Economic Equilibrium with Mobility: Two Illustrative Models

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IVT Seminar: Valuing Accessibility

ETH Zürich

7 May 2009

Outline

1. The general equilibrium modeling framework.

2. A general equilibrium model of the market for mules and porters in Lamjung district.

3. A model of traffic congestion, housing prices and compensating wage differentials

Complementarity is a feature of constrained optimization problems. In an optimal or equilibrium program:

- Every process in use makes a zero profit
- No process in the technology makes a positive profit
- Every good used below the limit of its availability has a zero price
- No good has a negative price

Credit for these insights are given to the contributions of Lerner, Samuelson, and Kantorovich.

The Arrow Debreu Framework

A general-equilibrium model consists of:

- Profit-maximizing firms.
- Markets, typically with supply and demand mediated through prices.
- Budget-constrained utility-maximizing households.

In policy analysis, numerically *calibrated* versions of these models are referred to as *Computable* or *Applied* General Equilibrium Models (CGE).

Firms and Production

Activities in the Arrow-Debreu framework transform some goods and factors into others goods. These may include trade activities which transform domestic into foreign goods, activities which transform leisure into labor supply, and more conventional production activities which transfer labor, capital and materials into products.

Activities are most usefully represented by their dual, or cost-functions.

Equilibrium conditions relate marginal cost to the value of output with *complementary slackness* between profit and activity level.

Markets and Prices

General-equilibrium models consist of market clearing conditions. A *commodity* is a general term that includes goods, factor of production, and even utility.

Market clearing conditions in a general equilibrium model relate supply and demand. Prices exhibit *complementary slackness* with excess supply.

Consumers (Households and Governments)

Consumers in the Arrow Debreu framework are endowed with goods (and possibily tax revenue), and they demand commodities. Quantities demanded arise from optimization subject to a budget constraint.

Strengths and Weaknesses

- Key *advantage* of the general equilibrium framework: transparency, logical coherence and consistent accounting of both direct and indirect effects.

- Key *disadvantage* of the approach: potential complexity, reliance on optimizing behaviour, and data requirements.

