Differentiated Pricing of Urban Transportation Networks

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Price Differentiation

Economic concept

- Defined by Dupuit (1894)
- Identical products sold at different prices

Examples in transportation pricing

- One two-way ticket is cheaper than two one-way tickets
- Senior citizens pay lower bus fare
- Trucks pay more toll than cars
- Area-based congestion pricing
- Differentiated pricing with respect to value of time

We investigate another third-degree differentiated pricing scheme that differentiates travelers with respect to their *travel characteristics*.

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We investigate another third-degree differentiated pricing scheme that differentiates travelers with respect to their *travel characteristics*.

How does the differentiated pricing scheme work?

Definitions

Anonymous scheme: Everyone pays the same amount of toll.
Differentiated scheme: Toll depends on travel characteristics.
Level of differentiation:

- L0: None $\rightarrow$ anonymous scheme
- L1: Origin $\rightarrow$ origin-specific scheme
- L2: Origin and destination $\rightarrow$ OD-specific scheme
- L3: Path $\rightarrow$ path-based scheme

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Level of Differentiation

Path	L0	L1	L2	L3
	π_9	π_9^2	$\pi_9^{2,3}$	$\pi_{4,8,9,11}$
	π_9	π_9^1	$\pi_9^{1,3}$	$\pi_{2,8,9,11}$
	π_9	π_9^1	$\pi_9^{1,4}$	$\pi_{2,8,9,12}$
	π_9	π_9^1	$\pi_9^{1,4}$	$\pi_{1,6,9,12}$

Benefits of Differentiation

Example: First-best Condition

- All links are tollable $\rightarrow$ System optimum is achievable
  - $x_a = \bar{x}_a \qquad \forall a \in A$
- Tolls that minimize travel time are not unique. Benefits of price differentiation can only be reflected on a secondary objective
- Toll revenue is a financial burden on travelers
- Higher toll revenue implies less public acceptance
- Choose the tolls with minimum revenue
  - min $\quad \sum_{w \in W} \sum_{p \in P_w} \pi_p f_p$

Example: First-best Condition

Table : First-best pricing for nine-nodes network

Tolling Scheme	Toll Revenue		OD Generalized Travel Cost			
	Amount	Reduction	[1, 3]	[1, 4]	[2, 3]	[2, 4]
Anonymous	887.6	0%	30.6	29.2	33.0	31.6
Origin-specific	311.6	65%	23.4	29.3	25.8	24.4
OD-specific	295.6	67%	23.4	22.0	25.8	27.6
Path-based	263.6	70%	23.4	22.0	29.0	24.4

Example: Second-best Condition

- Suppose links $a \in \bar{\Psi}$ are untollable
  - Origin-specific $\gamma_a^{o(w)} = 0 \quad \forall w \in W, a \in \bar{\Psi}$
  - OD-specific $\gamma_a^{w} = 0 \quad \forall w \in W, a \in \bar{\Psi}$
- Travel time as the performance measure
- Choose tolls that minimize total system travel time
  - min $\sum_{w \in W} \sum_{p \in P_w} t_p(f) f_p$

Example: Second-best Condition

Table : Second-best pricing for nine-node network

Tolling Scheme	Total Travel Time		OD Generalized Travel Cost			
	Amount	Saving	[1, 3]	[1, 4]	[2, 3]	[2, 4]
UE	2455.9	0%	24.9	23.8	24.3	25.1
Anonymous	2361.2	46.9%	25.8	24.9	25.1	25.9
Origin-specific	2306.1	74.2%	24.3	24.2	27.1	25.7
OD-specific	2281.7	86.2%	24.4	22.9	26.8	25.3
SO	2253.9	100%	-	-	-	-

