Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Static Traffic Assignment Problem. A comparison between Beckmann (1956) and Nesterov & de Palma (1998) models.

V. Dos Santos Eleuterio
ETH Zurich, Institute for Operations Research

Joint work with F.A. Chudak and Y. Nesterov

IVT - Gruppen-Seminare 2007/2008
5th November 2007

Outline

1. Static Traffic Assignment Problem
2. Beckmann ('56) and Nesterov & de Palma ('98) Models
3. Flow Distribution - Numerical Results
4. Final Remarks

Outline

1 Static Traffic Assignment Problem

2 Beckmann ('56) and Nesterov & de Palma ('98) Models

3 Flow Distribution - Numerical Results

4 Final Remarks

Static Traffic Assignment Problem

Given: A traffic network $G = (\mathcal{N}, \mathcal{A})$, $\mathcal{N}$ intersections, $\mathcal{A}$ roads, with

- **flow capacity** per road, $c_a > 0 \ \forall \ a \in \mathcal{A}$,
- **free travel time** per road, $\bar{t}_a > 0 \ \forall \ a \in \mathcal{A}$.

A set of **origin-destination pairs** each one with given demand, $\mathcal{OD} \subset \mathcal{N} \times \mathcal{N}$, $d_k > 0$ demand of $\mathcal{OD}$-pair k.

Find: An assignment of drivers on the network following a defined behavioral principle and satisfying the demands.

The current state of a traffic network is specified by **flow pattern** f, i.e. where cars are driving, and a **travel time pattern** t, i.e., how long it takes to cross roads.

Static Traffic Assignment Problem

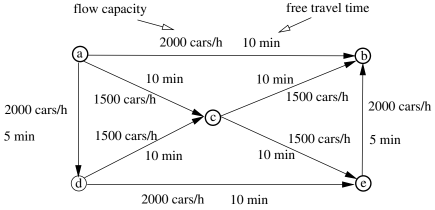
Given: A traffic network $G = (\mathcal{N}, \mathcal{A})$, $\mathcal{N}$ intersections, $\mathcal{A}$ roads, with

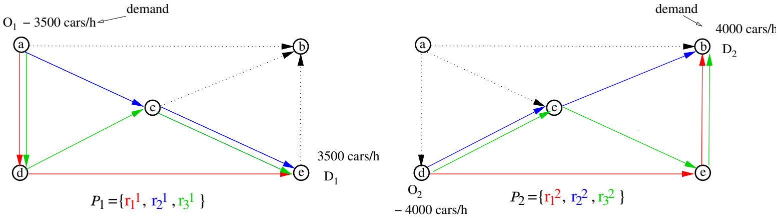
- **flow capacity** per road, $c_a > 0 \ \forall \ a \in \mathcal{A}$,
- **free travel time** per road, $\bar{t}_a > 0 \ \forall \ a \in \mathcal{A}$.

A set of **origin-destination pairs** each one with given demand, $\mathcal{OD} \subset \mathcal{N} \times \mathcal{N}$, $d_k > 0$ demand of $\mathcal{OD}$-pair k.

Find: An assignment of drivers on the network following a defined behavioral principle and satisfying the demands.

The current state of a traffic network is specified by **flow pattern** f, i.e. where cars are driving, and a **travel time pattern** t, i.e., how long it takes to cross roads.





Let $\mathcal{P}_k$ be the set of all routes for $\mathcal{OD}$-pair k

Drivers Behaviors Principle

User Equilibrium (UE): (First Wardrop principle '52)

At user equilibrium each driver selects the fastest route, i.e. no faster alternative is available.

Drivers are selfish.

Social Optimum (SO): (Second Wardrop principle '52)

At social optimum, the total travel time, i.e. the sum of all drivers' travel times, is minimized.

Central organization controls the traffic.

Outline

1 Static Traffic Assignment Problem

2 Beckmann ('56) and Nesterov & de Palma ('98) Models

3 Flow Distribution - Numerical Results

4 Final Remarks

Main Assumptions

Beckmann Model '56

- The travel time t_a on a road $a \in \mathcal{A}$ is given by a continuous, positive, and strictly increasing **latency function** that depends only on the total flow on these road, f_a, $l_a(f_a)$.
- Flow capacity restrictions are considered indirectly on the latency function.

Extended Beckmann Model '61

- Additional constraints are considered (e.g. flow capacity constraints, technical constraints, ...),
- Additional travel time's penalty (delay) has to be considered.

[Charnes and Cooper '61, Weigel and Cremeans '72, Ahuja '93]

Main Assumptions

Beckmann Model '56

- The travel time t_a on a road $a \in \mathcal{A}$ is given by a continuous, positive, and strictly increasing **latency function** that depends only on the total flow on these road, f_a, $l_a(f_a)$.
- Flow capacity restrictions are considered indirectly on the latency function.

