{0}$$

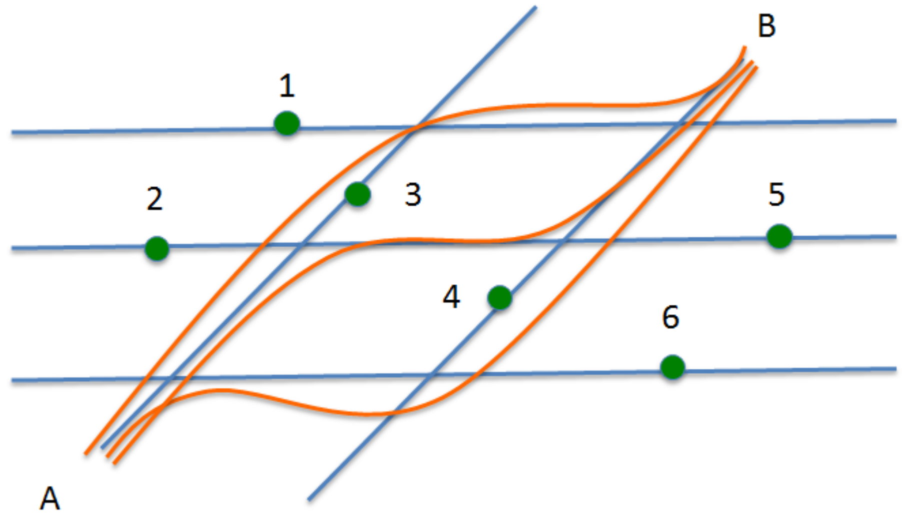
i.e. ignore deviations regarding prior values and deviations between simulated and observed measurement values in an ordinary least squares (OLS) way

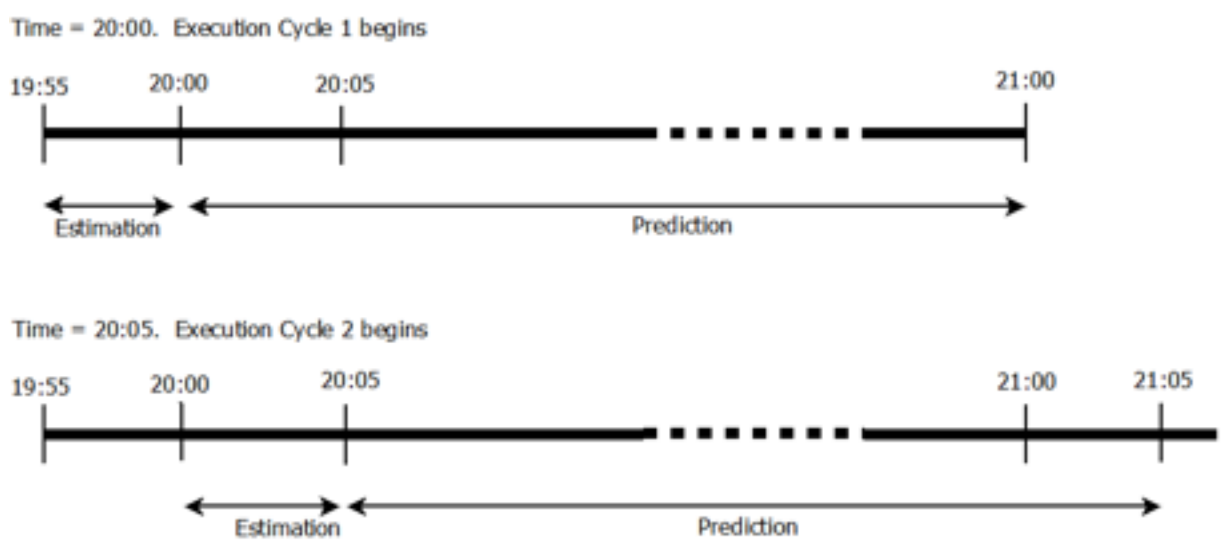
- Then

$$z(\theta) = \sum_{j=1}^{D} \epsilon_{M_j}^2$$

$$\hat{g}_{ki}(\hat{\boldsymbol{\theta}}_k) = \frac{\sum_{j=1}^{D}\left[\left(\epsilon_{M_{kj}}^{+}\right)^2 - \left(\epsilon_{M_{kj}}^{-}\right)^2\right]}{2c_k\Delta_{ki}}$$