Finding Optimal Differentiated Schemes

Feasible region of path-based pricing schemes defined as follows:

$$\sum_{p \in P_w} f_p = d_w \qquad \forall w \in W$$

$$f_p \left(t_p \left(f \right) + \pi_p - \lambda_w \right) = 0 \qquad \forall p \in P_w, w \in W$$

$$t_p \left(f \right) + \pi_p - \lambda_w \geq 0 \qquad \forall p \in P_w, w \in W$$

$$f_p \geq 0 \qquad \forall p \in P_w, w \in W$$

$$\pi_p \geq 0 \qquad \forall p \in P_w, w \in W$$

$$x_a = \sum_{w \in W} \sum_{p \in P_w} \delta_{ap} f_p \qquad \forall a \in A$$

Finding Optimal Differentiated Schemes

Additional constraints needed for link-based schemes:

- Origin-specific

$$\pi_p = \sum_{a \in A} \delta_{ap} \gamma_a^{o(w)} \qquad \forall p \in P_w, w \in W$$

$$\gamma_a^{o(w)} \geq 0 \qquad \forall a \in A, w \in W$$

- OD-specific

$$\pi_p = \sum_{a \in A} \delta_{ap} \gamma_a^{w} \qquad \forall p \in P_w, w \in W$$

$$\gamma_a^{w} \geq 0 \qquad \forall a \in A, w \in W$$

Finding Optimal Differentiated Schemes

First-best Condition:

- Additional constraint: $x_a = \bar{x}_a \qquad \forall a \in A$
- Objective function: min $\sum_{w \in W} \sum_{p \in P_w} \pi_p f_p$

Second-best Condition:

- Objective function: min $\sum_{w \in W} \sum_{p \in P_w} t_p(f) f_p$

Example: Path-based Scheme (First-best Condition)

$$\min \quad \sum_{w \in W} \sum_{p \in P_w} \pi_p f_p$$

$s.t.$

$$\sum_{p \in P_w} f_p = d_w \qquad \forall w \in W$$

$$f_p \left(t_p \left(f \right) + \pi_p - \lambda_w \right) = 0 \qquad \forall p \in P_w, w \in W$$

$$t_p \left(f \right) + \pi_p - \lambda_w \geq 0 \qquad \forall p \in P_w, w \in W$$

$$f_p \geq 0 \qquad \forall p \in P_w, w \in W$$

$$\pi_p \geq 0 \qquad \forall p \in P_w, w \in W$$

$$\bar{x}_a = \sum_{w \in W} \sum_{p \in P_w} \delta_{ap} f_p \qquad \forall a \in A$$

Sioux Falls Network

- 24 nodes
- 76 links
- 528 OD pairs
- 14 tollable links for second-best pricing

Implementation on Sioux Falls Network

Table : Results of differentiated pricing for Sioux Falls network

TollingScheme	First-best		Second-best	
	Min. Revenue	Reduction	Travel Time	Saving
Anonymous	23.441	0.00%	74.043	26.55%
Origin-specific	0.750	96.80%	73.060	60.93%
OD-specific	0.616	97.37%	72.997	63.13%
Path-based	0.182	99.23%	-	-
User Equilibrium	-	-	74.802	0.00%
System Optimum	-	-	71.943	100.00%

- Differentiated schemes perform significantly better than anonymous schemes
- For both the first-best and second-best conditions, higher levels of differentiation lead to more favorable results

Is there any issue associated with these schemes?

Privacy

- Location privacy: the ability to prevent other parties from learning one's current or past location
- Arises when providing a service requires location data
- Differentiated pricing requires the knowledge of travelers' travel characteristics such as the origin and destination of each trip for an OD-specific scheme
- The OD data may imply home/work locations. Even if they are anonymous, they can be used to identify individuals (Krumm, 2009).
- The sole fact of being tracked by the tolling system can also cause inconvenience or discomfort.

Value of Privacy

- Individuals value their privacy differently
- They can be grouped into categories of privacy unconcerneds, privacy pragmatists, and privacy fundamentalists
- Mathematically, we can use a distribution to represent different individual valuations of privacy across the population

Who benefits from differentiated schemes?

