Designing Stated Choice Experiments:

State-of-the-Art

John Rose

The University of Sydney, Australia
Institute of Transport & Logistics Studies
johnr@itls.usyd.edu.au

Michiel Bliemer

The University of Sydney, Australia
Institute of Transport & Logistics Studies
michielb@itls.usyd.edu.au

Delft University of Technology, The Netherlands
Department of Transport & Planning
m.c.j.bliemer@tudelft.nl

ETH Zurich

5th December 2007

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What are Stated Choice experiments?

Paper and Pencil Surveys

CAPI Surveys

Internet Surveys

Card Number **L02A**

Your Trip:	CAR TOLL ROAD	CAR NO TOLL
Travel time to work	45 min.	70 min.
Time variability	±1 min.	±1 min.
Toll (one way)	$6.00	free
Pay toll if you leave between these times (otherwise free)	6:30-9:00 am	—
Fuel cost (per day)	$6.00	$12.00
Parking cost (per day)	$20.00	$10.00

Your Trip:	BUSWAY	TRAIN
Total time in the vehicle (one way)	30 min.	30 min.
Time from home to your closest stop	Walk 25 min. Car/Bus 8 min.	Walk 5 min. Car/Bus 4 min.
Time to your workplace from the closest stop	Walk 25 min. Bus 8 min.	Walk 5 min. Bus 4 min.
Frequency of service	Every 25 min.	Every 5 min.
Return fare (per day)	$3.00	$3.00

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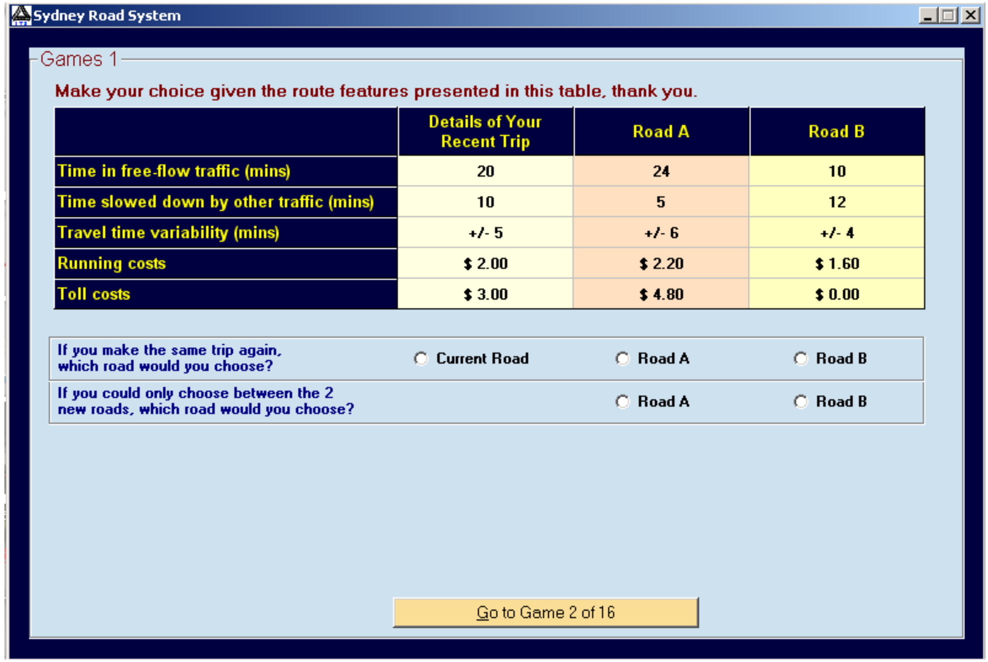
Internet Surveys


Sydney Road System

Games 1

Make your choice given the route features presented in this table, thank you.

	Details of Your Recent Trip	Road A	Road B
Time in free-flow traffic (mins)	20	24	10
Time slowed down by other traffic (mins)	10	5	12
Travel time variability (mins)	+/- 5	+/- 6	+/- 4
Running costs	$ 2.00	$ 2.20	$ 1.60
Toll costs	$ 3.00	$ 4.80	$ 0.00

If you make the same trip again, which road would you choose? ○ Current Road ○ Road A ○ Road B

If you could only choose between the 2 new roads, which road would you choose? ○ Road A ○ Road B

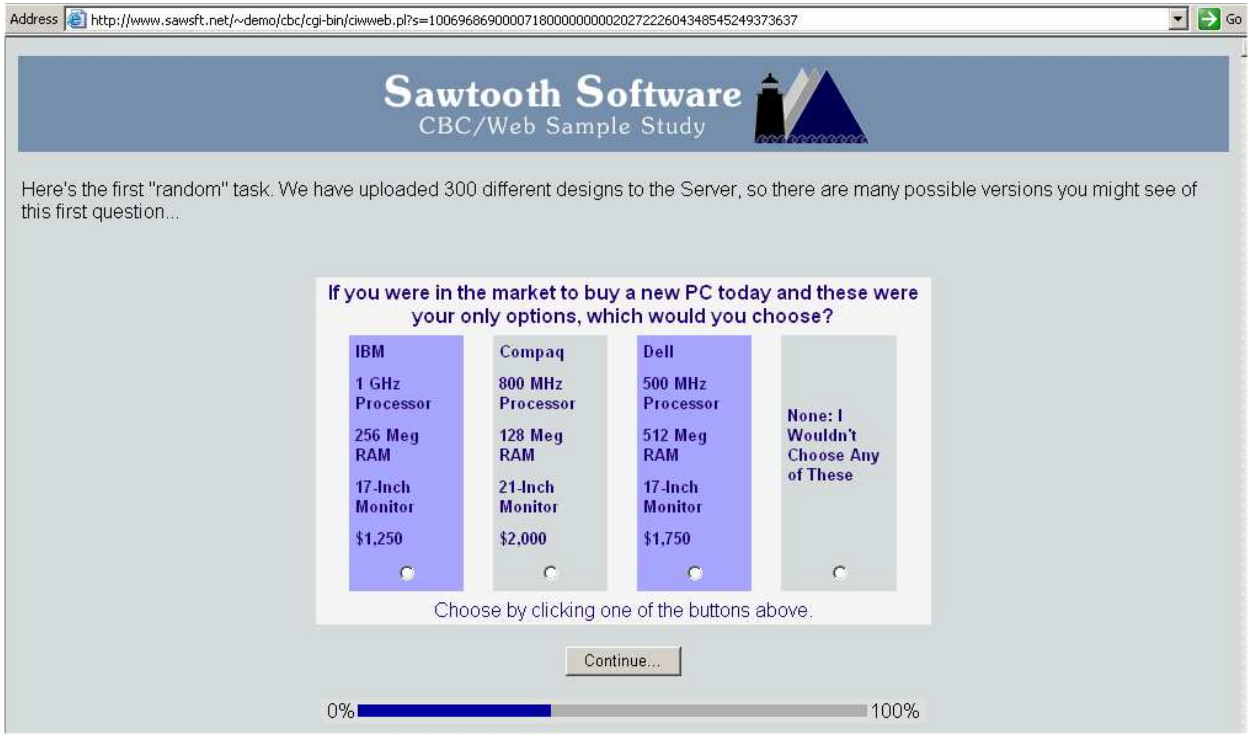
Go to Game 2 of 16

What are Stated Choice experiments?