Signal vs. noise





W-SPSA

Excludes the negative influence of irrelevant measurements in the gradient approximation process of SPSA

$$\hat{g}_{ki}(\widehat{\boldsymbol{\theta}}_k) = \frac{\mathbf{z}(\widehat{\boldsymbol{\theta}}_k + c_k \boldsymbol{\Delta}_k) - \mathbf{z}(\widehat{\boldsymbol{\theta}}_k - c_k \boldsymbol{\Delta}_k)}{2 c_k \Delta_{ki}} \boldsymbol{W}_i$$

For a change in parameter θ_i , w_{ji} represents the relative magnitude of change in measurement j.

$$\hat{g}_{ki}(\widehat{\boldsymbol{\theta}}_k) = \frac{\sum_{j=1}^{D} w_{ji} \left[\left(\epsilon_{M_{kj}}^{+} \right)^2 - \left(\epsilon_{M_{kj}}^{-} \right)^2 \right]}{2 c_k \Delta_{ki}}$$

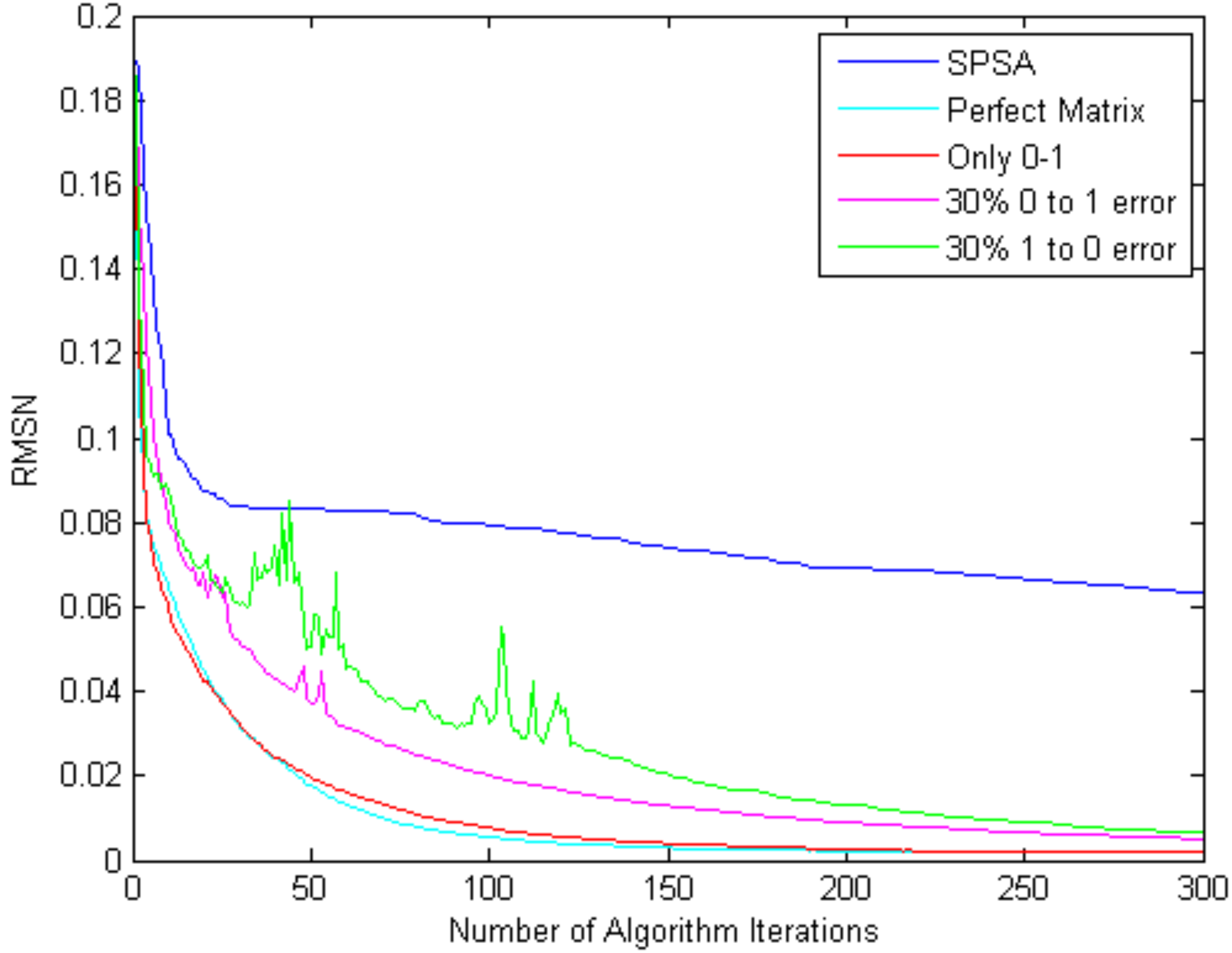
W-SPSA

For a change in parameter θ_i , w_{ji} represents the relative magnitude of change in measurement j.

$$\boldsymbol{W}|_{\hat{\boldsymbol{\theta}}_k} = \hat{\boldsymbol{J}}|_{\hat{\boldsymbol{\theta}}_k}$$

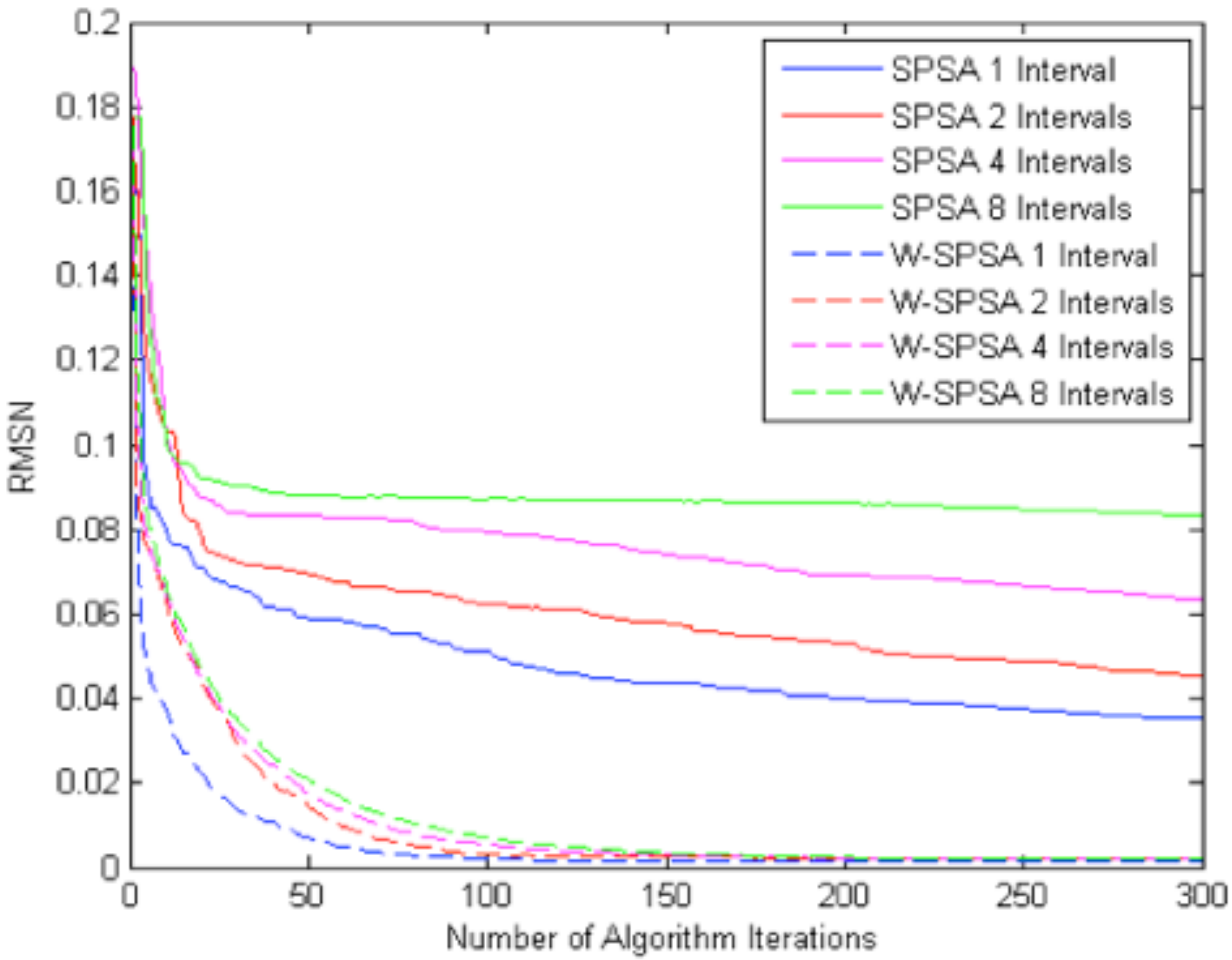
$$\hat{\boldsymbol{J}}_{i,j} = \left|\frac{\partial \epsilon_j}{\partial \theta_i}\right|$$