We now use origin-specific pricing as an example for modeling privacy. For each OD-pair w:

- Travel cost saving: $\lambda_{w,0} - \lambda_{w,1}$
- The percentage of motorists who will be better off under origin-specific pricing: $\int_0^{\lambda_{w,0}-\lambda_{w,1}} \xi(z)dz$

Example: Nine-node Network

Table : The percentage of travelers who benefit from origin-specific scheme

Network Condition	First-best				Second-best			
OD pair	[1,3]	[1,4]	[2,3]	[2,4]	[1,3]	[1,4]	[2,3]	[2,4]
Travel Cost Saving	7.2	-0.1	7.2	7.2	1.5	0.7	-2.0	0.2
$\beta \sim U(0,4)$	100.00	0.00	100.00	100.00	37.50	17.50	0.00	5.00
$\beta \sim U(0,8)$	90.00	0.00	90.00	90.00	18.75	8.75	0.00	2.50
$\beta \sim U(0,16)$	45.00	0.00	45.00	45.00	9.37	4.38	0.00	1.25
$\beta \sim EXP(0.500)$	97.27	0.00	97.27	97.27	52.76	29.53	0.00	9.52
$\beta \sim EXP(0.250)$	83.47	0.00	83.47	83.47	31.27	16.05	0.00	4.88
$\beta \sim EXP(0.125)$	59.34	0.00	59.34	59.34	17.10	8.38	0.00	2.47

- The savings of time and toll that some travelers enjoy from differentiated schemes are offset by the loss of their privacy
- When average value of privacy is high, there are still some travelers who benefit from origin-specific scheme

What is the solution to privacy issue?

Idea

We propose an incentive program that allows each traveler to opt in to differentiated schemes

- Self-selection mechanism: travelers who choose to reveal their private information will pay differentiated tolls while those who remain anonymous will pay uniform tolls
- Anonymous scheme preserves location privacy
- Differentiated scheme offers incentives
- In the simplest setting, the travel cost difference between differentiated and anonymous schemes can be viewed as incentive

Design of Incentive Program

- Demand split constraints:

$$d_{w,0} + d_{w,1} = d_w \qquad \forall w \in W$$

$$\sum_{p \in P_w} f_{p,c} = d_{w,c} \qquad \forall w \in W, c \in \{0,1\}$$

$$d_{w,1} = \Xi(\lambda_{w,0} - \lambda_{w,1}) d_w \qquad \forall w \in W$$

$$d_{w,0}, d_{w,1} \geq 0 \qquad \forall w \in W$$

Tolled User Equilibrium

- Tolled user equilibrium

$$
\begin{aligned}
& f_{p,c}\left(t_p\left(f\right)+\pi_{p,c}-\lambda_{w,c}\right)=0 && \forall p \in P_w, w \in W, c \in \{0,1\} \\
& t_p\left(f\right)+\pi_{p,c}-\lambda_{w,c} \geq 0 && \forall p \in P_w, w \in W, c \in \{0,1\} \\
& \pi_{p,c} \geq 0 && \forall p \in P_w, w \in W, c \in \{0,1\} \\
& \pi_{p,0}=\sum_{a \in A} \delta_{ap} \gamma_a && \forall p \in P_w, w \in W \\
& \gamma_a \geq 0 && \forall a \in A \\
& \pi_{p,1}=\sum_{a \in A} \delta_{ap} \gamma_a^{o(w)} && \forall p \in P_w, w \in W \\
& \gamma_a^{o(w)} \geq 0 && \forall a \in A, w \in W
\end{aligned}
$$

Performance Measures

- Travel time

$$\sum_{a \in A} x_a t_a(x_a) \tag{1}$$

- Toll revenue form OD pair w

$$\sum_{p \in P_w} (\pi_{p,0} f_{p,0} + \pi_{p,1} f_{p,1}) \tag{2}$$

- Privacy cost for OD pair w

$$\int_0^{\lambda_{w,0} - \lambda_{w,1}} d_w \xi(z) z dz \tag{3}$$

Objective functions

- First-best network conditions (all links are tollable)

$$\min \quad \sum_{w \in W} \left(\int_0^{\lambda_{w,0}-\lambda_{w,1}} d_w \xi(z) z dz + \sum_{p \in P_w} (\pi_{p,0} f_{p,0} + \pi_{p,1} f_{p,1}) \right)$$

