Modeling activity duration and travel choice from a common microeconomic framework

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Contents

1. Introduction

- Discrete travel choice models: key element is the utility of each alternative, V_i.
- V_i corresponds to a Conditional Indirect Utility Function (CIUF).
- V_i is derived from consumer behavior models that include time assignment.

Train and McFadden (1978)

- The individual behaves as if:

$$\left.\begin{aligned} & Max_{S.to}\ \ U(G,L) \\ & G + c_i \le wW \\ & L + W + t_i = \tau \\ & i \in M \end{aligned}\right\} \quad \begin{aligned} & U[(wW - c_i),(t - W - t_i)] \\ & \frac{\partial U}{\partial W} = 0 \ \Rightarrow \ W^*(c_i, w, t_i) \end{aligned}$$

$$\therefore V_i = U[c_i, w, t_i]$$

Discrete analogy of Becker (1965)

SVT=*w*

2. A General Microeconomic Framework

Johnson (1966), Oort (1969), DeSerpa (1971), Evans (1972), Bates (1987), Juster (1990) →

$$\underset{subject\ \ to}{Max} \quad U(T, X)$$

Income constraint (l)
Total time constraint (m)
Tecnological constraints (k)

Yields $$V_i = U\left[T^*(c_i, w, t_i), X^*(c_i, w, t_i)\right]$$

∴ travel, activity time assignment and goods consumption models should be compatible.

Values of time

SVTTS $\Rightarrow \quad \dfrac{k_i}{l} = \dfrac{m}{l} - \dfrac{\partial U / \partial T_i}{l} = w + \dfrac{\partial U / \partial T_w}{l} - \dfrac{\partial U / \partial T_i}{l} = \dfrac{\partial V_i / \partial t_i}{\partial V_i / \partial c_i}$

$\dfrac{k_i}{l} =$ Subjective value of travel time savings in mode *i* (constrained activity)

$\dfrac{m}{l} =$ Value of time as a resource (value of leisure)

$\dfrac{\partial U / \partial T_i}{l} =$ Value of time assigned to activity *i* (travel in this case)

3. A Labor Supply and Travel Model.

(Jara-Díaz and Guevara, 2002)

- If U is Cobb-Douglas

$$\underset{\text{subject to}}{Max} \quad U = \Omega\, T_w^{q_w} T_t^{q_t} \prod_{i\in I} T_i^{q_i} \prod_{k\in K} X_k^{h_k}$$

$$\sum_{k\in K} P_k X_k + c_t \le w T_w \quad \leftarrow \lambda$$

$$\sum_{i\in I} T_i + T_w + T_t = \tau \quad \leftarrow \mu$$

$$T_t \ge T_t^{Min.} \quad \leftarrow \kappa$$

- First order conditions (Disaggregated)
  - Good k: $\frac{\lambda}{U} = \frac{\eta_k}{P_k X_k}$
  - Activity i: $\frac{\mu}{U} = \frac{\theta_i}{T_i}$
  - Work: $\frac{m}{U} = \frac{l}{U} w + \frac{q_w}{T_w}$

Aggregated Version

Goods: $\frac{l}{U} = \frac{B}{(wT_w - c_t)}$

Leisure: $\frac{m}{U} = \frac{A}{(t - T_w - T_t)}$

Labor Supply Model

$$T_w^* = b\left(t - T_t\right) + a\,\frac{c_t}{w} + \sqrt{\left[b\left(t - T_t\right) + a\,\frac{c_t}{w}\right]^2 - \left(2\left(a + b\right) - 1\right)\left(t - T_t\right)\frac{c_t}{w}}$$

$$A = \sum_i \theta_i; \quad B = \sum_k \eta_k; \quad \alpha = \frac{A + \theta_w}{2\left(A + B + \theta_w\right)}; \quad \beta = \frac{B + \theta_w}{2\left(A + B + \theta_w\right)}$$

Mode Choice Model

$$V_i \approx g_i + g_c c_i + g_t t_i \quad \Rightarrow \quad SVTTS = \frac{\kappa_i}{\lambda} = \frac{\gamma_t}{\gamma_c}$$