W-SPSA robustness



Lu, L., Y. Xu, C. Antoniou and M. Ben-Akiva (2015). An Enhanced SPSA Algorithm for the Calibration of Dynamic Traffic Assignment Models. Transportation Research Part C, 51, pp. 149-166.

SPSA vs. W-SPSA



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The Weight Matrix

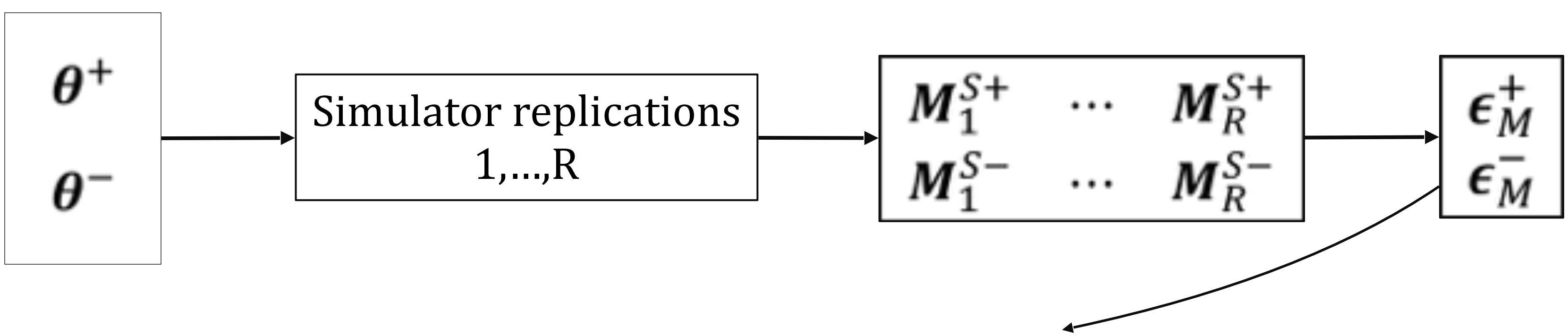
How to estimate W ?