Paper and Pencil Surveys

CAPI Surveys

Internet Surveys



Stated choice experiments

	car	train
Travel time (mins)	20	30
Fuel costs / fare ($)	5	4
Which alternative do you prefer?	○	●

next ...

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Stated choice experiments

	car	train
Travel time (mins)	20	30
Fuel costs / fare ($)	5	4

	car	train
Travel time (mins)	25	25
Fuel costs / fare ($)	6	5

	car	train
Travel time (mins)	25	30
Fuel costs / fare ($)	5	3

Stated choice experiments

Questionnaire

	car	train
Travel time (mins)	20	30
Fuel costs / fare ($)	5	4

	car	train
Travel time (mins)	25	25
Fuel costs / fare ($)	6	5

	car	train
Travel time (mins)	25	30
Fuel costs / fare ($)	5	3

Experimental design

car		train	
Time	Cost	Time	Cost
...	...	...	...
...	...	...	...
...	...	...	...

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Creating stated choice experiments

- Step 1: Specify model
  - which alternatives?
  - which attributes?
  - generic or alternative-specific param
  - which model type (MNL, NL, ML)?

$$U^{car} = \beta_0 + \beta_1 \cdot Time^{car} + \beta_2 \cdot Cost^{car}$$
$$U^{train} = \beta_3 \cdot Time^{train} + \beta_2 \cdot Cost^{train}$$

alternative-specific parameters

generic parameter

Creating stated choice experiments

- Step 2: Generate experimental design
  - how many attribute levels?
  - which attribute levels (level range)?
  - how many choice situations?
  - which attribute level combinations?

car		train	
Time	Cost	Time	Cost
20	3	15	2
25	1	20	4
30	5	25	4
25	3	40	2
30	1	35	4
20	5	30	2

Creating stated choice experiments

- Step 3: Construct questionnaire

	car	train
Travel time (mins)	20	30
Fuel costs / fare (€)	5	4

Which alternative do you prefer? ○ car ◉ train

next …

Experimental design

Given:
number of alternatives, attributes, attribute levels/range

There are 3 x 3 x 6 x 2 = 108 possible different choice situations.

Full factorial design
Complete set of all 108 choice situations.
(typically too many for a single respondent)

Fractional factorial design
Select e.g. 6 choice situations from these possible 108 (gives $1{,}38 \cdot 10^{12}$ potential designs)
- orthogonal designs
- efficient designs
- other designs (constrained, pivot, …)

car		train	
Time	Cost	Time	Cost
20	3	15	2
25	1	20	4
30	5	25	4
25	3	40	2
30	1	35	4
20	5	30	2

Full factorial designs

1	20	1	15	2
2	20	1	15	4
3	20	1	20	2
4	20	1	20	4
5	20	1	25	2
6	20	1	25	4
7	20	1	30	2
8	20	1	30	4
9	20	1	35	2
10	20	1	35	4
11	20	1	40	2
12	20	1	40	4
13	20	3	15	2
14	20	3	15	4
15	20	3	20	2
16	20	3	20	4
17	20	3	25	2
18	20	3	25	4
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
100	30	5	20	4
101	30	5	25	2
102	30	5	25	4
103	30	5	30	2
104	30	5	30	4
105	30	5	35	2
106	30	5	35	4
107	30	5	40	2
108	30	5	40	4

Advantages:
- Includes all possible combinations of attribute levels
- It can be used to estimate all main effects and interaction effects
- Orthogonal (no correlations between attribute levels)

Disadvantages:
- Too many questions for a single respondent
- May contain “useless” choice situations

Orthogonal designs (traditional)

Advantages:
- Orthogonal (no correlations between attribute levels)
- Fractional factorial, so only a subset of choice situations

Disadvantages:
- There may still be too many questions for a single respondent (the number of choice situations cannot be freely chosen) This problem may be solved by *blocking*.
- It may not be possible to find an orthogonal design
- May contain "useless" choice situations

1	20	3	35	2
2	25	5	15	2
3	30	1	25	2
4	25	3	25	4
5	30	5	35	4
6	20	1	15	4
7	20	1	30	2
8	25	3	40	2
9	30	5	20	2
10	30	1	20	4
11	20	3	30	4
12	25	5	40	4
13	25	1	15	2
14	30	3	25	2
15	20	5	35	2
16	30	5	40	4
17	20	1	20	4
18	25	3	30	4
19	25	5	30	4
20	30	1	40	4
21	20	3	20	4
22	30	3	35	4
23	20	5	15	4
24	25	1	25	4
25	25	5	20	2
26	30	1	30	2
27	20	3	40	2
28	20	1	40	2
29	25	3	20	2
30	30	5	30	2
31	30	3	15	4
32	20	5	25	4
33	25	1	35	4
34	20	5	25	2
35	25	1	35	2
36	30	3	15	2

Orthogonal designs (traditional)

Orthogonality may not be that important!

- orthogonality is usually lost in the data anyway, due to
  - missing blocks of observations
  - covariates (socio-economics, such as income or gender)
- orthogonality may not be important in estimating logit models, as it is the *differences* between the attribute levels that count
- non-orthogonal designs can yield more reliable parameter estimates

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Optimal Orthogonal Choice Designs

- Optimal Orthogonal Designs (OOC) has been pioneered by Street (UTS)
- The aim of OOC designs is to:
  - Maintain orthogonality in the design
    - Within alternatives, not between alternatives
  - Maximise the differences in the attribute levels across alternatives
    - Force trade-offs between all attributes in every choice situation of the design

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1			Design characteristics			Levels										
2		S	8		A1	A2	A3		D-eff	=	100.00%					
3		J	2		2	2	2									
4																
5					Gen:	1	1	1			A1	A2	A3	B1	B2	B3
6		S	A1	A2	A3	B1	B2	B3		A1	1					
7		1	0	0	1	1	1	0		A2	0	1				
8		2	0	0	0	1	1	1		A3	0	0	1			
9		3	1	0	0	0	1	1		B1	-1	0	0	1		
10		4	1	0	1	0	1	0		B2	0	-1	0	0	1	
11		5	0	1	1	1	0	0		B3	0	0	-1	0	0	1
12		6	0	1	0	1	0	1								
13		7	1	1	0	0	0	1								
14		8	1	1	1	0	0	0								

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1		Design characteristics				Levels										
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4																
5					Gen:	1	1	1			A1	A2	A3	B1	B2	B3
6		S	A1	A2	A3	B1	B2	B3		A1	1					
7		1	0	0	1	1	1	0		A2	0	1				
8		2	0	0	0	1	1	1		A3	0	0	1			
9		3	1	0	0	0	1	1		B1	-1	0	0	1		
10		4	1	0	1	0	1	0		B2	0	-1	0	0	1	
11		5	0	1	1	1	0	0		B3	0	0	-1	0	0	1
12		6	0	1	0	1	0	1								
13		7	1	1	0	0	0	1								
14		8	1	1	1	0	0	0								

Optimal Orthogonal Choice Designs

Practice Scenario

Please choose your preferred dating option:

	Person 1	**Person 2**	
Drinker:	Yes	No	
Children:	Yes	No	
Wants Children:	No	Yes	
Personality Type:	Introvert	Extrovert	
Smoking Habits:	Does not Smoke	Smokes	
Body Type:	Large	Slim	
Hair Colour:	Black	Red	
Highest Education:	Degree	Post Grad	
Political Persuasion:	Labor	Liberal	
Pets:	None	Dog	
Looks:	2 Stars	3 Stars	
Price to Contact:	$40	$80	
Your Choice:			
If you were in the dating market and were looking through a dating website and had a choice between the above two people, based on the descriptions listed which person would you choose?	○ **Person 1**	○ **Person 2**	○ **Neither Person**
You must also make a choice between the two people only.	○ **Person 1**	○ **Person 2**	

Next

http://survey.itls.usyd.edu.au/dating/SurveyController.php

Efficient designs

Advantages:
- Fractional factorial, so only a subset of choice situations
- More or less free choice in the number of choice situations (possibility to create smaller designs)
- Aim to avoid "useless" choice situations
- Improve the reliability of the parameter estimates

Disadvantages:
- In general not orthogonal (not that important)
- Prior parameter estimates (or prior distributions) are needed
- Needs more computation power

1	20	5	35	2
2	25	5	25	2
3	30	3	20	2
4	25	1	40	4
5	30	3	30	4
6	20	1	15	4

1	30	3	15	4
2	30	1	35	4
3	30	1	20	4
4	20	1	25	4
5	25	5	30	2
6	20	3	35	2
7	20	1	20	4
8	25	3	40	2
9	25	5	25	2
10	20	5	15	2
11	30	5	30	2
12	25	3	40	4

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Efficient designs

A design is more ***efficient*** if (with the same number of respondents) it generates data on which the model parameters can be estimated with a greater expected reliability (i.e. lower expected standard errors).

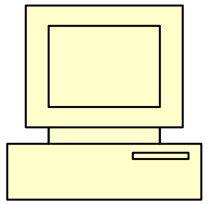
1	30	3	15	4
2	30	1	35	4
3	30	1	20	4
4	20	1	25	4
5	25	5	30	2
6	20	3	35	2
7	20	1	20	4
8	25	3	40	2
9	25	5	25	2
10	20	5	15	2
11	30	5	30	2
12	25	3	40	4

experimental design

respondents

0 0 1 0
0 1 0 0
0 1 0 0
0 0 0 1
1 0 0 0
0 1 0 0
1 0 0 0

data



estimation

$\hat{\beta}$

$se(\hat{\beta})$

results

Efficient designs

Efficiency can be determined using the true parameter values.

Problem: true parameter values are unknown.

Solution: use prior parameter values as an indication

Prior parameter values can be obtained from
e.g. literature and pilot studies.

Note:
Using prior parameter values equal to zero (i.e. no information, not even the sign) has a close correspondence with using an orthogonal design.

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Efficient designs

Which choice situation will provide the most information?

$$U^{car} = 0.2 - 0.05 \cdot Time^{car} - 0.1 \cdot Cost^{car}$$
$$U^{train} = -0.04 \cdot Time^{train} - 0.1 \cdot Cost^{train}$$

	car	train
Travel time (mins)	20	30
Fuel costs / fare ($)	3	6

$$U^{car} = -0.9 \quad (77\%)$$
$$U^{train} = -2.1 \quad (23\%)$$

	car	train
Travel time (mins)	25	20
Fuel costs / fare ($)	5	3

$$U^{car} = -1.3 \quad (50\%)$$
$$U^{train} = -1.3 \quad (50\%)$$

	car	train
Travel time (mins)	30	25
Fuel costs / fare ($)	3	4

$$U^{car} = -1.2 \quad (55\%)$$
$$U^{train} = -1.4 \quad (45\%)$$

Efficient designs

$\Omega_N(X,\tilde{\beta})$ = (asymptotic) variance-covariance matrix of the parameter estimates using experimental design X, prior parameters $\tilde{\beta}$, and a sample size of N respondents
[Note: the standard errors are the roots of the diagonals]

D-error = $\det(\Omega_N)^{1/K}$

A-error $\mathrm{tr}(\Omega_N)/K$

The lower the *D*-error,
the higher the efficiency of the experimental design.

Aim: Determine experimental design X that generates the lowest *D*-error.

Efficient designs

$$\Omega_N(X,\tilde{\beta}) = -\left[I_N(X,\tilde{\beta}) \right]^{-1}$$

$$I_N(X,\beta) = \frac{\partial^2 L_N(X,\beta)}{\partial\beta\partial\beta'}$$

Determined (a) using Monte Carlo simulation, or
(b) analytically

$L_N(X,\beta) =$ Log-likelihood function

Efficient designs

$$L_N(X,\beta) = \sum_n \sum_s \sum_i y_{isn} \log P_{isn}$$

$y_{isn} = 1,$ if respondent n chooses alternative i in choice situation s

$= 0,$ otherwise

$P_{isn} =$ probability that respondent n chooses alternative i in choice situation s

Efficient designs

MNL
$$P_{isn} = \frac{\exp\left(V_i(X_{sn},\beta)\right)}{\sum_j \exp\left(V_j(X_{sn},\beta)\right)}$$

NL
$$P_{isn} = \frac{\left(\sum_{j\in J_m} \exp\left(V_{jsn|m}(X,\beta)\right)\right)^{\lambda_m}}{\sum_k \left(\sum_{j\in J_k} \exp\left(V_{jsn|k}(X,\beta)\right)\right)^{\lambda_{nk}}} \cdot \frac{\exp\left(V_{isn|m}(X,\beta)\right)}{\sum_{j\in J_m} \exp\left(V_{isn|m}(X,\beta)\right)}$$

ML
$$P_{isn} = \iiint_\beta \frac{\exp\left(V_i(X_{sn},\beta)\right)}{\sum_j \exp\left(V_j(X_{sn},\beta)\right)} g(\beta \mid \theta) d\beta$$

Efficient designs

MNL

$$\frac{\partial^2 L(X,\beta)}{\partial \beta_{k_1}^* \partial \beta_{k_2}^*} = -\sum_n \sum_s \sum_i x_{ik_1sn}^* P_{isn} \left(x_{ik_2sn}^* - \sum_j x_{jk_2sn}^* P_{jsn} \right)$$

McFadden (1974)

$$\frac{\partial^2 L(X,\beta)}{\partial \beta_{i_1k_1} \partial \beta_{k_2}^*} = -\sum_n \sum_s x_{i_1k_1sn} P_{i_1sn} \left(x_{i_1k_2sn}^* - \sum_j x_{jk_2sn}^* P_{jsn} \right)$$

Bliemer and Rose (2005)

$$\frac{\partial^2 L(X,\beta)}{\partial \beta_{i_1k_1} \partial \beta_{i_2k_2}} = \begin{cases} \sum_n \sum_s x_{i_1k_1sn} x_{i_2k_2sn} P_{i_1sn} P_{i_2sn} & \text{if } i_1 \neq i_2 \\ -\sum_n \sum_s x_{i_1k_1sn} x_{i_2k_2sn} P_{i_1sn} \left(1 - P_{i_2sn}\right) & \text{if } i_1 = i_2 \end{cases}$$