Extended Beckmann Model '61

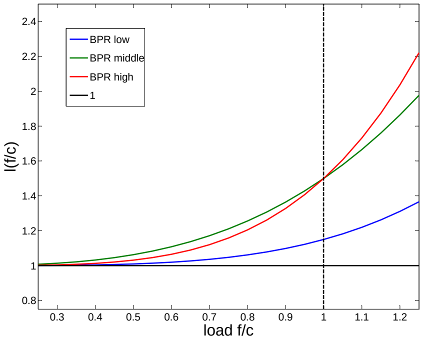
- Additional constraints are considered (e.g. flow capacity constraints, technical constraints, ...),
- Additional travel time's penalty (delay) has to be considered.

[Charnes and Cooper '61, Weigel and Cremeans '72, Ahuja '93]

Latency Function - Travel Time

Example: US. Bureau of Public Roads function '64, BPR,

$$l_a(f_a) = \bar{t}_a\big(1 + \alpha \left(\frac{f_a}{c_a}\right)^{\beta}\big), \quad \alpha, \beta > 0.$$



Main Assumptions (2)

Nesterov & de Palma Model '98

- The travel time t_a on a road $a \in \mathcal{A}$ is a variable, which has to satisfy

$$\begin{aligned} \text{if } f_a < c_a &\quad\Rightarrow\quad t_a = \bar{t}_a, \\ \text{if } f_a = c_a &\quad\Rightarrow\quad t_a \geq \bar{t}_a. \end{aligned}$$

- Flow capacity cannot be violated.

Beckmann Mathematical Model

f^k	$:= (f_r^k)_{r \in \mathcal{P}_k}$	$\forall\ k \in \mathcal{OD}$	flow paths vector
f_a	$:= \sum_{k \in \mathcal{OD}} \sum_{r \in \mathcal{P}_k} \delta_a^r f_r^k$	$\forall\ a \in \mathcal{A}$	

Social Optimum

B-SO	min	$\sum_{a \in \mathcal{A}}$	$f_a \cdot l_a(f_a)$		total travel time
	s.t.	$\sum_{r \in \mathcal{P}_k}$	$f_r^k = d_k$	$\forall\ k \in \mathcal{OD}$	demand is satisfied
		$f_r^k \geq 0$		$\forall\ k \in \mathcal{OD}$, $\forall\ r \in \mathcal{P}_k$	

Convex optimization problem.

Beckmann Mathematical Model

$$f^k := (f_r^k)_{r \in \mathcal{P}_k} \qquad \forall\ k \in \mathcal{OD} \qquad \text{flow paths vector}$$

$$f_a := \sum_{k \in \mathcal{OD}} \sum_{r \in \mathcal{P}_k} \delta_a^r f_r^k \qquad \forall\ a \in \mathcal{A}$$

Social Optimum

B-SO

$$\min \sum_{a \in \mathcal{A}} f_a \cdot l_a(f_a) \qquad \text{total travel time}$$

$$\text{s.t.} \sum_{r \in \mathcal{P}_k} f_r^k = d_k \qquad \forall\ k \in \mathcal{OD} \qquad \text{demand is satisfied}$$

$$f_r^k \geq 0 \qquad \forall\ k \in \mathcal{OD}, \ \forall\ r \in \mathcal{P}_k$$

Convex optimization problem.

Beckmann Mathematical Model (2)

User Equilibrium

For each $\mathcal{OD}$ pair k, the flow f^k is at user equilibrium if and only if

$$f_s^k > 0 \quad \Rightarrow \quad t_s(f) = \min_{r \in \mathcal{P}_k} t_r(f),$$

$$f_s^k = 0 \quad \Rightarrow \quad t_s(f) \geq \min_{r \in \mathcal{P}_k} t_r(f),$$

where $t_r(f) = \sum_{a \in r} l_a(f_a)$, i.e. the travel time of route r.

Beckmann Mathematical Model (3)

Optimality conditions of the following convex optimization problem

User Equilibrium

B-UE min $\sum_{a \in \mathcal{A}} \int_0^{f_a} l_a(x)dx$

s.t. $\sum_{r \in \mathcal{P}_k} f_r^k = d_k \quad \forall\ k \in \mathcal{OD}$

$f_r^k \geq 0 \quad \forall\ k \in \mathcal{OD},\ \forall\ r \in \mathcal{P}_k$

Beckmann Mathematical Model (3)

Optimality conditions of the following convex optimization problem

User Equilibrium

$$\begin{array}{llll} \textbf{B-UE} & \min & \sum_{a \in \mathcal{A}} \int_0^{f_a} l_a(x) dx & \\ & \text{s.t.} & \sum_{r \in \mathcal{P}_k} f_r^k = d_k & \forall\ k \in \mathcal{OD} \\ & & f_r^k \geq 0 & \forall\ k \in \mathcal{OD},\ \forall\ r \in \mathcal{P}_k \end{array}$$

Remark: Flow pattern f and travel time pattern t are in general different at Social Optimum and at User Equilibrium.