The ***components*** of SVTTS can be calculated

- Value of time as a resource

$$SVT_{res} = \frac{\mu}{\lambda} = \frac{1-2\beta}{1-2\alpha}\frac{\left(wT_w^* - c_t\right)}{\left(\tau - T_w^* - T_t\right)}$$

Value of time assigned to work

$$SVT_{assig\_w} = \frac{\partial U / \partial T_w}{\lambda} = \frac{\mu}{\lambda} - w$$

Value of time assigned to travel

$$SVT_{assig\_t} = \frac{\partial U / \partial T_t}{\lambda} = \frac{\mu}{\lambda} - \frac{\kappa_i}{\lambda}$$

Application of the Jara-Díaz and Guevara model (2002)

- Data from Santiago 1991 OD survey; 366 workers.
- Simple activity structure:
  - Home - work travel - work - home travel - home.
- Two income strata

Results

Resultados					
$/min	VSATV	Valor Ocio	w	VATT	VATV
Estrato medio	16.08	0.87	22.77	-21.90	-15.21
	(4,26)	(8,53)	(-)	(-)	(-)
Estrato Alto	45.90	2.35	50.41	-48.06	-43.55
	(2,26)	(8,09)	(-)	(-)	(-)

Error correlation

- Error Structure

$$T_w = T_w^*\left(\frac{c_t}{w}, T_t\right) + \varepsilon_w \quad \text{with} \quad \varepsilon_w \sim \text{i.i.d. Normal}(0.\sigma^2)$$

$$V_i = V_i^*(c_i, t_i) + \xi_i \quad \text{with} \quad \xi_i \sim \text{i.i.d. Gumbel}(0.1)$$

$$\text{Cov}(\varepsilon_w, \xi_i) \neq 0$$

Values of time

Resultados Estrato Medio					
$/min	VSATV	Valor Ocio	w	VATT	VATV
Independiente	16.08	0.87	22.77	-21.90	-15.21
	(4,26)	(8,53)	(-)	(-)	(-)
Con Correlaciones	10.52	0.64	22.77	-22.13	-9.88
	(4,09)	(8,70)	(-)	(-)	(-)

Resultados Estrato Alto					
$/min	VSATV	Valor Ocio	w	VATT	VATV
Independiente	45.90	2.35	50.41	-48.06	-43.55
	(2,26)	(8,09)	(-)	(-)	(-)
Con Correlaciones	33.65	1.69	50.41	-48.72	-31.96
	(2,24)	(7,95)	(-)	(-)	(-)

❧Statistical comparison

Log Verosimilitud Independiente	-2646,34
Log Verosimilitud Mejor Modelo	-2629,57
Test L.R	33,53
Chi Cuadrado (4gdl, 99,9%)	18,5

rho Achof	0,6232	(5,750)
rho Bus	0,3011	(1,926)
rho Cami	-0,4699	(-3,168)
rho Tax	0,4905	(3,982)

4. Model of activities, consumption and travel.

$$T_w^* = \beta(\tau - T_t) + \alpha\frac{c_t}{w} + \sqrt{\left[\beta(\tau - T_t) + \alpha\frac{c_t}{w}\right]^2 - (2(\alpha+\beta)-1)(\tau - T_t)\frac{c_t}{w}}$$

$$T_i^* = \frac{q_i}{A}\left(t - T_w^*\left(\frac{c_t}{w}, T_t\right) - T_t\right) = \frac{J_{\mathrm{i}}}{(1-2b)}\left(t - T_w^*\left(\frac{c_t}{w}, T_t\right) - T_t\right) \quad \forall i \in I$$

$$X_k^* = \frac{h_k}{B}\frac{w}{P_k}\left(T_w^*\left(\frac{c_t}{w}, T_t\right) - \frac{c_t}{w}\right) = \frac{g_k}{(1-2a)}\frac{w}{P_k}\left(T_w^*\left(\frac{c_t}{w}, T_t\right) - \frac{c_t}{w}\right) \quad \forall k \in K$$

$$V_i = \left(\frac{\Omega}{A^A B^B}\prod_{k\in K}\left(\frac{h_k}{P_k}\right)^{h_k}\prod_{i\in I}(q_i)^{q_i}\right)\left(wT_w^*\left(\frac{c_i}{w}, t_i\right) - c_i\right)^B\left(t - T_w^*\left(\frac{c_i}{w}, t_i\right) - t_i\right)^A T_w^*\left(\frac{c_i}{w}, t_i\right)^{q_w} t_i^{q_t}$$