Note: y drops out

Efficient designs

NL Bliemer, Rose and Hensher (2006)

$$\frac{\partial L(X,(\beta,\lambda))}{\partial \beta_{k_1}\partial \beta_{k_2}} = \sum_n \sum_{s=1}^{S} \Bigg\{ \sum_{m=1}^{M} P_{ms} \Bigg[(\lambda_m - 1) \Bigg(\sum_{i \in J_{mk_1k_2}} x_{mik_1s} P_{is|m} x_{mik_2s} - \sum_{i \in J_{mk_1}} x_{mik_1s} P_{is|m} \sum_{j \in J_{mk_2}} P_{js|m} x_{mjk_2s} \Bigg) \Bigg]$$

$$- \Bigg(\sum_{m=1}^{M} \lambda_m P_{ms} \Bigg[\Bigg(\lambda_m \sum_{i \in J_{mk_2}} P_{is|m} x_{mik_2s} - \sum_{n=1}^{M} \lambda_n P_{ns} \sum_{i \in J_{nk_2}} P_{is|n} x_{nik_2s} \Bigg) \sum_{i \in J_{mk_1}} x_{mik_1s} P_{is|m} + \sum_{i \in J_{mk_1k_2}} x_{mik_1s} P_{is|m} x_{mik_2s} - \sum_{i \in J_{mk_1}} x_{mik_1s} P_{is|m} \sum_{j \in J_{mk_2}} P_{js|m} x_{mjk_2s} \Bigg] \Bigg) \Bigg\}$$

$$\frac{\partial L(X,(\beta,\lambda))}{\partial \lambda_{m_1}\partial \beta_{k_2}} = -\sum_n \sum_{s=1}^{S} \Bigg[\log \sum_{i \in J_{m_1}} \exp\Bigg(\sum_{k \in K_{m_1 i}} \beta_k x_{m_1 iks} \Bigg) P_{m_1 s} \Bigg(\lambda_{m_1} \sum_{i \in J_{m_1k_2}} P_{is|m_1} x_{m_1ik_2s} - \sum_{n=1}^{M} \lambda_n P_{ns} \sum_{i \in J_{nk_2}} P_{is|n} x_{nik_2s} \Bigg) \Bigg]$$

$$\frac{\partial^2 L(X,(\beta,\lambda))}{\partial \lambda_{m_1}\partial \lambda_{m_2}} = \begin{cases} -\sum_n \sum_{s=1}^{S} P_{m_1 s} \left(1 - P_{m_1 s}\right) \left(\log\left[\sum_{i \in J_{m_1}} \exp\left(\sum_{k \in K_{m_1 i}} \beta_k x_{m_1 iks} \right) \right] \right)^2, & \text{if } m_1 = m_2 \\ \sum_n \sum_{s=1}^{S} P_{m_1 s} P_{m_2 s} \log\left[\sum_{i \in J_{m_1}} \exp\left(\sum_{k \in K_{m_1 i}} \beta_k x_{m_1 iks} \right) \right] \log\left[\sum_{i \in J_{m_2}} \exp\left(\sum_{k \in K_{m_2 i}} \beta_k x_{m_2 iks} \right) \right], & \text{if } m_1 \neq m_2 \end{cases}$$

Note: y drops out

Efficient designs

ML Bliemer and Rose (2006)

$$\frac{\partial^2 L(X,(\mu,\sigma))}{\partial\mu_{k_1}\partial\mu_{k_2}} = \sum_s\sum_j\int_\varphi P_{js}\left(x_{jk_1s}-\sum_{i\in J_{k_1}}P_{is}x_{ik_1s}\right)\left(x_{jk_2s}-\sum_{i\in J_{k_2}}P_{is}x_{ik_2s}\right)-P_{js}\sum_{i\in J_{k_1}}x_{ik_1s}P_{is}\left(x_{ik_2s}-\sum_{h\in J_{k_2}}P_{hs}x_{hk_2s}\right)\varphi d\varphi$$

$$-\frac{\int_\varphi P_{js}\left(x_{jk_1s}-\sum_{i\in J_{k_1}}P_{is}x_{ik_1s}\right)\varphi d\varphi\cdot\int_\varphi P_{js}\left(x_{jk_2s}-\sum_{i\in J_{k_2}}P_{is}x_{ik_2s}\right)\varphi d\varphi}{\int_\varphi P_{js}\varphi d\varphi}$$

$$\frac{\partial^2 L(X,(\mu,\sigma))}{\partial\mu_{k_1}\partial\sigma_{k_2}} = \sum_s\sum_j\int_\varphi P_{js}\left(x_{jk_1s}-\sum_{i\in J_{k_1}}P_{is}x_{ik_1s}\right)\left(x_{jk_2s}-\sum_{i\in J_{k_2}}P_{is}x_{ik_2s}\right)-P_{js}\sum_{i\in J_{k_1}}x_{ik_1s}P_{is}\left(x_{ik_2s}-\sum_{h\in J_{k_2}}P_{hs}x_{hk_2s}\right)\varphi_{k_2}\varphi d\varphi$$

$$-\frac{\int_\varphi P_{js}\left(x_{jk_1s}-\sum_{i\in J_{k_1}}P_{is}x_{ik_1s}\right)\varphi d\varphi\cdot\int_\varphi P_{js}\left(x_{jk_2s}-\sum_{i\in J_{k_2}}P_{is}x_{ik_2s}\right)\varphi_{k_2}\varphi d\varphi}{\int_\varphi P_{js}\varphi d\varphi}$$

$$\frac{\partial^2 L(X,(\mu,\sigma))}{\partial\sigma_{k_1}\partial\sigma_{k_2}} = \sum_s\sum_j\int_\varphi P_{js}\left(x_{jk_1s}-\sum_{i\in J_{k_1}}P_{is}x_{ik_1s}\right)\left(x_{jk_2s}-\sum_{i\in J_{k_2}}P_{is}x_{ik_2s}\right)-P_{js}\sum_{i\in J_{k_1}}x_{ik_1s}P_{is}\left(x_{ik_2s}-\sum_{h\in J_{k_2}}P_{hs}x_{hk_2s}\right)\varphi_{k_1}\varphi_{k_2}\varphi d\varphi$$

$$-\frac{\int_\varphi P_{js}\left(x_{jk_1s}-\sum_{i\in J_{k_1}}P_{is}x_{ik_1s}\right)\varphi_{k_1}d\varphi\cdot\int_\varphi P_{js}\left(x_{jk_2s}-\sum_{i\in J_{k_2}}P_{is}x_{ik_2s}\right)\varphi_{k_2}\varphi d\varphi}{\int_\varphi P_{js}\varphi d\varphi}$$

Note: y drops out

Efficient designs

Interesting observation:

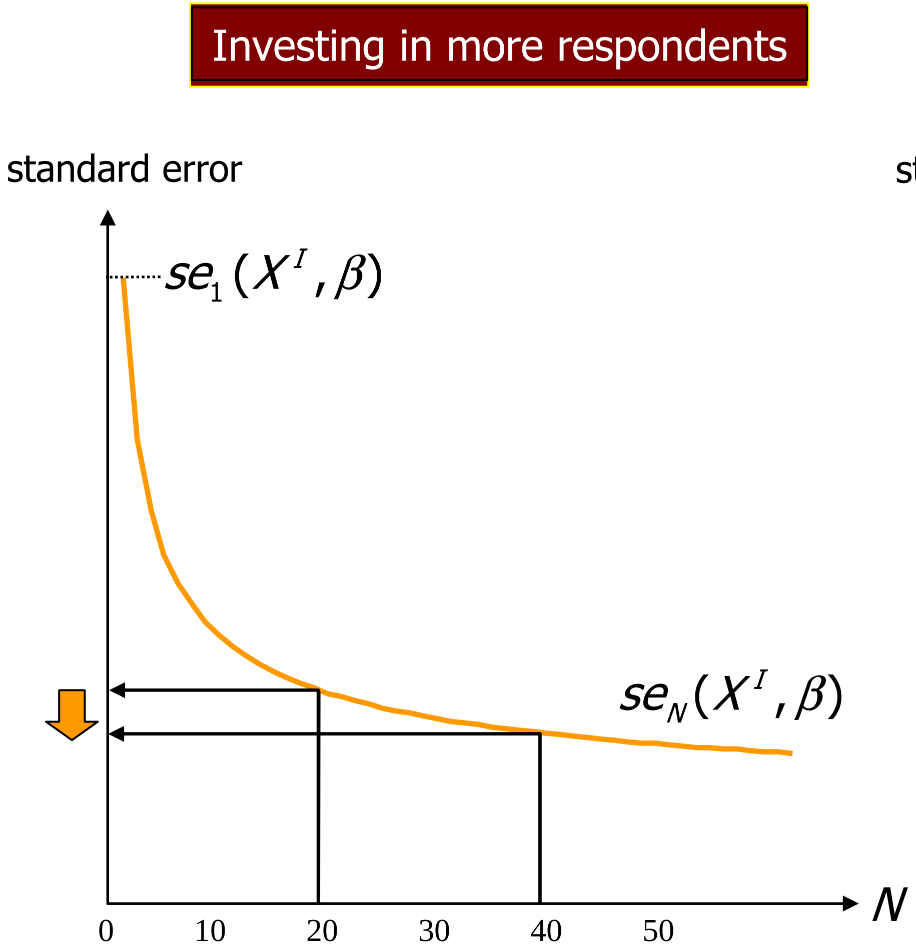
If all respondents face the same choice situations, then

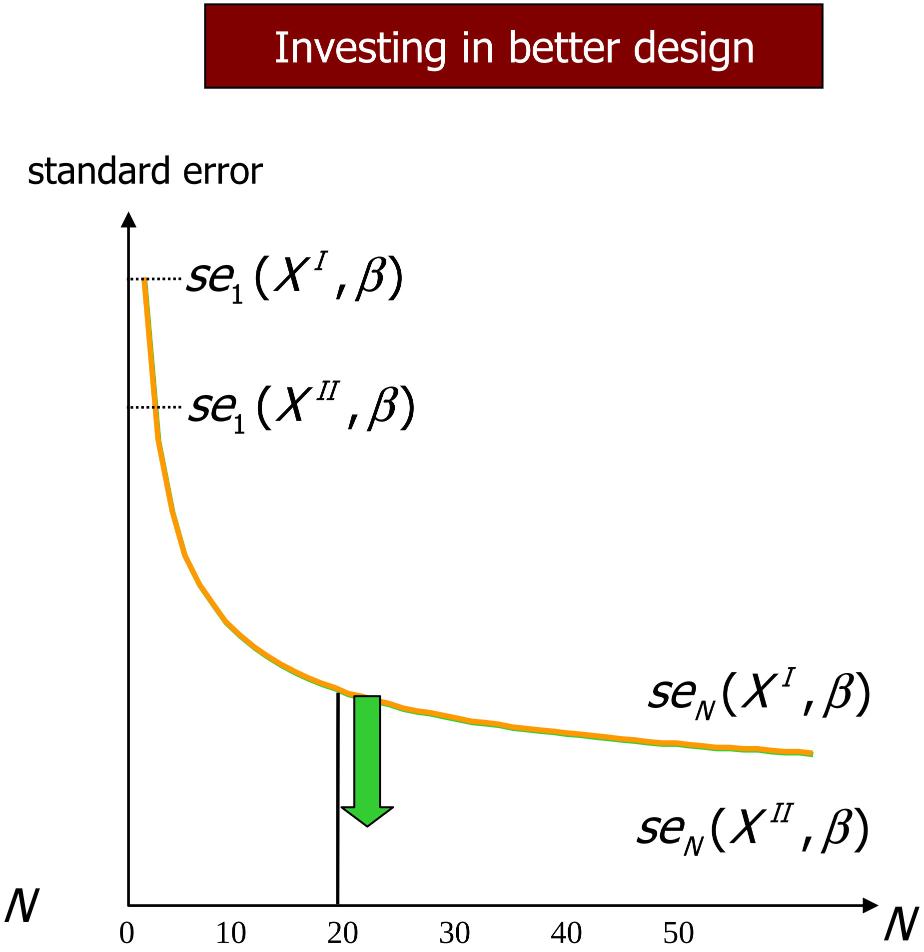
Hence, we can derive the asymptotic variance-covariance (AVC) matrix with N respondents from the AVC matrix from a single respondent.

Furthermore:

$$se_N(X,\tilde{\beta}) = \frac{1}{\sqrt{N}} se_1(X,\tilde{\beta})$$

Efficient designs





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What if the priors are unreliable?

Instead of assumed fixed prior parameter values, we can assume prior parameter distributions.

$$U^{car} = \tilde{\beta}_0 + \tilde{\beta}_1 Time^{car} + \tilde{\beta}_2 .Cost^{car}$$

$$U^{train} = \tilde{\beta}_3 Time^{train} + \tilde{\beta}_2 .Cost^{train}$$

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What if the priors are unreliable?

Efficient designs:

Minimize D-error = $\det\left(\Omega_N(X,\tilde{\beta})\right)^{1/K}$

Bayesian efficient designs:

Minimize Expected D-error = $\iiint_{\tilde{\beta}} \det\left(\Omega_N(X,\tilde{\beta})\right)^{1/K} f(\tilde{\beta}\,|\,\omega)d\tilde{\beta}$

This integral can be approximated by
- pseudo-Monte Carlo simulation
- Modified Latin Hybercube sampling
- quasi-Monte Carlo simulation (e.g., Halton, Sobol draws)
- Guassian quadrature

A Bayesian efficient design is a more “stable” design that will be relatively efficient over a range of prior parameter values.

Other designs

Constrained designs
Some attribute level combinations may not occur

Pivot designs
Attribute levels pivoted from a knowledge base, so the design is optimized for each individual or the whole population.