Structure of a Simple CGE Model

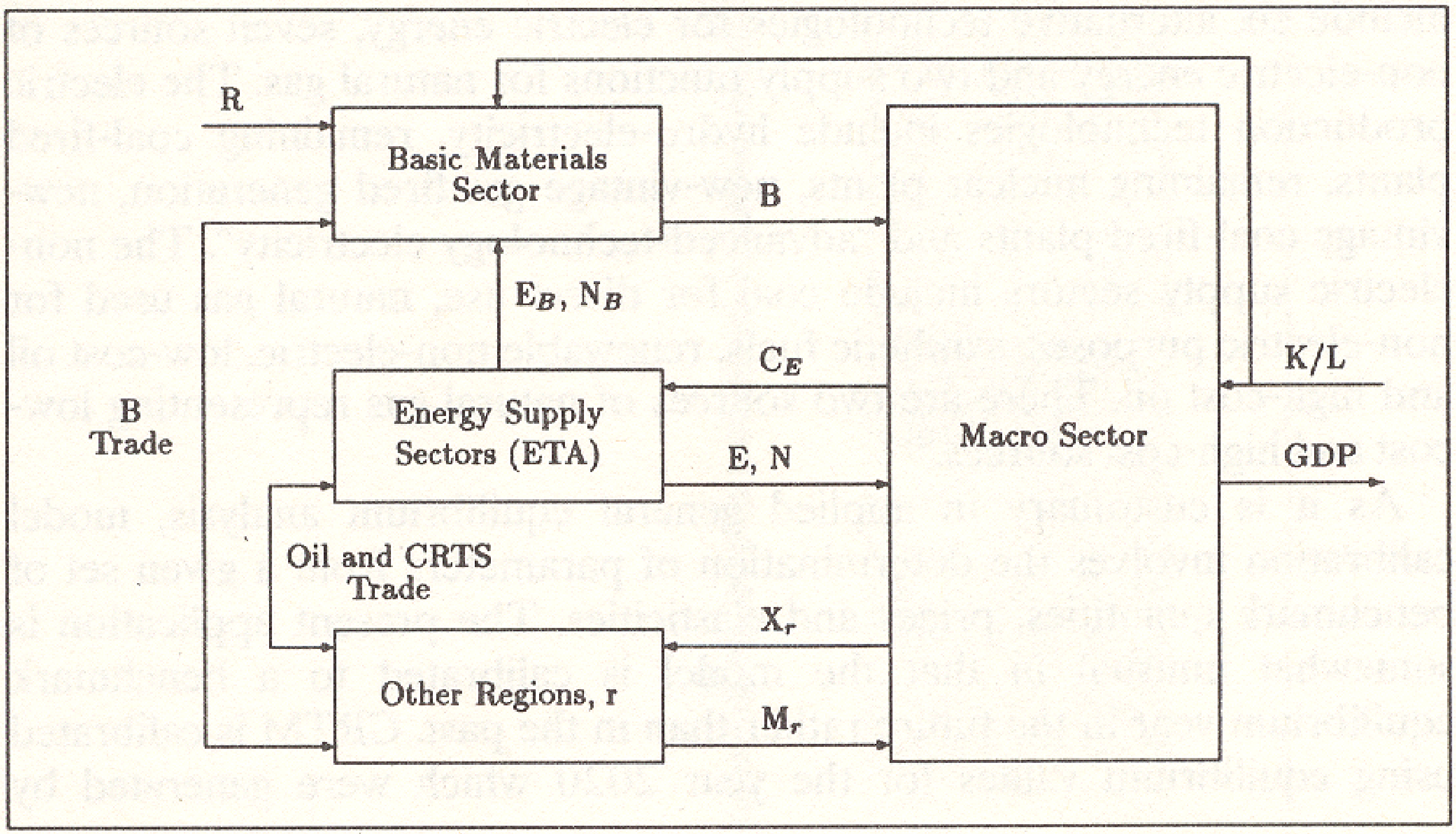
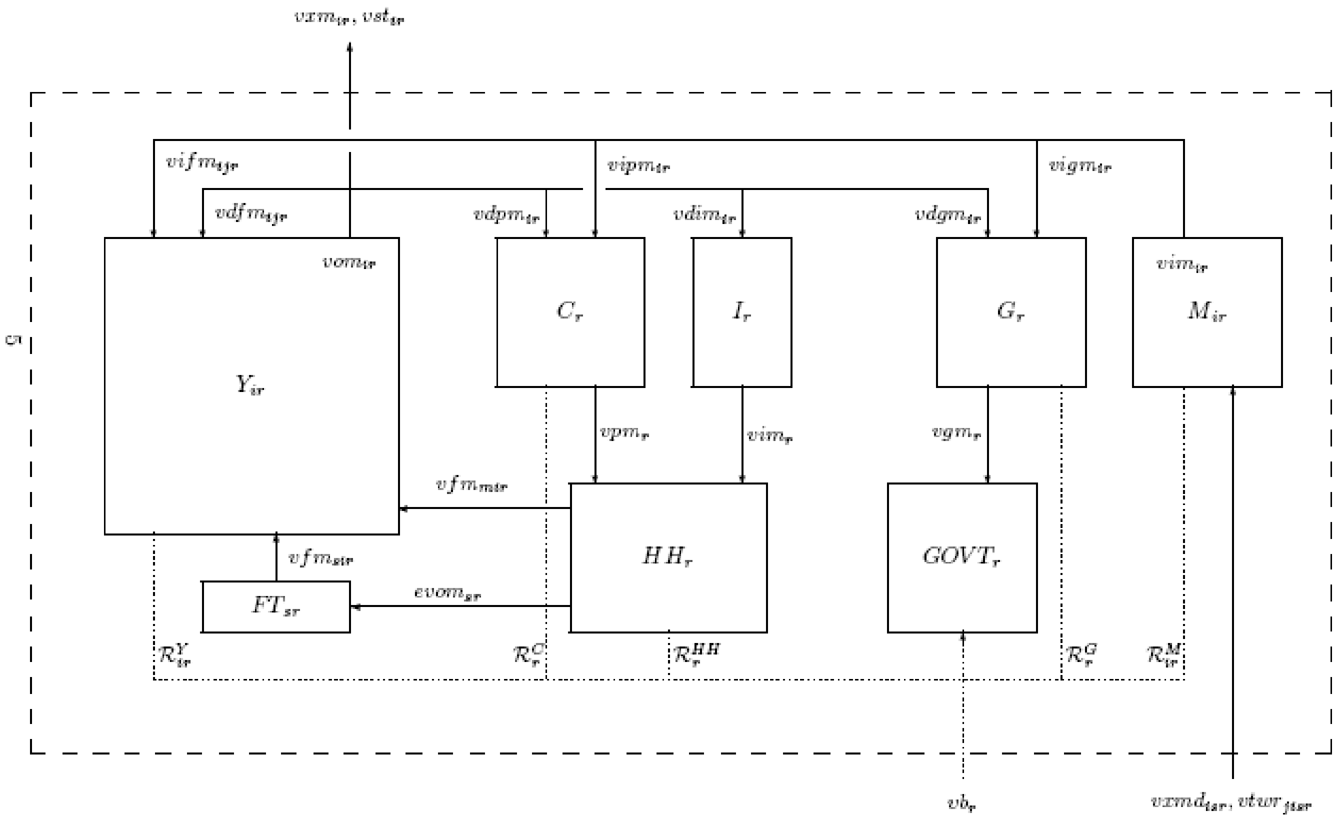


Fig. 1. Single region submodel.

Structure of a More Complex CGE Model

Figure 1: Regional Economic Structure



Usefulness of CGE Models

CGE models are commonly employed in a wide range of economic policy debates:

- Cost-benefit assessment of climate policy (integrated assessment)
- Trade policy
- Analysis of tax reform proposals
- Health care financing
- Assessment of the industrial impacts of climate policy (carbon leakage)

Empirical Foundations?