Extended Beckmann Model

User Equilibrium

Bext-UE min $\sum_{a\in\mathcal{A}} \int_0^{f_a} l_a(x)dx$

s.t. $g_i(f) \leq 0 \quad \forall\, i \in \mathcal{I}$ additional constraints

$\sum_{r\in\mathcal{P}_k} f_r^k = d_k \quad \forall\, k \in \mathcal{OD}$

$f_r^k \geq 0 \quad \forall\, k \in \mathcal{OD}, \forall\, r \in \mathcal{P}_k$

where $\mathcal{I}$ indices of arcs, nodes or $\mathcal{OD}$ pairs,
$g_i(f)$ convex and continuous differential functions.

Extended Beckmann Model (2)

User Equilibrium

The optimal conditions of **Bext-UE** correspond to User Equilibrium with generalized travel times,

$$t_r(f^*, \zeta^*) := \sum_{a \in r} l_a(f_a^*) + \sum_{i \in \mathcal{I}} \zeta_i^* \left(\sum_{a \in r} \frac{\partial g_i(f^*)}{\partial f_a} \right)$$

$$\forall\ r \in \mathcal{P}_k,\ \forall\ k \in \mathcal{OD},$$

where f^* is an optimal solution of Bext-UE
ζ^* are the Lagrange multipliers corresponding to the additional constraints.

(f^*, t^*) **traffic assignment at User Equilibrium**

Nesterov & de Palma Mathematical Model

Notation: $f_a^k := \sum_{r \in \mathcal{P}_k} \delta_a^r f_r^k \quad \forall\ a \in \mathcal{A}$

Social Optimum

$$\textbf{NdP-SO} \quad \min \quad \sum_{a \in \mathcal{A}} \quad f_a \cdot \bar{t}_a$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{OD}} \quad f_a^k \leq c_a \quad \forall\ a \in \mathcal{A}$$

capacity constraints

$$\sum_{r \in \mathcal{P}_k} \quad f_r^k = d_k \quad \forall\ k \in \mathcal{OD}$$

$$f_r^k \geq 0 \quad \forall\ k \in \mathcal{OD},\ \forall\ r \in \mathcal{P}_k$$

Minimum cost multicommodity flow problem !

Nesterov & de Palma Mathematical Model

Notation: $f_a^k := \sum_{r \in \mathcal{P}_k} \delta_a^r f_r^k \quad \forall\ a \in \mathcal{A}$

Social Optimum

NdP-SO min $\sum_{a \in \mathcal{A}} f_a \cdot \bar{t}_a$

s.t. $\sum_{k \in \mathcal{OD}} f_a^k \leq c_a \quad \forall\ a \in \mathcal{A} \qquad \Longleftarrow \lambda \geq 0$

$\sum_{r \in \mathcal{P}_k} f_r^k = d_k \quad \forall\ k \in \mathcal{OD}$

$f_r^k \geq 0 \quad \forall\ k \in \mathcal{OD}, \forall\ r \in \mathcal{P}_k$

λ_a "travel time penalty (delay) for getting one additional unit of flow capacity".

Nesterov & de Palma Mathematical Model (2)

Social Optimum

$$\textbf{NdP-SO} \quad \min \sum_{a\in\mathcal{A}} f_a \cdot \bar{t}_a$$

$$\text{s.t.} \quad \sum_{k\in\mathcal{OD}} f_a^k \leq c_a \quad \forall\, a \in \mathcal{A} \qquad \Longleftarrow \lambda \geq 0$$

$$\sum_{r\in\mathcal{P}_k} f_r^k = d_k \quad \forall\, k \in \mathcal{OD}$$

$$f_r^k \geq 0 \quad \forall\, k \in \mathcal{OD}, \ \forall\, r \in \mathcal{P}_k$$

NdP-SO $=$ Bext-UE

$$l_a(f_a) := \bar{t}_a \ \forall\, a \in \mathcal{A} \quad t_r(f^*, \lambda^*) = \sum_{a\in r} (\bar{t}_a + \lambda_a^*) \ \ \forall\, r \in \mathcal{P}_k, \forall\, k \in \mathcal{OD}.$$

$(f^*, \bar{t} + \lambda^*)$ **traffic assignment at User Equilibrium**

Nesterov & de Palma Mathematical Model (3)

Langrange dual problem

$$\max_{\lambda \geq 0} \quad -\langle \lambda, c \rangle + \sum_{k \in \mathcal{OD}} \quad \min \langle f^k, \bar{t} + \lambda \rangle$$

$$\sum_{r \in \mathcal{P}_k} f_r^k = d_k$$

$$f_r^k \geq 0 \quad \forall\ r \in \mathcal{P}_k$$

separable per $\mathcal{OD}$ pair !