E.g. levels: [-50%, 0%, +50%]
Respondent 1: travel time = 60 min. levels = {30, 60, 90}
Respondent 2: travel time = 10 min. levels = {5, 10, 15}

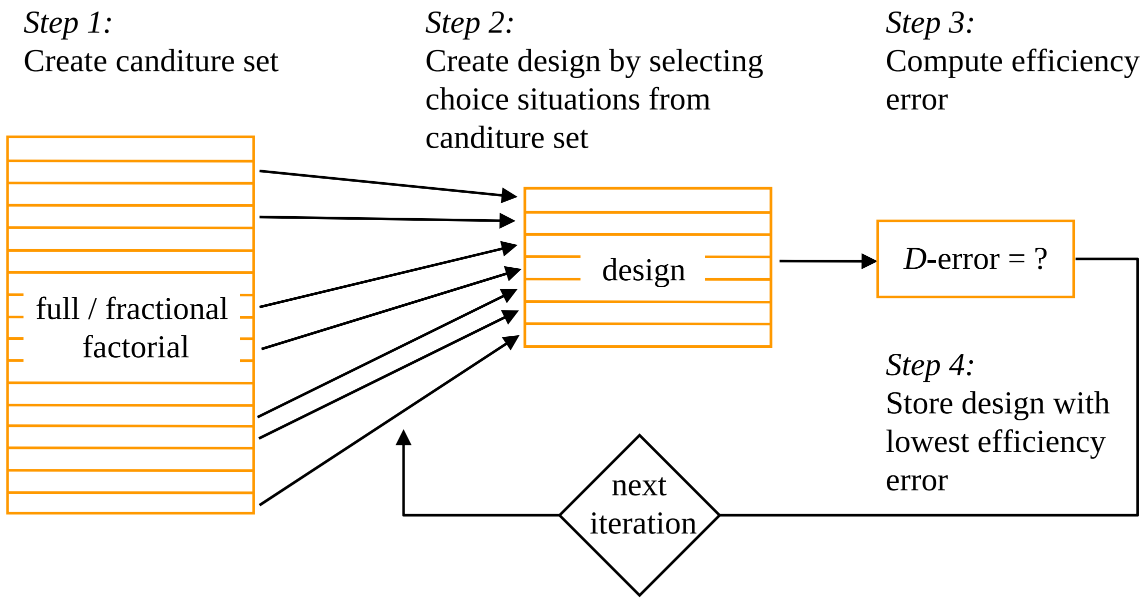
Designs with covariates
Adding covariates (e.g. income, gender) to utility function changes the efficiency of the design. One can create designs optimal for each individual or the whole population.

How to generate designs?

Algorithms for finding efficient designs:

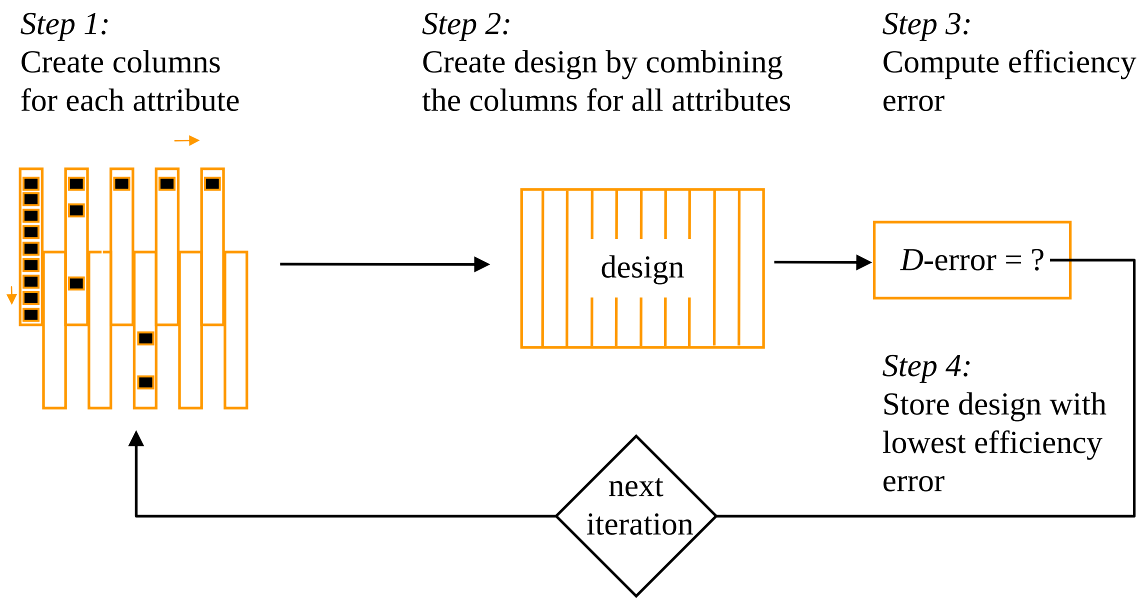
- Modified Federov algorithms
- RSC (relabeling, swapping, cycling) algorithms
- …

How to generate designs?



Modified Federov Algorithm (Cook and Nachtsheim, 1980)

How to generate designs?



RSC Algorithm (Huber and Zwerina, 1996)

Example

MNL model:

$$V_A = \beta_1 X_{A1} + \beta_2 X_{A2} + \beta_{A1} X_{A3} + \beta_{A2} X_{A4}$$
$$V_B = \beta_{B0} + \beta_1 X_{B1} + \beta_2 X_{B2} + \beta_{B1} X_{B3} + \beta_{B2} X_{B4}$$

Priors:

$$\tilde{\beta}_1 = 0.4 \qquad \tilde{\beta}_2 = 0.3$$
$$\tilde{\beta}_{A1} = 0.3 \qquad \tilde{\beta}_{A2} = 0.6$$
$$\tilde{\beta}_{B0} = -1.2 \qquad \tilde{\beta}_{B1} = 0.4 \qquad \tilde{\beta}_{B2} = 0.7$$

Attribute levels:

$$X_{A1} = 2,4,6; \quad X_{A2} = 1,3,5; \quad X_{A3} = 2\tfrac{1}{2}, 3, 3\tfrac{1}{2}; \quad X_{A4} = 4,6,8;$$
$$X_{B1} = 2,4,6; \quad X_{B2} = 1,3,5; \quad X_{B3} = 2\tfrac{1}{2}, 4, 5\tfrac{1}{2}; \quad X_{B4} = 4,6,8.$$

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Example

"Random" design

	G1	G2	A1	A2	G1	G2	B0	B1	B2
1	4	3	2.5	4	6	3	1	4	8
2	2	3	3	6	4	1	1	5.5	6
3	2	5	3.5	4	6	5	1	2.5	8
4	4	1	2.5	8	2	3	1	4	4
5	6	1	2.5	8	2	3	1	5.5	6
6	6	5	3.5	6	6	5	1	2.5	4
7	2	5	2.5	4	4	5	1	5.5	8
8	4	1	3.5	4	6	1	1	2.5	6
9	2	3	3	6	2	1	1	5.5	8
10	6	1	3	8	2	1	1	4	4
11	4	5	3.5	6	4	5	1	2.5	4
12	6	3	3	8	4	3	1	4	6

correlation matrix:

	G1	*G2*	*A1*	*A2*	*G1*	*G2*	*B1*	*B2*
G1	1.00							
G2	-0.38	1.00						
A1	0.00	0.38	1.00					
A2	0.63	-0.50	-0.25	1.00				
G1	-0.13	0.50	0.50	-0.75	1.00			
G2	0.00	0.75	0.13	-0.25	0.38	1.00		
B1	-0.25	-0.25	-0.75	0.25	-0.63	-0.38	1.00	
B2	-0.63	0.25	-0.25	-0.63	0.25	0.00	0.38	1.00

D-error = 1.7470

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Example

Orthogonal design

	G1	G2	A1	A2	G1	G2	B0	B1	B2
1	4	1	3.5	6	2	3	1	4	4
2	2	1	3.5	8	6	5	1	4	8
3	2	3	3	4	6	1	1	5.5	4
4	2	3	3	6	2	3	1	2.5	4
5	4	3	3	6	2	3	1	5.5	8
6	6	1	2.5	4	4	5	1	5.5	6
7	6	5	3.5	8	4	1	1	5.5	6
8	6	1	2.5	8	4	1	1	2.5	6
9	4	5	2.5	8	6	5	1	4	4
10	6	5	3.5	4	4	5	1	2.5	6
11	4	3	3	4	6	1	1	2.5	8
12	2	5	2.5	6	2	3	1	4	8

correlation matrix:

	G1	*G2*	*A1*	*A2*	*G1*	*G2*	*B1*	*B2*
G1	1.00							
G2	0.00	1.00						
A1	0.00	0.00	1.00					
A2	0.00	0.00	0.00	1.00				
G1	0.00	0.00	0.00	0.00	1.00			
G2	0.00	0.00	0.00	0.00	0.00	1.00		
B1	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
B2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

D-error = 0.4251

Example

Efficient design

	G1	G2	A1	A2	G1	G2	B0	B1	B2
1	4	3	3	4	6	3	1	4	6
2	2	3	3	6	4	3	1	5.5	6
3	6	1	2.5	8	2	5	1	4	8
4	4	1	3.5	4	4	5	1	2.5	4
5	4	5	3.5	6	4	1	1	5.5	6
6	6	5	2.5	4	2	1	1	2.5	8
7	6	3	3	6	4	3	1	4	4
8	2	5	2.5	6	6	1	1	5.5	4
9	2	5	3.5	8	6	1	1	2.5	8
10	4	1	3.5	8	2	5	1	5.5	6
11	2	1	2.5	8	6	5	1	2.5	4
12	6	3	3	4	2	3	1	4	8

correlation matrix:

	G1	*G2*	*A1*	*A2*	*G1*	*G2*	*B1*	*B2*
G1	1.00							
G2	-0.13	1.00						
A1	-0.13	0.00	1.00					
A2	-0.38	-0.25	0.00	1.00				
G1	-0.75	0.25	0.00	0.13	1.00			
G2	0.13	-1.00	0.00	0.25	-0.25	1.00		
B1	-0.13	0.13	0.13	0.13	-0.13	-0.13	1.00	
B2	0.38	0.25	0.00	0.00	-0.50	-0.25	-0.13	1.00

D-error = 0.1949

Example

Orthogonal efficient design

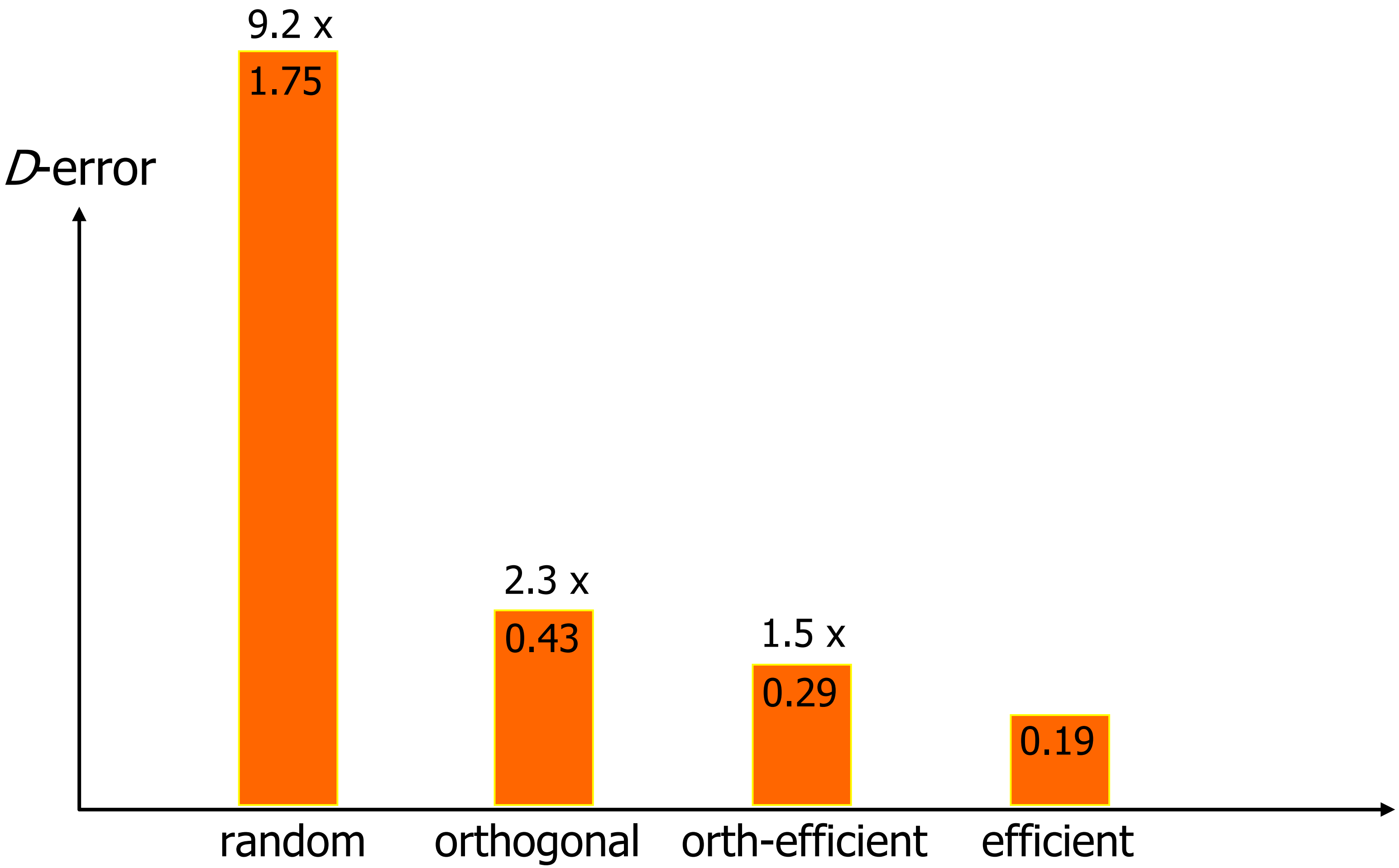
	G1	G2	A1	A2	G1	G2	B0	B1	B2
1	4	5	3.5	8	6	1	1	4	8
2	4	3	3	4	6	5	1	2.5	4
3	2	5	3.5	6	2	3	1	4	4
4	6	1	3.5	8	4	5	1	2.5	6
5	4	1	2.5	6	2	3	1	4	8
6	6	5	2.5	4	4	1	1	2.5	6
7	6	1	3.5	4	4	1	1	5.5	6
8	6	5	2.5	8	4	5	1	5.5	6
9	2	3	3	4	6	5	1	5.5	8
10	2	3	3	6	2	3	1	2.5	8
11	4	3	3	6	2	3	1	5.5	4
12	2	1	2.5	8	6	1	1	4	4

correlation matrix:

	G1	*G2*	*A1*	*A2*	*G1*	*G2*	*B1*	*B2*
G1	1.00							
G2	0.00	1.00						
A1	0.00	0.00	1.00					
A2	0.00	0.00	0.00	1.00				
G1	0.00	0.00	0.00	0.00	1.00			
G2	0.00	0.00	0.00	0.00	0.00	1.00		
B1	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
B2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

D-error = 0.2918

Example


9.2 x
1.75
2.3 x
0.43
1.5 x
0.29
0.19
D-error
random
orthogonal
orth-efficient
efficient