1. Analytical derivation

Direct observation of parameter-measurement relationships

2. Simulation–based approximation

Simulator can be used to estimate W (e.g. assignment matrices)



local linear approximation of the weight matrix

The Weight Matrix

3. Numerical approximation

Similar to previous but,

several experiments $n = 1, \dots, N$ at each k^{th} iteration are carried out

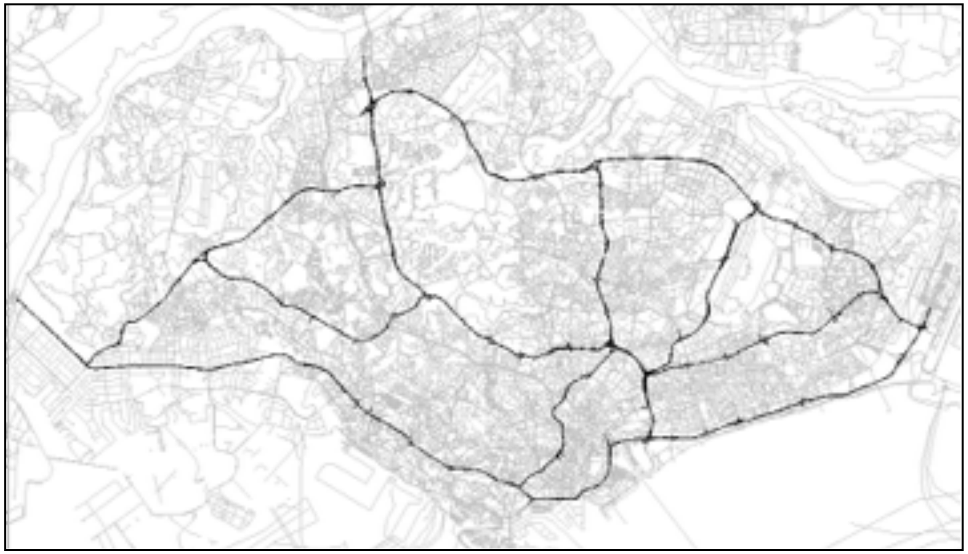
$$w_{ij} = \frac{\partial \epsilon_{M_j}}{\partial \theta_i} \sim \frac{\sum_{n=1}^{N} \frac{\epsilon_{M_n^S j}}{\Delta \theta_{ni}}}{N}$$

4. Heuristics–based approximation
5. Hybrid & composite methods

$$\boldsymbol{W_h} = \lambda_a \boldsymbol{W_a} + \lambda_n \boldsymbol{W_n} + \lambda_o \boldsymbol{W_o}$$

Case-study 1: Large-scale Mesoscopic DTA

- DynaMIT
  - A simulation–based mesoscopic DTA demand and supply simulator
  - Demand based on a path-size logit for route-choice & dynamic OD
  - Supply based on speed-density functions + capacity
- Singapore Expressway network
  - 3388 segments, 4106 OD pairs



Case-study 1: DynaMIT and Simulation Set-up

Off-line calibration

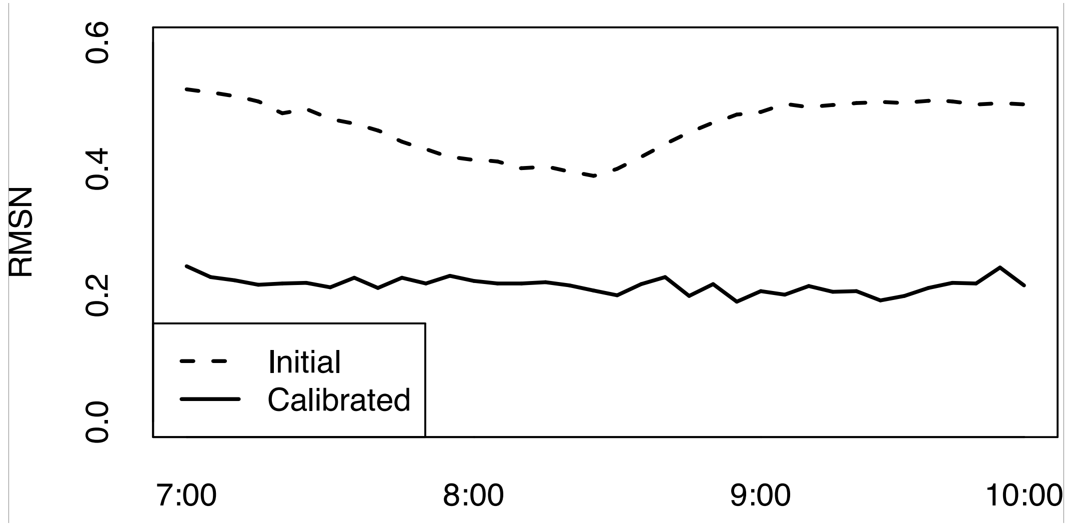
- 3 hour simulation (AM peak) with dynamic OD by 5mins
- 4 106 × 36 = 147 816 time dependent OD flows
- 1 route choice model parameter (travel time)
- 6 speed-density function parameters + 1 capacity parameter (20 328 +3 388)
- 216 measurement locations (counts and speeds) × 36 (average for August 2011 – ok for off-line calibration)