Nesterov & de Palma Mathematical Model (3)

Langrange dual problem

$$\max_{\lambda \geq 0} \quad -\langle \lambda, c \rangle + \sum_{k \in \mathcal{OD}} \quad \min \langle f^k, \bar{t} + \lambda \rangle$$

separable per $\mathcal{OD}$ pair !

$$\sum_{r \in \mathcal{P}_k} f_r^k = d_k$$

$$f_r^k \geq 0 \quad \forall\ r \in \mathcal{P}_k$$

minimum cost flow
without capacity
constraints !

For each $\mathcal{OD}$ pair k the flow is distributed along the shortest paths given the travel time $\bar{t} + \lambda$.

Nesterov & de Palma Mathematical Model (3)

Langrange dual problem

$$\max_{\lambda \geq 0} \quad -\langle \lambda, c \rangle + \sum_{k \in \mathcal{OD}} \quad \min \langle f^k, \bar{t} + \lambda \rangle$$

$$\sum_{r \in \mathcal{P}_k} f_r^k = d_k$$

$$f_r^k \geq 0 \quad \forall\ r \in \mathcal{P}_k$$

separable per $\mathcal{OD}$ pair !

minimum cost flow without capacity constraints !

For each $\mathcal{OD}$ pair k the flow is distributed along the shortest paths given the travel time $\bar{t} + \lambda$. $\Longrightarrow$ **User Equilibrium**

Remark

Nesterov & de Palma Model:

Let f^*, λ^* primal and dual optimal solutions. Then,

- $(f^*, \overline{t})$ is a traffic assignment at Social Optimum, $(f^*, \overline{t} + \lambda^*)$ is a traffic assignment at User Equilibrium.
- λ^* can be used as an incentive for drivers to reach the Social Optimum (new free travel times, toll).

Remark

Nesterov & de Palma Model:

Let f^*, λ^* primal and dual optimal solutions. Then,

- $(f^*, \overline{t})$ is a traffic assignment at Social Optimum,
  $(f^*, \overline{t} + \lambda^*)$ is a traffic assignment at User Equilibrium.
- λ^* can be used as an incentive for drivers to reach the Social Optimum (new free travel times, toll).

Beckmann and Extended Beckmann Model:

Flow pattern f and travel time pattern t are in general different at Social Optimum and at User Equilibrium.

Outline

1. Static Traffic Assignment Problem
2. Beckmann ('56) and Nesterov & de Palma ('98) Models
3. **Flow Distribution - Numerical Results**
4. Final Remarks

Flow Distribution - Numerical Results

Since the models base the travel times on different assumptions, a direct comparison of the travel times is not suitable.

We focus on the flow distribution in both models,

- Where does congestion occur at Social Optimum? At User Equilibrium?
- How many paths are used per $\mathcal{OD}$ pair?
- How far away from the best possible use of the network is an assignment at User Equilibrium (price of anarchy)?
- Do both models detect Braess phenomena?

We present numerical results based on a small network and a large network.

Flow Distribution - Numerical Results

Since the models base the travel times on different assumptions, a direct comparison of the travel times is not suitable.

We focus on the flow distribution in both models,

- Where does congestion occur at Social Optimum? At User Equilibrium?
- How many paths are used per $\mathcal{OD}$ pair?
- How far away from the best possible use of the network is an assignment at User Equilibrium (price of anarchy)?
- Do both models detect Braess phenomena?

We present numerical results based on a small network and a large network.

Flow Distribution - Numerical Results

Since the models base the travel times on different assumptions, a direct comparison of the travel times is not suitable.

We focus on the flow distribution in both models,

- Where does congestion occur at Social Optimum? At User Equilibrium?
- How many paths are used per $\mathcal{OD}$ pair?
- How far away from the best possible use of the network is an assignment at User Equilibrium (price of anarchy)?
- Do both models detect Braess phenomena?

We present numerical results based on a small network and a large network.

Small Network - Sioux Falls

Sioux Falls: 24 nodes, 76 arcs and 528 $\mathcal{OD}$ pairs, solved with high accuracy using standard commercial solvers.

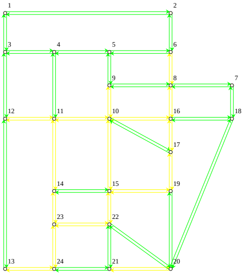
*provided by Dr. Hillel Bar-Gera at www.bgu.ac.il/∼bargera/tntp/

Small Network - Sioux Falls

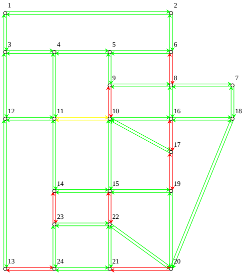
Sioux Falls: 24 nodes, 76 arcs and 528 $\mathcal{OD}$ pairs, solved with high accuracy using standard commercial solvers.