Example

$$V_A = \beta_1 X_{A1} + \beta_2 X_{A2} + \beta_{A1} X_{A3} + \beta_{A2} X_{A4}$$

$$V_B = \beta_{B0} + \beta_1 X_{B1} + \beta_2 X_{B2} + \beta_{B1} X_{B3} + \beta_{B2} X_{B4}$$

narrow ← attribute level range → wide

attribute levels

number of levels

X_{A1}, X_{B1}, X_{B3} X_{A2}, X_{A3}, X_{B2} X_{A4}, X_{B4}	0.28	0.09	0.04
X_{A1}, X_{B1}, X_{B3} X_{A2}, X_{A3}, X_{B2} X_{A4}, X_{B4}	0.38	0.11	0.06
X_{A1}, X_{B1}, X_{B3} X_{A2}, X_{A3}, X_{B2} X_{A4}, X_{B4}	0.43	0.13	0.07

Optimal Choice Probability Designs

- Optimal choice percentage designs are basically *D*-efficient designs that are made more efficient by assuming that one attribute has continuous attribute levels (e.g., price)
- Pre-determined attribute levels (e.g., {1,3,5}) put a constraint on the efficiency of a design; the efficiency could be improved if the attribute level is assumed continuous on a range (e.g., [1,5])

Optimal Choice Probability Designs (cont'd)

- Which choice situation yields the lowest *D*-error?

Model: $U_A = \beta_1 A_1 + \beta_2 A_2$ Priors: $\beta_1 = 0.1$

$U_B = \beta_1 B_1 + \beta_2 B_2$ $\beta_2 = 0.2$

A1	A2	B1	B2
8	2	2	4

$P_A = 0.50, \quad P_B = 0.50$

A1	A2	B1	B2
0	2	40	4

$P_A = 0.01, \quad P_B = 0.99$

A1	A2	B1	B2
1	2	10	4

$P_A = 0.18, \quad P_B = 0.82$

A1	A2	B1	B2
5	2	3	4

$P_A = 0.40, \quad P_B = 0.60$

Optimal Choice Probability Designs (cont'd)

- In case of a generic model with two alternatives, the following optimal choice probabilities hold:

Number of attributes	Optimal choice probabilties
2	0.82 / 0.18
3	0.77 / 0.23
4	0.74 / 0.26
5	0.72 / 0.28
6	0.70 / 0.30
7	0.68 / 0.32
8	0.67 / 0.33

Source: Johnson *et al.* (2006)

These probabilities are sometimes called Magic P values

- What are the optimal probabilities for 3 or more alternatives?
- What if the model is not generic?

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Generating Optimal Choice Prob. Designs

- Step 1: Generate orthogonal design for first alternative
- Step 2: Generate orthogonal designs for second alternative using a fold-over (reversing attribute levels)
- Step 3: Select attribute with continuous levels
- Step 4: Look up optimal choice probabilities from table
- Step 5: Change attribute levels of the continuous attribute such that in each choice situation these optimal choice probabilities are matched

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Generating Optimal Choice Prob. Designs

- Consider generic model with 2 alternatives and 5 attributes (assume first attribute has continuous levels)

	A1	B1	C1	D1	E1	A2	B2	C2	D2	E2	PA	PB
1	1	4	0	1	1	10	8	1	0	0	1.00	0.00
2	10	4	0	0	0	1	8	1	1	1	0.01	0.99
3	1	8	0	0	1	10	4	1	1	0	0.90	0.10
4	10	8	0	1	0	1	4	1	0	1	0.00	1.00
5	1	4	1	1	0	10	8	0	0	1	1.00	0.00
6	10	4	1	0	1	1	8	0	1	0	0.01	0.99
7	1	8	1	0	0	10	4	0	1	1	0.98	0.02
8	10	8	1	1	1	1	4	0	0	0	0.00	1.00

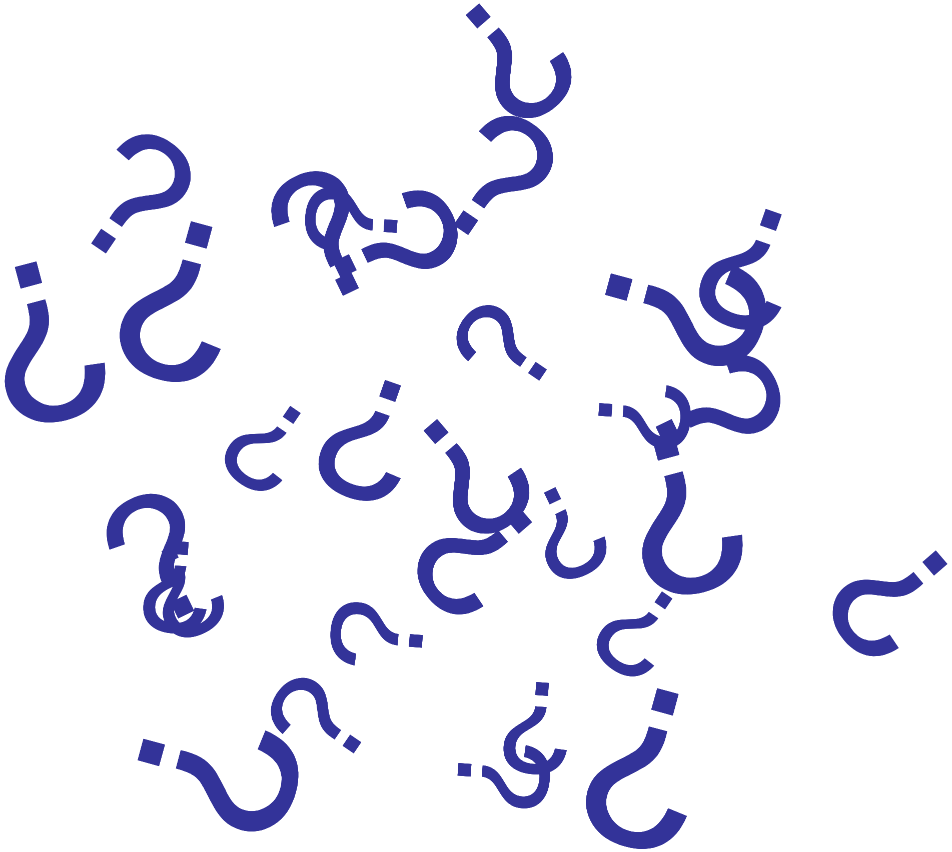
D-error = 3.5846

	A1	B1	C1	D1	E1	A2	B2	C2	D2	E2	PA	PB
1	10	4	0	1	1	8.876	8	1	0	0	0.72	0.28
2	4.209	4	0	0	0	1	8	1	1	1	0.28	0.72
3	3.124	8	0	0	1	10	4	1	1	0	0.72	0.28
4	1.542	8	0	1	0	1	4	1	0	1	0.28	0.72
5	10	4	1	1	0	6.209	8	0	0	1	0.72	0.28
6	3.542	4	1	0	1	1	8	0	1	0	0.28	0.72
7	5.791	8	1	0	0	10	4	0	1	1	0.72	0.28
8	1	8	1	1	1	1.124	4	0	0	0	0.28	0.72

D-error = 0.2969

lowest *D*-error with fixed levels: 0.3750

Questions?




Thank you!