Composite method

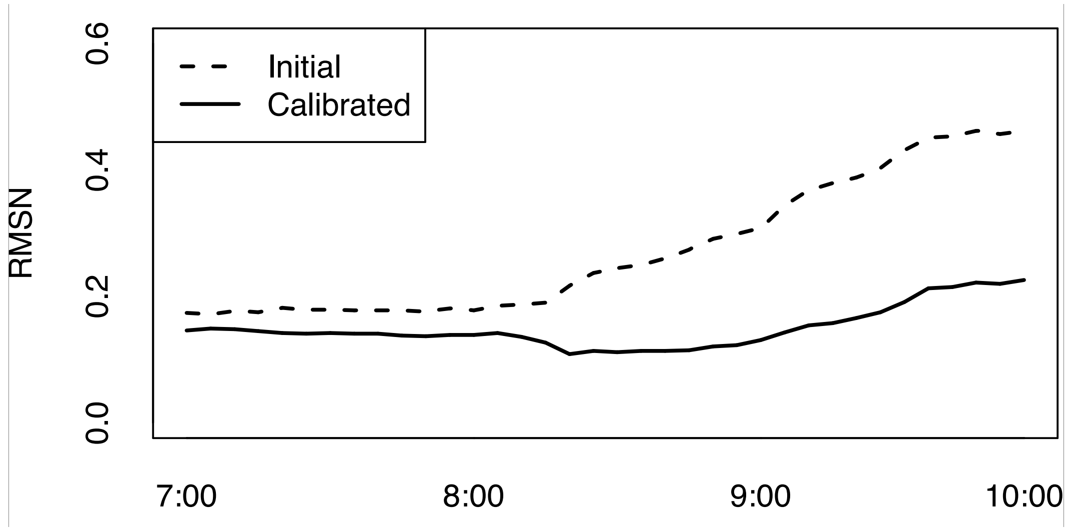
- W for OD/counts is based on dynamic route-choice proportions & travel times (simulation approximation)
- W for supply parameters/counts+speeds is based on heuristics {0,1}