*provided by Dr. Hillel Bar-Gera at www.bgu.ac.il/∼bargera/tntp/

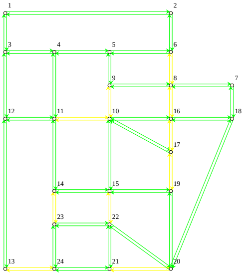
Social Optimum



Nesterov & de Palma



Beckmann BPR low

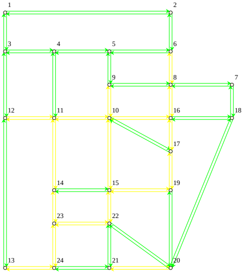


Ext. Beckmann BPR low

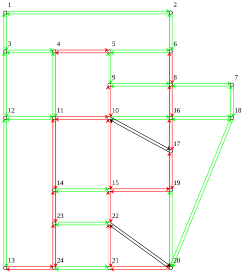
– unused road – not congested road – congestion – overflow

Small Network - Sioux Falls (2)

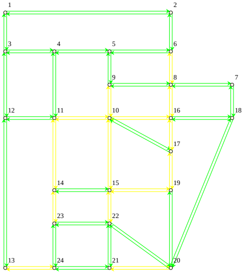
User Equilibrium



Nesterov & de Palma
44.7% congested roads



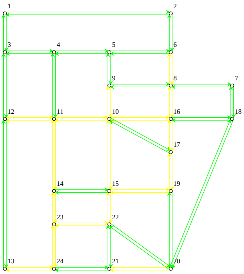
Beckmann BPR low
0% congested roads



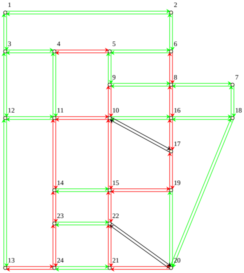
Ext. Beckmann BPR low
39.5% congested roads

Small Network - Sioux Falls (2)

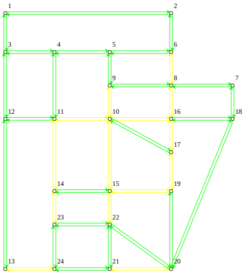
User Equilibrium



Nesterov & de Palma
44.7% congested roads



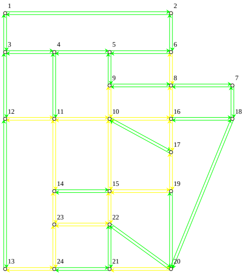
Beckmann BPR low
0% congested roads
42.1% roads with overflow
16.4% average overflow



Ext. Beckmann BPR low
39.5% congested roads

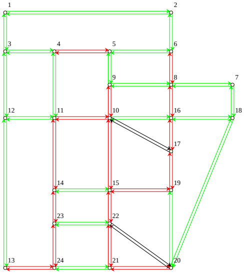
Small Network - Sioux Falls (2)

User Equilibrium

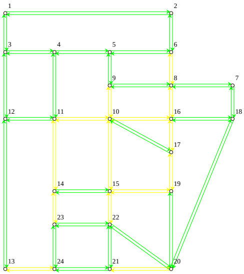


Nesterov & de Palma
44.7% congested roads

1.06 average paths used



Beckmann BPR low
0% congested roads
42.1% roads with overflow
16.4% average overflow
1.3 average paths used



Ext. Beckmann BPR low
39.5% congested roads

1.6 average paths used

Small Network - Sioux Falls (3)

Remarks

- set of congested roads in Nesterov & de Palma model includes those of Beckmann model almost ($\sim$ 85 %),
- drivers are less spread out in Nesterov & de Palma model than in Beckmann model,
- latency function BPR low duplicates the Nesterov & de Palma model best, it corresponds to BPR function with the standard parameters $\alpha = 0.15$ and $\beta = 4$.

Small Scale Network - Sioux Falls (4)

Price of Anarchy [Koutsoupias and Papadimitriou '99]

$$\text{price of anarchy} = \frac{\text{Total Travel Time at UE}}{\text{Total Travel Time at SO}}$$

$$\text{price of anarchy per } \mathcal{OD} \text{ pair} = \frac{\text{Shortest Travel Time at UE}}{\text{Shortest Travel Time at SO}}$$

Model	Nesterov & de Palma	BPR low		BPR high	
		Beckmann	Ext. Beck.	Beckmann	Ext. Beck.
price of anarchy	1.38	1.03	1.003	1.05	1.03
average price of anarchy per $\mathcal{OD}$ pair	1.48	1.06	1.008	1.09	1.05

Small Scale Network - Sioux Falls (4)

Price of Anarchy [Koutsoupias and Papadimitriou '99]

$$\text{price of anarchy} = \frac{\text{Total Travel Time at UE}}{\text{Total Travel Time at SO}}$$

$$\text{price of anarchy per } \mathcal{OD} \text{ pair} = \frac{\text{Shortest Travel Time at UE}}{\text{Shortest Travel Time at SO}}$$

Model	Nesterov & de Palma	BPR low		BPR high	
		Beckmann	Ext. Beck.	Beckmann	Ext. Beck.
price of anarchy	1.38	1.03	1.003	1.05	1.03
average price of anarchy per $\mathcal{OD}$ pair	1.48	1.06	1.008	1.09	1.05

Price of Anarchy

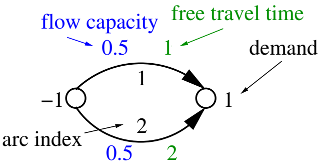
Remarks:

- Depending on the choice of the latency function, the price of anarchy is bounded for the Beckmann model.
  [Tardos and Roughgarden '02, Correa, Schulz and Stier Moses '03]

Price of Anarchy

Remarks:

- Depending on the choice of the latency function, the price of anarchy is bounded for the Beckmann model.
  [Tardos and Roughgarden '02, Correa, Schulz and Stier Moses '03]
- No bound is possible for the price of anarchy for the Nesterov & de Palma model.