Existence of a substantive CGE calculation should always be viewed as a *necessary* but not a *sufficient* condition for justifying the merits of a particular policy proposal.

Mathematics

- The nonlinear complementarity problem:

  Given $F : R^N \to R^N$

  Find $z \in R^N$ such that

$$F(z) \geq 0, \quad z \geq 0, \quad z^T F(z) = 0$$

- The Arrow-Debreu equilibrium problem cast as a complementarity problem:

  Given:

  - $\Pi_j(p)$ unit profit functions corresponding to constant returns to scale sectors $j \in \{1, \ldots, m\}$

  - $d_h(p, M)$ ordinary demand functions for households $h \in \{1, \ldots, H\}$, functions of market prices and income.

  - $\omega_h \in R^n$ vectors of household endowments

Find: $p \in R^n$, $y \in R^m$, $M \in R^H$ such that

- Firms earn zero profit:

$$\Pi_j(p) \geq 0 \quad \perp y_j \geq 0 \quad \forall j$$

- Markets clear

$$\sum_j \frac{\partial \Pi_j(p)}{\partial p_i} y_j + \sum_h \omega_{ih} \geq \sum_h d_{ih}(p, M_h) \quad \perp p_i \geq 0 \quad \forall i$$

- Incomes balance with expenditure:

$$M_h = \sum_i p_i \omega_{ih}$$

Engineering and Economics

Academics have never been in complete agreement about the appropriate role of economics in academic and policy discourse.

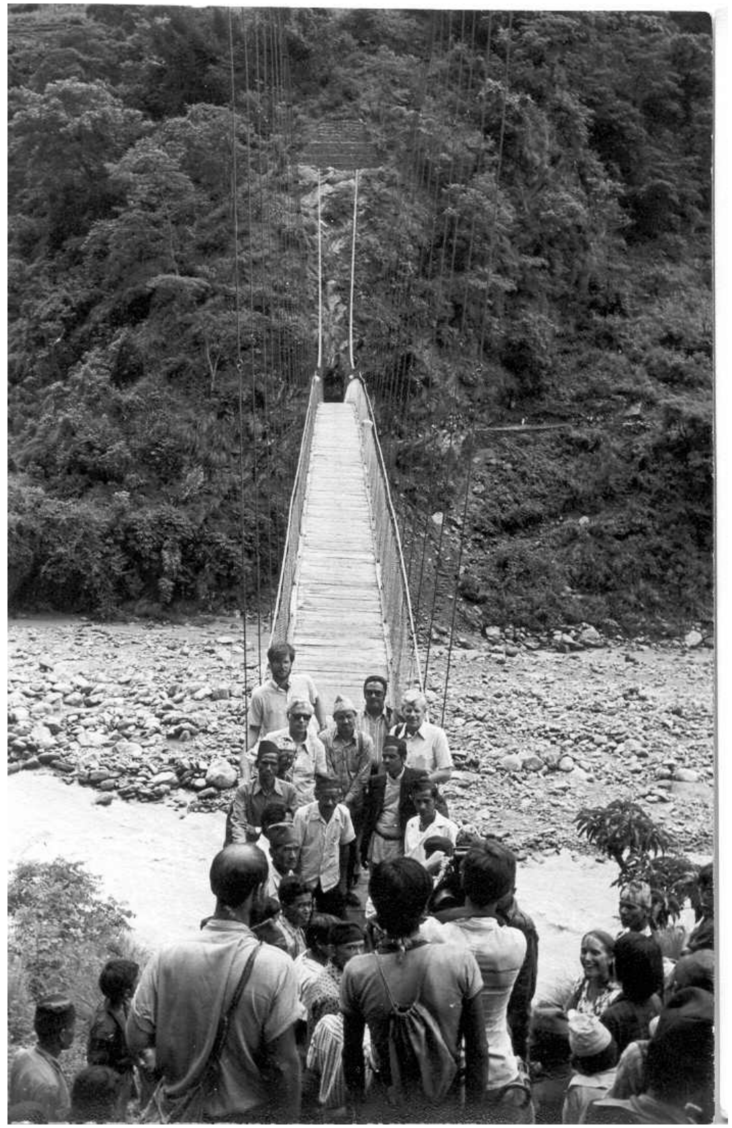
British economist A. C. Pigou (1920) as quoted by Koopmans:

“. . . it is not the business of economists to teach woolen manufacturers how to make and sell wool, or brewers how to make and sell beer . . . ”.

Many European economists, particularly German, Dutch and Scandinavians, disagreed. Models of production planning and economic efficiency were perceived as valuable contributions to both the theory of the firm and public economics.