- Second-best network conditions (some of the links are untollable)

$$\min \quad \sum_{w \in W} \left(\int_0^{\lambda_{w,0}-\lambda_{w,1}} d_w \xi(z) z dz + \sum_{p \in P_w} t_p(f) f_p \right)$$

Implementation on Nine-node Network

Table : Incentive program under first-best conditions

Pricing Scheme	Distribution of β	$E(\beta)$	Toll Rev.	Privacy Cost	Total User Cost
Anonymous	-	-	887.60	0.00	887.60
Origin-specific	-	2	311.60	200.00	511.60
	-	4	311.60	400.00	711.60
	-	8	311.60	800.00	1111.60
Hybrid	$U(0,4)$	2	247.82	28.46	276.28
	$U(0,8)$	4	235.25	58.47	293.72
	$U(0,16)$	8	220.76	116.96	337.72
	$EXP(0.500)$	2	249.84	17.49	267.33
	$EXP(0.250)$	4	237.43	35.52	272.95
	$EXP(0.125)$	8	213.08	71.02	284.10

- Anonymous scheme yields highest toll revenue; origin-specific leads to highest privacy cost while hybrid scheme results in smallest user cost
- Higher average value of privacy leads to higher total user cost

Implementation on Nine-node Network

Table : Incentive program under second-best conditions

Pricing Scheme	Distribution of β	$E(\beta)$	Travel Time	Privacy Cost	Total System Cost
Anonymous	-	-	2361.16	0.00	2361.16
Origin-specific	-	2	2306.10	200.00	2506.10
	-	4	2306.10	400.00	2706.10
	-	8	2306.10	800.00	3106.10
Hybrid	$U(0,4)$	2	2291.79	9.13	2300.92
	$U(0,8)$	4	2296.76	13.08	2309.84
	$U(0,16)$	8	2304.63	17.57	2322.20
	$EXP(0.500)$	2	2291.45	5.82	2297.27
	$EXP(0.250)$	4	2293.47	9.56	2303.04
	$EXP(0.125)$	8	2299.10	13.30	2312.40

- Anonymous scheme yields highest travel time; origin-specific leads to highest privacy cost and hybrid scheme results in smallest total system cost
- Higher average value of privacy results in higher total system cost

Implementation on Sioux Falls Network

Table : Incentive program under second-best conditions

Pricing Scheme	Distribution of β	$E(\beta)$	Travel Time	Privacy Cost	Total System Cost
Anonymous	-	-	74.043	0.000	74.043
Origin-specific	-	0.02	73.060	7.212	80.272
	-	0.04	73.060	14.424	87.474
	-	0.08	73.060	28.848	101.908
Hybrid	$U(0, 0.04)$	0.02	73.294	0.118	73.412
	$U(0, 0.08)$	0.04	73.421	0.138	73.421
	$U(0, 0.16)$	0.08	73.591	0.163	73.753
	$EXP(50.0)$	0.02	73.272	0.086	73.357
	$EXP(25.0)$	0.04	73.355	0.106	73.461
	$EXP(12.5)$	0.08	73.455	0.163	73.618

- The model works for a more realistic network
- Similar observations can be made

Summary

Contributions:

- Explored a new class of tolling schemes that charge different amount of toll for users with different origins, destinations, or paths.
- Developed a modeling approach for travelers location privacy
- Proposed an incentive program that allows the tolling agency to take advantage of the potentials of differentiated pricing without doing harm to travelers privacy rights.

Conclusion:

- Differentiated pricing may not be appealing for everyone
- Distribution of value of privacy has a significant effect on the acceptability of differentiated schemes
- Incentive program can create a win-win situation for all travelers

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