$f_1 = 0.5$ $f_2 = 0.5$

$t_1^{SO} = 1$ $t_2^{SO} = 2$ $t_1^{UE} >= 2$ $t_2^{UE} >= 2$

Total Travel Time at UE $>= 2$

Total Travel Time at SO $= \frac{3}{2}$

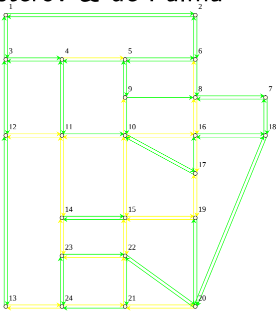
Braess Paradox

The **Braess paradox** is a situation in which the addition of more resources, i.e. roads, increases the total travel time at User Equilibrium.

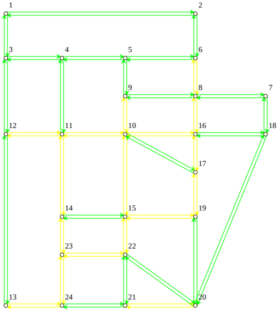
Road	Nesterov & de Palma	Beckmann		Extend Beckmann	
		BPR low	BPR high	BPR low	BPR high
$5 \longrightarrow 6$	2.02	-	-	-	-
$6 \longrightarrow 5$	2.02	-	-	-	-
$8 \longrightarrow 9$	3.52	-	1.36	-	-
$9 \longrightarrow 8$	3.52	-	1.36	-	-

Braess Paradox (2)

Nesterov & de Palma



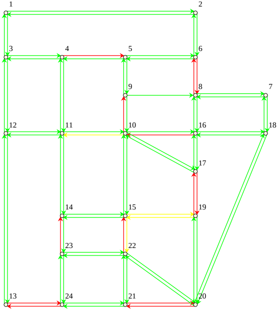
without arc 8 → 9



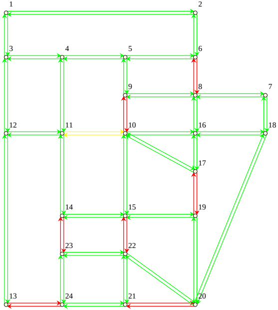
with arc 8 → 9

Braess Paradox (3)

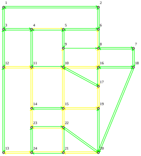
Beckmann BPR high



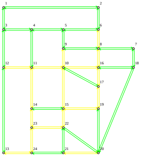
without arc 8 $\longrightarrow$ 9



with arc 8 $\longrightarrow$ 9

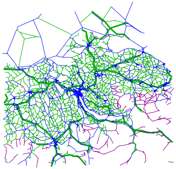


N&dP
without

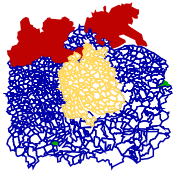


with

Large Network - Zurich Regional



Map of Roads



Map of Zones

Zurich Regional*: 784 zones, 7'009 nodes and 16'936 roads

	$\mathcal{OD}$-pairs	Total Demand
12:00 - 13:00	369'449	176'222.85
17:00 - 18:00	443'622	252'871.96
18:00 - 19:00	439'660	175'669.58

*provided by Prof. K.W. Axhausen, IVT ETH Zurich and M. Arendt, ARE Bern.

Large Network - Zurich Regional (2)

Beckmann Model Commercial software

- successive shortest path assignment
- flow balance

stopping criteria : maximal relative paths travel time difference 0.05

Nesterov & de Palma Model Primal dual subgradient techniques [Nesterov 03/05]

- computation of subgradients $\sim$ shortest path
- easy projections (minimizing quadratic functions over a box)

stopping criteria: relative gap 0.005

Large Network - Zurich Regional (2)

Beckmann Model Commercial software
- successive shortest path assignment
- flow balance

stopping criteria : maximal relative paths travel time difference 0.05

Nesterov & de Palma Model Primal dual subgradient techniques [Nesterov 03/05]
- computation of subgradients $\sim$ shortest path
- easy projections (minimizing quadratic functions over a box)

stopping criteria: relative gap 0.005

Large Network - Zurich Regional (2)