Case-study 1: Results

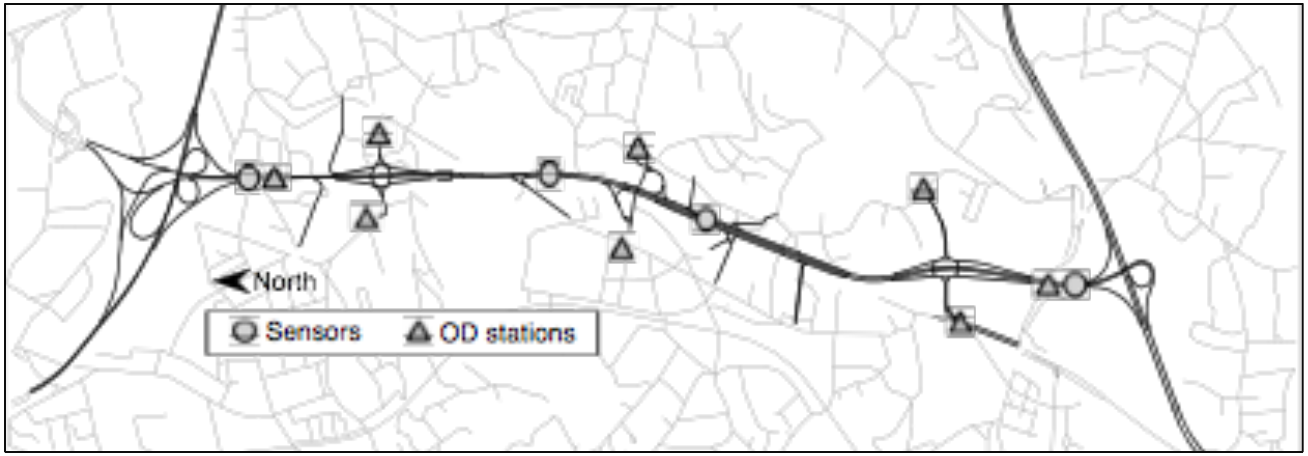
Counts



Speeds



Case-study 2: Microscopic Traffic Simulator



- Objective: calibrate the simulator for a large-set of scenarios targeting **surrogate safety analysis**
  - 6544 calibration scenarios (6400 non-acc. +144 acc.)
- A44 (Portugal) network with 100 OD pairs
- MITSIMLab
  - Microscopic simulator (acceleration, lane changing, …)

Case-study 2: MITSIMLab and Simulation Set-up

- 30 min simulation with dynamic OD by 15 min for each scenario
- 200 time dependent OD flows
- 11 driving behaviour parameters
- 192 sensor measurements (count & average speed)

Composite method

- W for OD/counts+speeds is based on simulated assignment matrix (simulation approximation)
- W for driving behavior/counts+speeds based on heuristics {0,1}
- OLS assumption was assumed (variance-covariance are block diagonal)