Background Material on Model # 1

A Nepal Porter

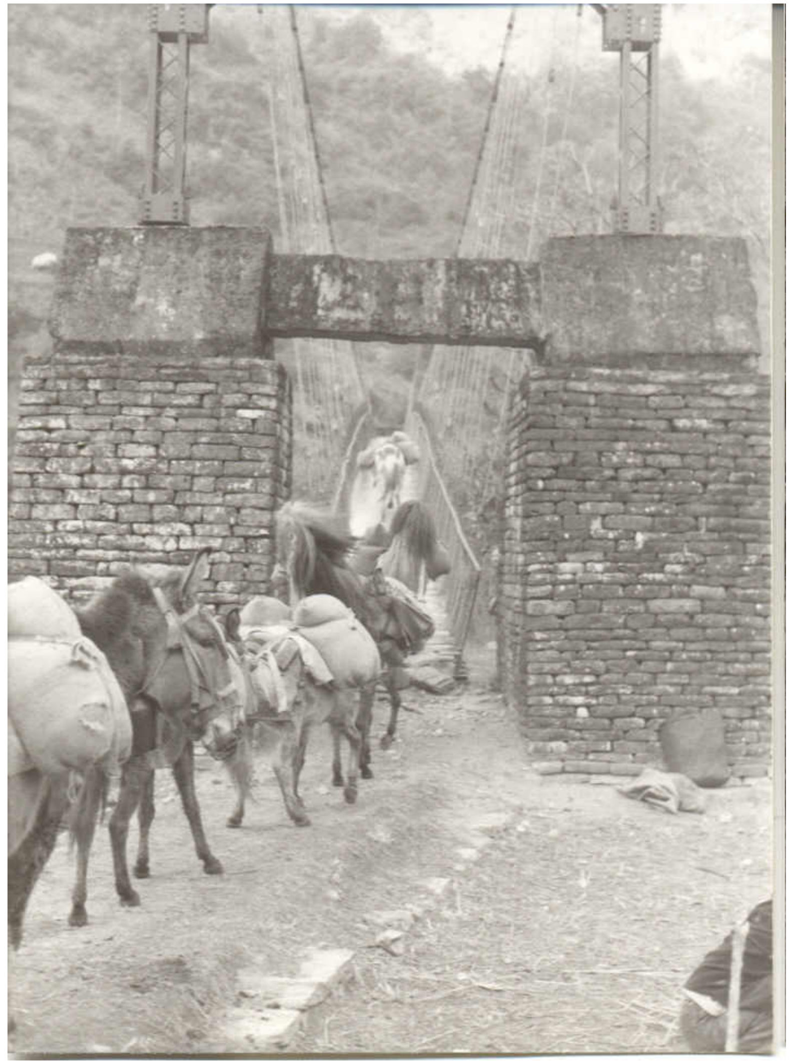


Six Years Later . . .

- The price of rice in Manang has falled by 70%

- 200-300 mules on the trail from Dumre to Manang

- Apart from porters working for trekking agencies, almost no porters to be seen on the trail.



An Arrow-Debreu Model of the Market for Porters

- Villages (r) are uniformly distributed on a square district.

- Commodities (g) are endowed to villages in random amounts.

- Representative consumers in each village are endowed with random quantities of goods and unit allocation of time.

- Cobb-Douglas preferences extend over consumption of goods (c_i) and leisure (ℓ):

$$U(C, \ell) = \ell \prod_g C_g$$

- Portering services are required to deliver goods from one village to neighboring villages.

- The shadow price of portering services differs on all routes depending on differences in commodity endowments and the availability of porters.

- Equilibrium prices clear all markets:

$$\underbrace{\omega_{gr}}_{\text{Initial Endowment}} + \underbrace{\sum_{r'} E_{gr'r}}_{\text{Imports}} = \underbrace{C_{gr}}_{\text{Consumption}} + \underbrace{\sum_{r'} E_{grr'}}_{\text{Exports}}$$

- Individuals allocate their time to leisure and portering:

$$\bar{L} = \underbrace{\ell_r}_{\text{Leisure}} + \underbrace{\sum_{g,r'} X_{rgr'}}_{\text{Portering}}$$

- Budgets are determined by prices and endowments:

$$\underbrace{M_r}_{\text{Income}} = \underbrace{P_r^{\ell}\bar{L}}_{\text{Value of Time}} + \underbrace{\sum_g P_{gr}\omega_{gr}}_{\text{Value of Endowment}}$$

- Individual choices are optimizing:

$$C_{gr} = \theta \frac{M_r}{P_{gr}}, \quad \ell_r = \theta \frac{M_r}{P_r^{\ell}}$$