Beckmann Model Commercial software

- successive shortest path assignment
- flow balance

stopping criteria : maximal relative paths travel time difference 0.05

Nesterov & de Palma Model Primal dual subgradient techniques [Nesterov 03/05]

- computation of subgradients $\sim$ shortest path
- easy projections (minimizing quadratic functions over a box)

stopping criteria: relative gap 0.005

Large Network - Zurich Regional (2)

Beckmann Model Commercial software
- successive shortest path assignment
- flow balance

stopping criteria : maximal relative paths travel time difference 0.05

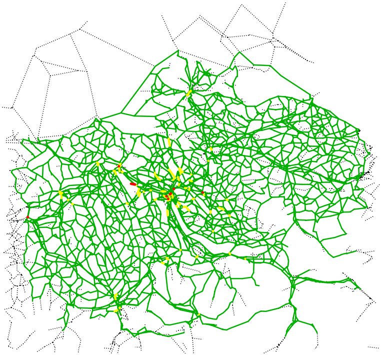
Nesterov & de Palma Model Primal dual subgradient techniques [Nesterov 03/05]
- computation of subgradients ∼ shortest path
- easy projections (minimizing quadratic functions over a box)

stopping criteria: relative gap 0.005

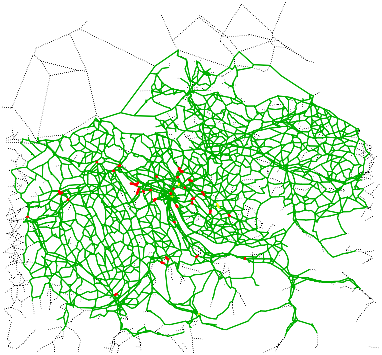
Large Network - Zurich Regional (3)

User Equilibrium

Nesterov & de Palma model



Beckmann model



17:00 - 18:00

Large Network - Zurich Regional (4)

	Nesterov & de Palma		Beckmann	
	Number of Congested and Overflow Roads (%)	Average Overflow (%)	Number of Congested and Overflow Roads (%)	Average Overflow (%)
12:00 - 13:00	0.08	0.74	0.02	5.68
17:00 - 18:00	0.89	2.17	0.41	12.23
18:00 - 19:00	0.27	0.75	0.05	3.84

- Set of congested roads and of roads with overflow in Nesterov & de Palma model includes those of Beckmann model almost ($\sim$ 80 %)

Large Network - Zurich Regional (5)

	Nesterov & de Palma		Beckmann	
	Average Number of Paths Used per $\mathcal{OD}$-pair	Maximal Relative Travel Time Difference	Average Number of Paths Used per $\mathcal{OD}$-pair	Maximal Relative Travel Time Difference
12:00 - 13:00	1.057	0.04	1.162	≤ 0.05
17:00 - 18:00	2.138	0.15	1.329	≤ 0.05
18:00 - 19:00	1.320	0.06	1.264	≤ 0.05

- For low demand, the drivers are less spread out in Nesterov & de Palma model than in Beckmann model.
- For high demand, the drivers are more spread out in Nesterov & de Palma model than in Beckmann model (feasibility of the instances?).

Outline

1 Static Traffic Assignment Problem

2 Beckmann ('56) and Nesterov & de Palma ('98) Models

3 Flow Distribution - Numerical Results

4 Final Remarks

Final Remarks

- We compared a new approach with a well established model for the traffic assignment problem,
  - defining new free travel times to reach Social Optimum is easier in Nesterov & de Palma model (duality).
- A direct comparison of the travel times is not suitable $\Rightarrow$ focus on flow distribution:
  - set of congested roads in Nesterov & de Palma model includes those of Beckmann model,
  - for the small networks the drivers are less spread out in Nesterov & de Palma model than in Beckmann model,
  - for the large networks with high total demand the drivers are more spread out in the Nesterov & de Palma model than in the Beckmann model (feasibility of these instances?).

Final Remarks

- We compared a new approach with a well established model for the traffic assignment problem,
  - defining new free travel times to reach Social Optimum is easier in Nesterov & de Palma model (duality).
- A direct comparison of the travel times is not suitable $\Rightarrow$ focus on flow distribution:
  - set of congested roads in Nesterov & de Palma model includes those of Beckmann model,
  - for the small networks the drivers are less spread out in Nesterov & de Palma model than in Beckmann model,
  - for the large networks with high total demand the drivers are more spread out in the Nesterov & de Palma model than in the Beckmann model (feasibility of these instances?).

Final Remarks (2)

- The Beckmann models detect Braess phenomena depending on the latency function used.
- At the moment our results do not enable us to say which better model predicts real traffic flow.
- A comparison with real traffic counters' data must be done.
- A comprehensive investigation of the extended Beckmann model, using large scale instances, should also be done to clarify the difference to the Nesterov & de Palma model.

Final Remarks (2)

- The Beckmann models detect Braess phenomena depending on the latency function used.
- At the moment our results do not enable us to say which better model predicts real traffic flow.
- A comparison with real traffic counters' data must be done.
- A comprehensive investigation of the extended Beckmann model, using large scale instances, should also be done to clarify the difference to the Nesterov & de Palma model.