Case-study 2: Individual scenario results

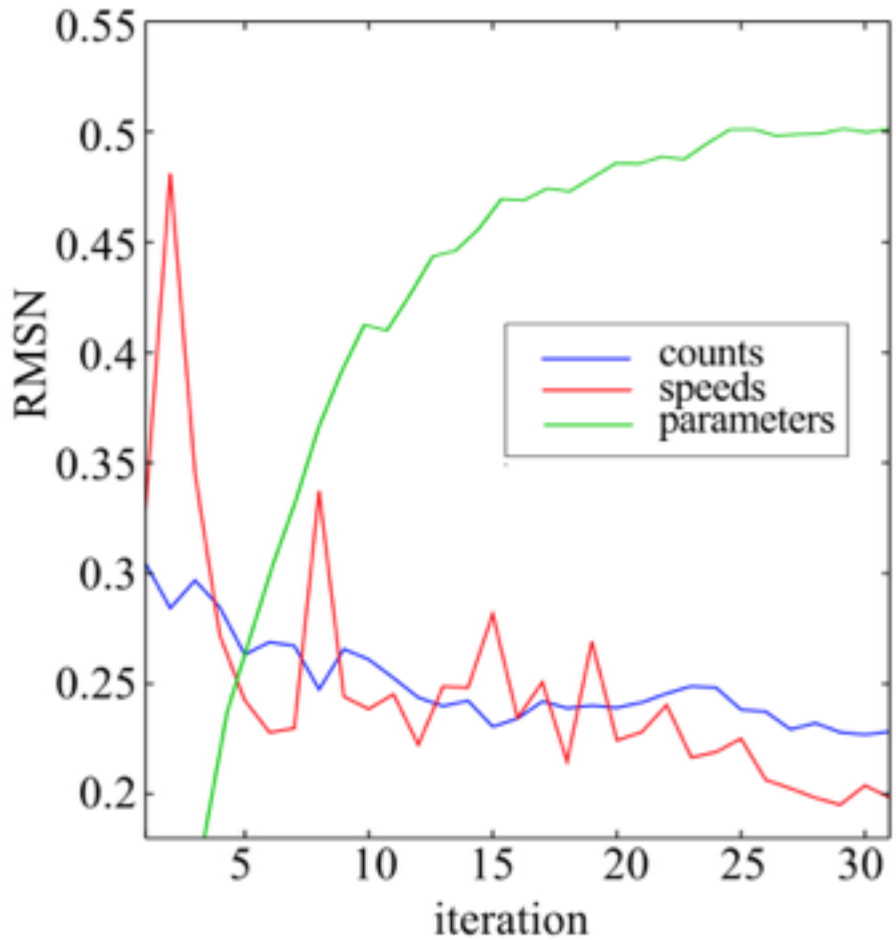
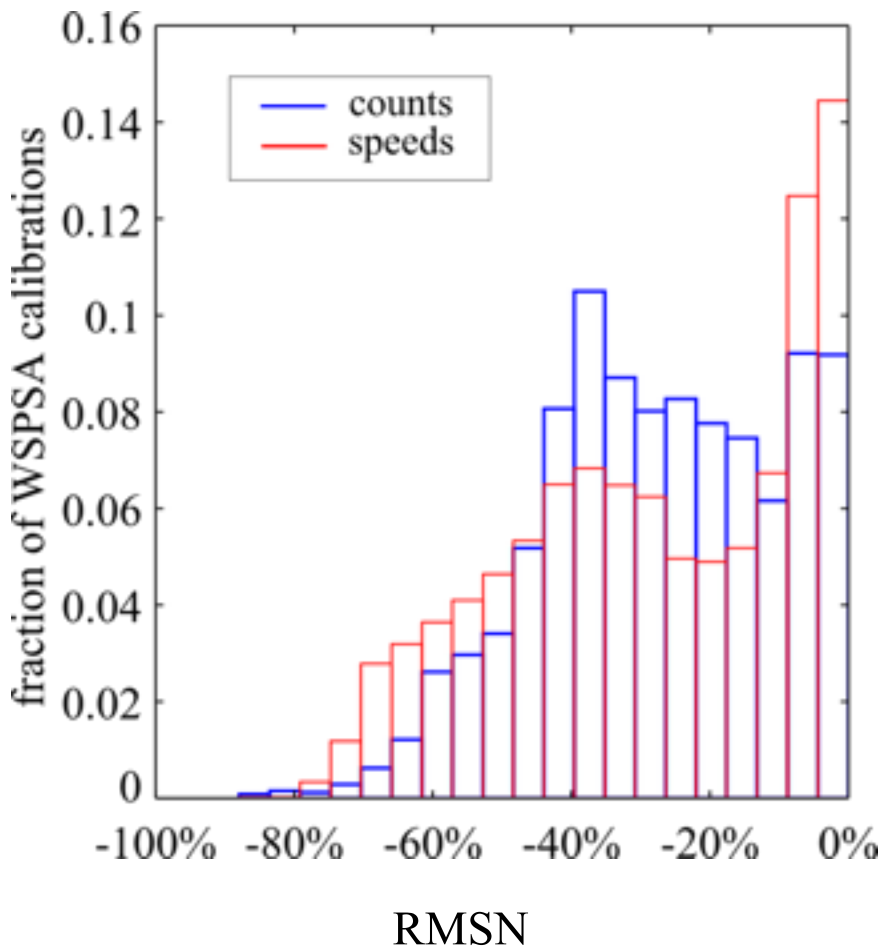


Fig. 6: W–SPSA test performance for a specific scenario calibration

Case-study 2: Aggregate results



Conclusions

- Alternative methods to estimate W were presented
- Heuristic and composite approaches can extend its applicability and two case-studies were used as demonstrations
- Results depend on each case → boundaries during calibration

Research streams

- Ratio perturbation
- Selective perturbation
- Simultaneous perturbation on a subset
- On-line calibration

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