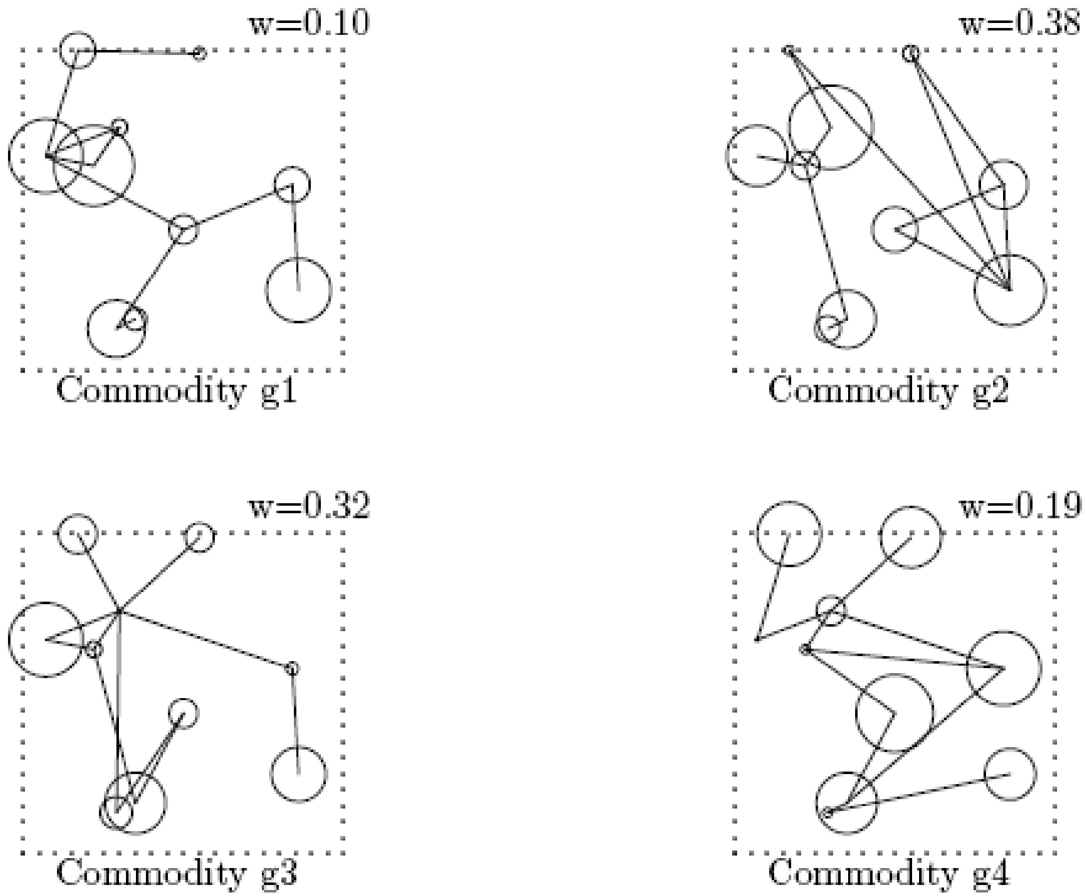
- Arbitrage constraints relate commodity prices, transportation arbitrage constraints to neighboring villages:

$$\underbrace{P_{gr}}_{\text{Purchase Price}} + \underbrace{PT_{rr'}\phi_{grr'}}_{\text{Transport Cost}} \geq \underbrace{P_{gr'}}_{\text{Sales Price}} \qquad \forall r' \in \mathcal{N}_r$$

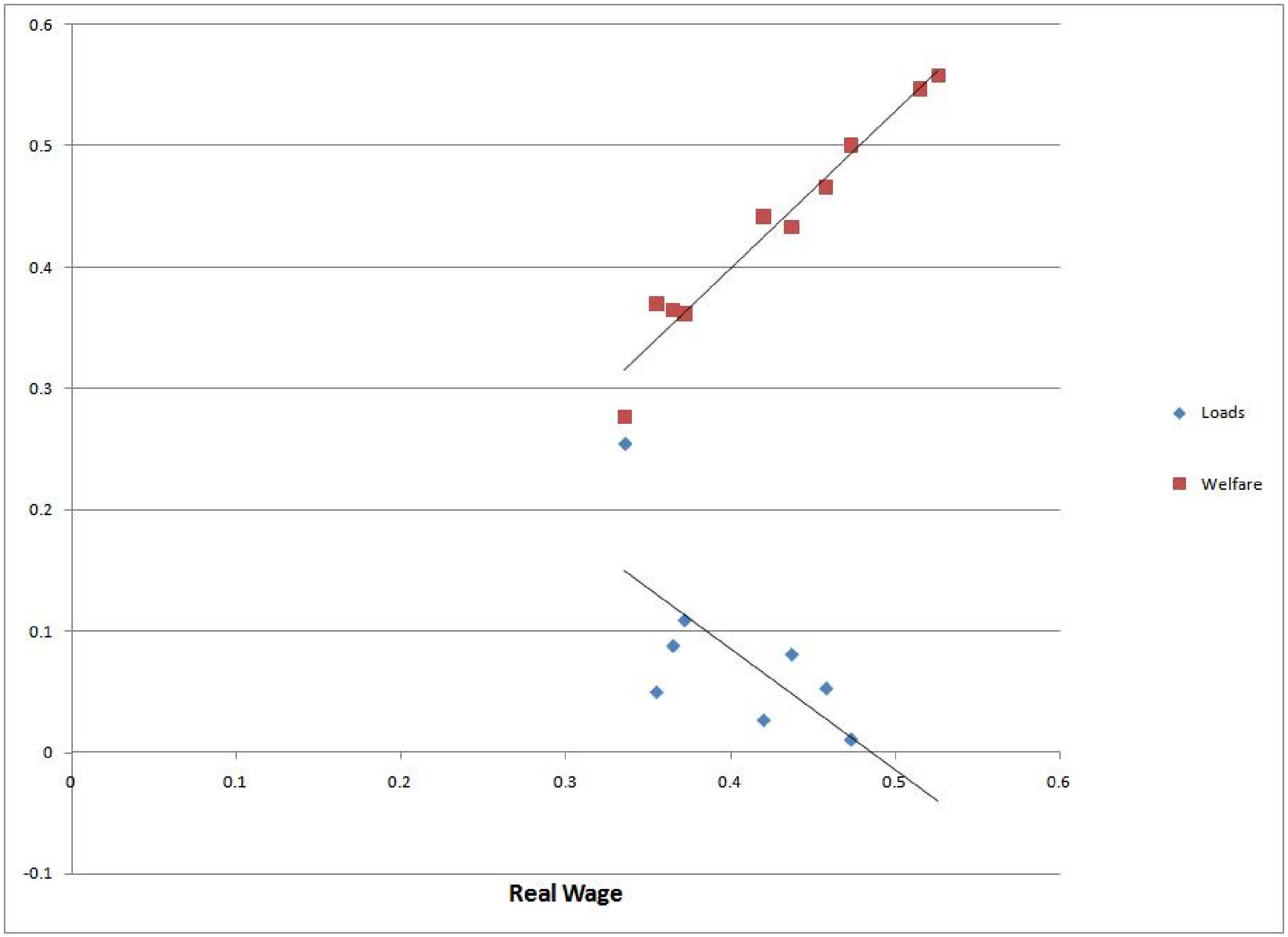
- When delivering a load of good g from r to r', the porter returns with no load if there are no goods to be transported on the return. The decision to porter loads thus depends on the shadow value of leisure and the market price of transportation services on neighborhood routes:

$$\underbrace{P_r^{\ell}}_{\text{Value of Time}} \geq \underbrace{PT_{rr'} + PT_{r'r}}_{\text{Portering Wages}} \quad \forall r' \in \mathcal{N}_r$$

Figure 1: Initial Endowments and Equilibrium Trade Flows



No Surprise: Poorer People Work as Porters



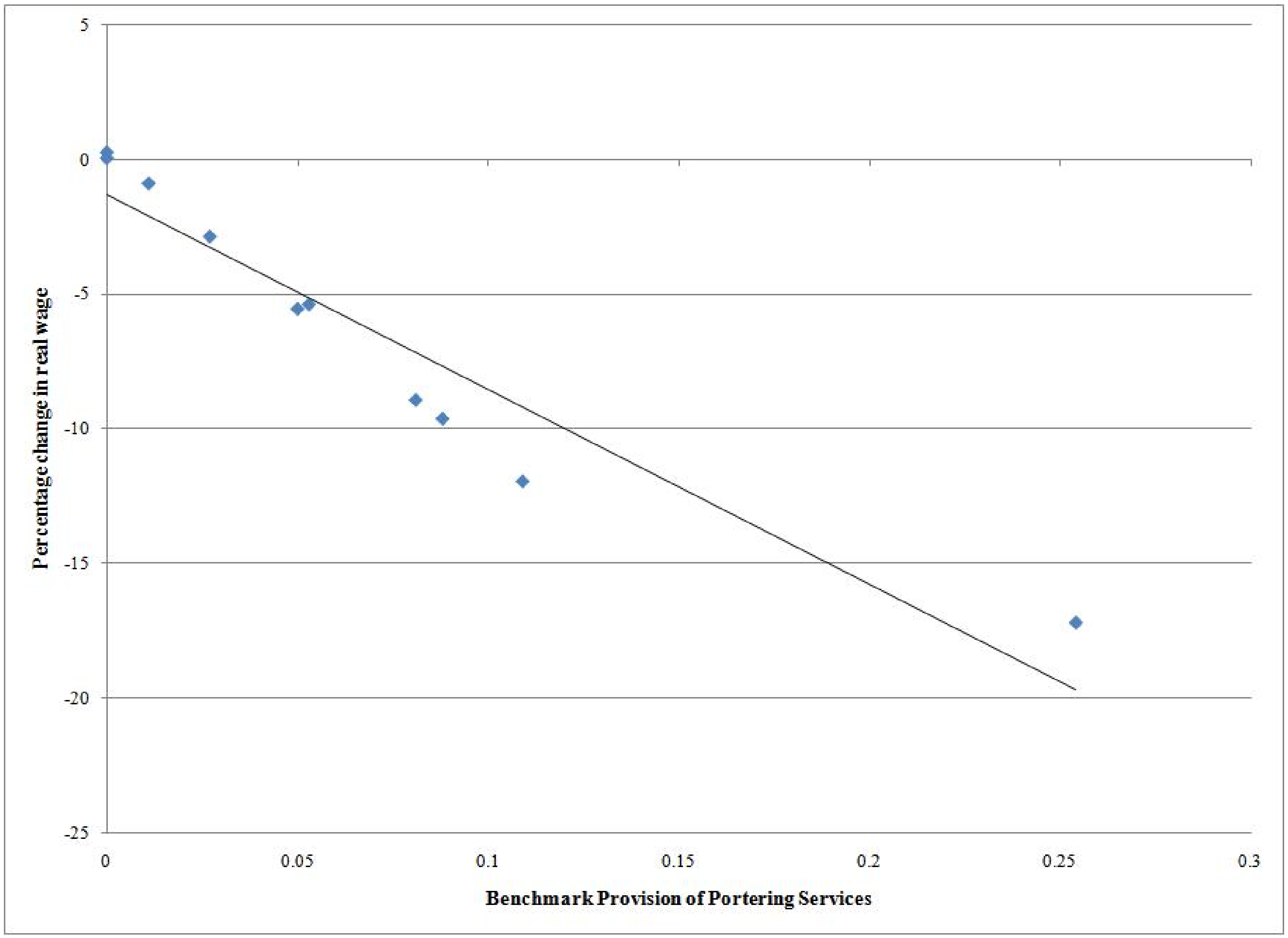
Introducing Mules

Mules based in region r carry loads in return for compensating