$$V_i = \tilde{\Omega} w^{1-2a}\left(T_w^*\left(\frac{c_i}{w}, t_i\right) - \frac{c_i}{w}\right)^{1-2a}\left(t - T_w^*\left(\frac{c_i}{w}, t_i\right) - t_i\right)^{1-2b} T_w^*\left(\frac{c_i}{w}, t_i\right)^{2a+2b-1} t_i^{J_t}$$

The components of SVTTS can be calculated ***directly***

- Value of time as a resource

$$SVT_{res} = \frac{\mu}{\lambda} = \frac{1-2\beta}{1-2\alpha}\frac{\left(wT_w^* - c_t\right)}{\left(\tau - T_w^* - T_t\right)} = \frac{\vartheta_i}{1-2\alpha}\frac{\left(wT_w^* - c_t\right)}{T_i^*}$$

Value of time assigned to work

$$SVT_{assig\_w} = \frac{\partial U / \partial T_w}{\lambda} = \frac{2\alpha + 2\beta - 1}{1-2\alpha}\frac{\left(wT_w^* - c_t\right)}{T_w^*}$$

Value of time assigned to travel

$$SVT_{assig\_t} = \frac{\partial U / \partial T_t}{\lambda} = \frac{\vartheta_t}{1-2\alpha}\frac{\left(wT_w^* - c_t\right)}{T_t^{Min.}}$$

SVTTS= $\frac{k_t}{l} = \frac{1-2b}{1-2a}\frac{\left(wT_w^* - c_t\right)}{\left(t - T_w^* - T_t^{Min.}\right)} - \frac{J_t}{1-2a}\frac{\left(wT_w^* - c_t\right)}{T_t^{Min.}}$

The General System

- Considerations
  - Sets of restricted variables
    - J : goods with minimum consumption.
    - R : activities with minimum time assignment.
  - Fixed Income component I_f
  - Definitions

$$G_f = \left(\sum_{j \in J} P_j X_j^{Min.} - I_f \right) \qquad T_f = \sum_{r \in R} T_r^{Min.}$$

Conditional demands and CIUF

$$T_w^* = b\left(t - T_f\right) + a\frac{G_f}{w} + \sqrt{\left(b\left(t - T_f\right) + a\frac{G_f}{w}\right) - \left(2a + 2b - 1\right)\left(t - T_f\right)\frac{G_f}{w}}$$

$$T_i^* = \frac{J_i}{\left(1 - 2b\right)}\left(t - T_w^*\left(\frac{G_f}{w}, T_f\right) - T_f\right) \quad \forall i \in I$$

$$X_k^* = \frac{g_k}{\left(1 - 2a\right)}\frac{w}{P_k}\left(T_w^*\left(\frac{G_f}{w}, T_f\right) - \frac{G_f}{w}\right) \quad \forall k \in K$$

$$V = \tilde{\Omega} w^{1-2a}\left(T_w^* - \frac{G_f}{w}\right)^{1-2a}\left(t - T_w^* - T_f\right)^{1-2b} T_w^{*2a+2b-1}\prod_{r \in R} T_r^{Min J_r}\prod_{j \in J} X_j^{Min. g_j}$$

6. Synthesis, conclusions and future work

- Discrete travel choice, time assignment and goods consumption models have a common microeconomic framework.

$$V_i \quad \rightarrow V_i \text{ and } T_w^* \rightarrow V_i, T_j^* \text{ and } X_k^*$$

The complete system provides richer information for the estimation of common parameters.

- The components of SVTTS can be calculated explicitly.
- Detailed information is necessary to experiment with the complete system (+)
- Need to analyze the error structure: Propagation? (-) Correlation?(+)
- Experiment with new formulations (technical constraints?) and sets of restricted variables (-)

Explain numerical results from individual and social behavioral sciences (- -)

Expand empirical work to system of continuous models (goods consumption, activity time assignment) and discrete choices (travel, work, food, marriage) (--)

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