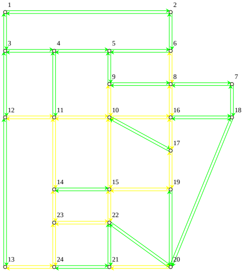
Thank You!

Outline

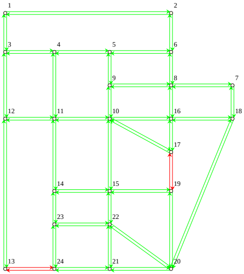
5 Backup

Small Network - Sioux Falls

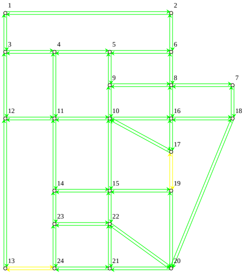
Social Optimum



Nesterov & de Palma



Beckmann BPR high

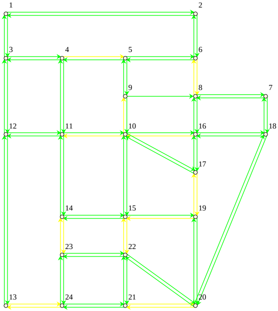


Ext. Beckmann BPR high

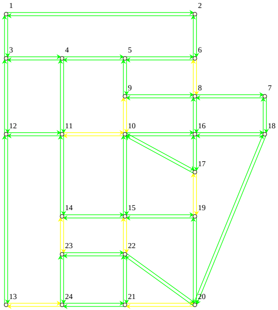
– unused road – not congested road – congestion – overflow

Braess Paradox

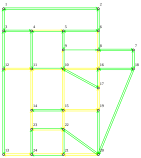
Extended Beckmann BPR high



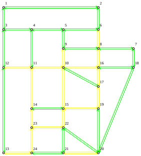
without arc 8 → 9



with arc 8 → 9



N&dP
without



with

Generation of Data - Zurich Regional

1. With VISUM assign the drivers in National model, save the load of the roads.
2. With VISUM assign the drivers of Zurich Regional in national model, save the load of the roads.
3. Update capacities: remove from arc's capacity the load at UE for the National model and add the load at UE for Zurich Regional model.
4. Delete roads not in Zurich Regional model.

Zurich Regional - cpu time

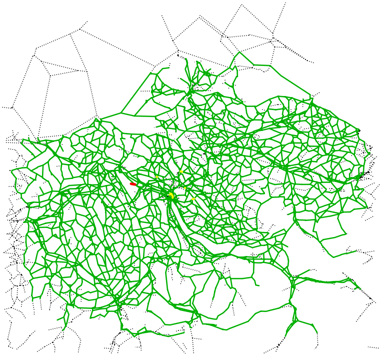
Visum : $\sim$ 30 minutes for each instance

instances	00-01	07-08	08-09	12-13	17-18	18-19
cpu time [min]	54	404 (7 h)	107	96	137	112
# iterations	202	202	202	202	202	202

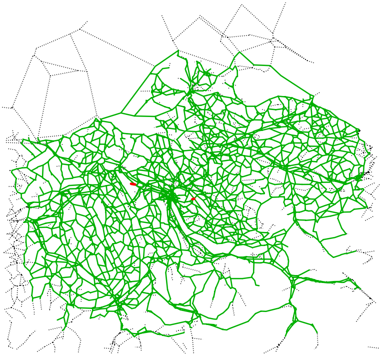
Large Network - Zurich Regional

User Equilibrium

Nesterov & de Palma model



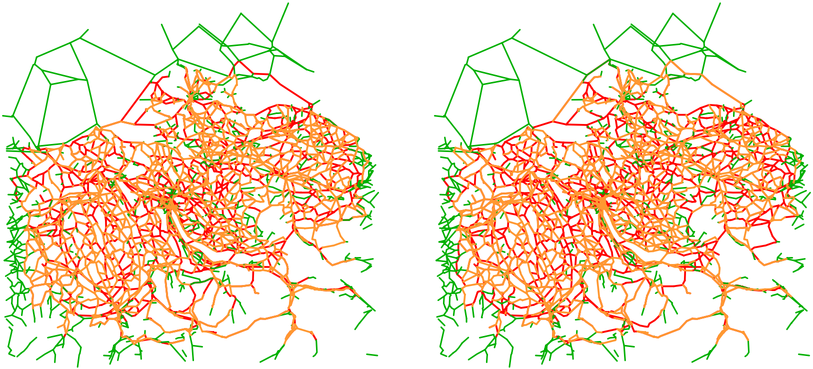
Beckmann model



12:00 - 13:00

Large Network - Zurich Regional

Flow differences at User Equilibrium



12:00 - 13:00 17:00 - 18:00

Beckmann + Beckmann = Nesterov & de Palma Nesterov & de Palma +