payment in goods. The arbitrage conditions for mules operating from region r is:

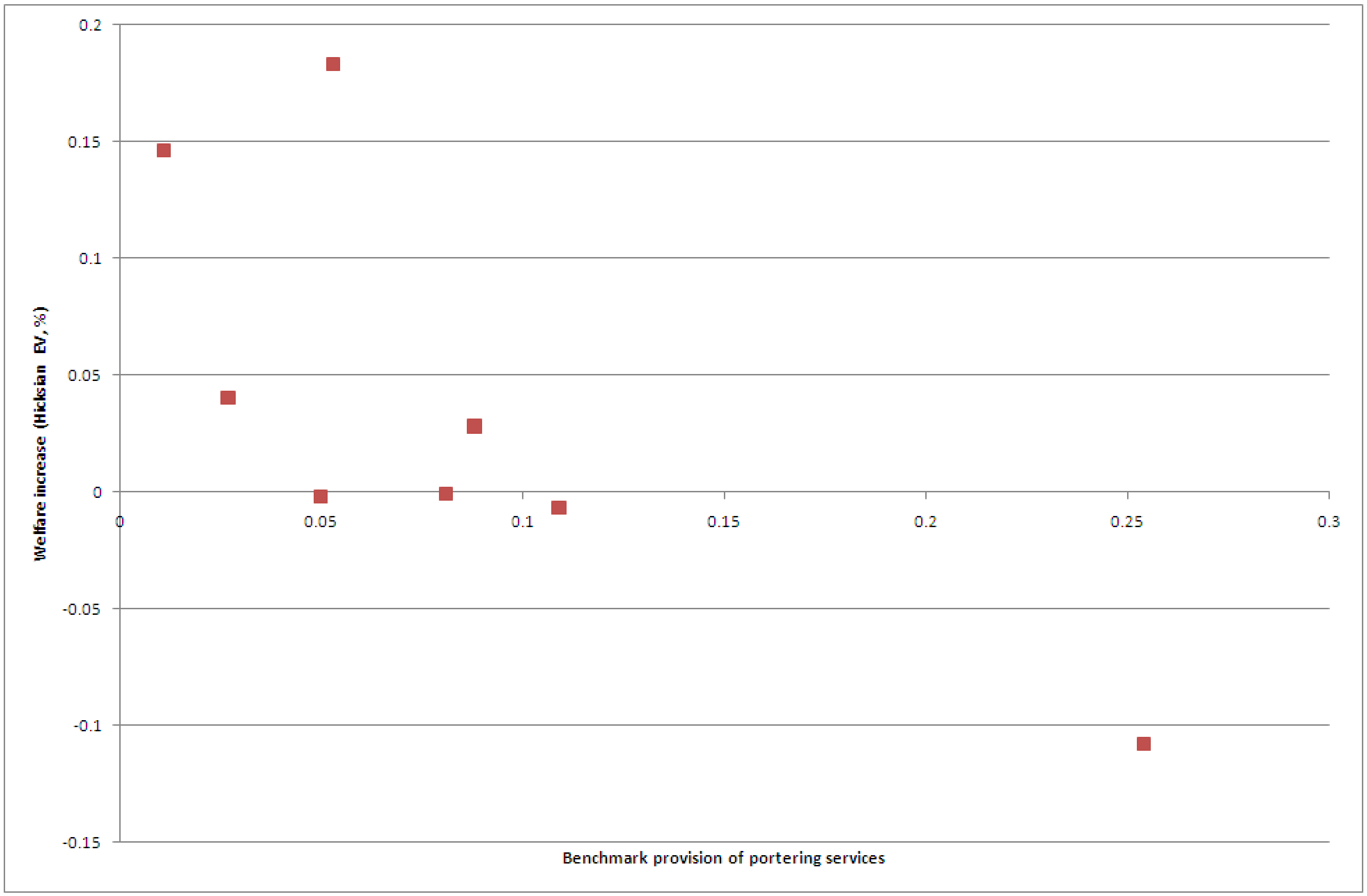
$$\underbrace{\mu \sum_g P_{gr}}_{\text{FeedingCosts}} \geq \underbrace{PT_{rr'} + PT_{r'r}}_{\text{Earnings}} \quad \forall r' \in \mathcal{N}_r$$

When the cost of mules (μ) is sufficiently low, porters are driven from the market and equilibrium wages fall.

Mules Lower Wages



Welfare Increase for Most (but not all) Villages



Illustrative Model #2

Extending the standard CGE paradigm: traffic congestion in a general equilibrium framework.

Merging ideas from engineering and economics we model traffic congestion and urban sortin in a general equilibrium framework. Vickrey's grim assessment:

"... traffic often behaves like population. It has been said that if nothing stops the growth of population but misery and starvation, then the population will grow until it is miserable and starves."

Wardropian Equilibria

Wardrop, J. G. (1952), 'Some theoretical aspects of road traffic research', *Proceeding of the Institute of Civil Engineers, Part II* pp. 325–378.

Key idea: people aren't stupid. Drivers take the shortest route, taking decisions of other drivers as given.

A *multicommodity formulation* of Wardrop's model can provide a compact and efficient representation of the model, permitting direct solution with "off-the-shelf" algorithms. (Ferris, Meeraus and Rutherford, 1999.)

A Spatial Equilibrium Model

Current program seeks to formulate spatial equilibrium models of regional housing markets which accounts for characteristics of roads, the housing stock and employment demand.

Our objective is to produce model which can be used to study the geography of a major metropolitan area through the representation of the locations for employment, housing and the connecting transportation arteries.

We will produce a model which can be used to study the interplay between the road system, the pattern and level of employment and the pattern and value of the housing stock.

Notation

$i, j, k \in \mathcal{N}$ are indices which will be used to describe "nodes" in the network. Each node has associated employment and housing stock. In a small-scale network, each node might represent a specific intersection. In a larger-scale application, a node might represent a major interchange in a freeway system.

a_{ij} denotes "transportation arcs" in the network. These arcs correspond to specific roads or major arteries in the road system.

$c_a(F_a)$ denotes the congestion function on arc a. Following the logic of Vickery's multiple interaction model, the time to traverse an arc is a function of the number of cars on that arc as follows:

$$c_a(F_a) = \alpha_a + \beta_a F_a^4$$

Logic of the Economic Model

$U(c,T,H)$ denotes househould utility which depends on consumption, travel time and housing. In our illustrative calculations we represent this with the following parametric form:

$$U(c,T,H) = \left(\gamma(\bar{T}-T)^{\rho} + (1-\gamma)\left(c^{\theta}H^{1-\theta}\right)^{\rho}\right)^{1/\rho}$$

$w_j \geq 0$ represents the wage paid by the employer at location j. This includes a premium which compensates for travel cost.

$D_j(w_j)$ represents the labor demand function by employers at node j, given by:

$$D_j(w_j) = \phi_j w_j^{-\sigma_j}$$

The equilibrium travel time from location i to j is T_{ij}.

A household is willing to live in location i and commute to j if the wage is sufficient to compensate for the loss of leisure required for the commute to work.

Budget-constrained utility maximization allocates income between goods and housing:

$$\max \quad U(c, T_{ij}, H) \quad \text{s.t.} \quad C + p_i^H H = w_j$$

Note that we have formulated this model as though all individuals are renters. The price of housing together with consumption then exhaust wage income.

Associated demand functions per household who lives at i and works at j may then be computed as functions of the housing price and the wage:

$$c_{ij}(w_j) = \theta w_j$$

and

$$H_{ij}\left(w_j, p_i^H\right) = (1-\theta)\frac{w_j}{p_i^H}$$

Multicommodity Formulation of Wardrop's Model

Travel time from node i to j satisfies the following arbitrage condition:

$$T_{ik} \leq c_{ij}(F_{ij}) + T_{jk}$$

i.e., commuting time from location i to location k must be no greater than the travel time from i to j plus the travel time from j to k.

N.B. Whenever $c_{ij}(F_{ij}) + T_{jk} > T_{ik}$ it immediately flows that $x_{ij}^k = 0$.

Flow conservation at location j for persons commuting to work at location k is given by:

$$N_{jk} + \sum_i x_{ij}^k = \sum_i x_{ji}^k \quad j \neq k$$

and

$$N_{jj} + \sum_i x_{ij}^j = D_j(w_j)$$

Equilibrium Sorting

Substituting for c, T and H in the direct utility function then yields an indirect utility function, $V_{ij}(w_j, p_i^H, T_{ij})$.

The following arbitrage condition determines the number of households living at location i and working at location j:

$$\widehat{U} \geq V_{ij}(w_j, p_i^H, T_{ij}) \quad \perp \quad N_{ij} \geq 0$$

N.B. N_{ij} will be zero whenever $V_{ij} < \widehat{U}$.

In equilibrium the utility level of all households are equal to $\widehat{U}$. The number of households living at i and working at j is given by N_{ij}. This number is greater then zero only in the event that the realized utility level offered by that location is sufficient to entice households to locate there.

Housing Market

In equilibrium the housing market is cleared as:

$$\sum_j N_{ij} H_{ij}(w_j, p_i^H) = \bar{H}_i$$

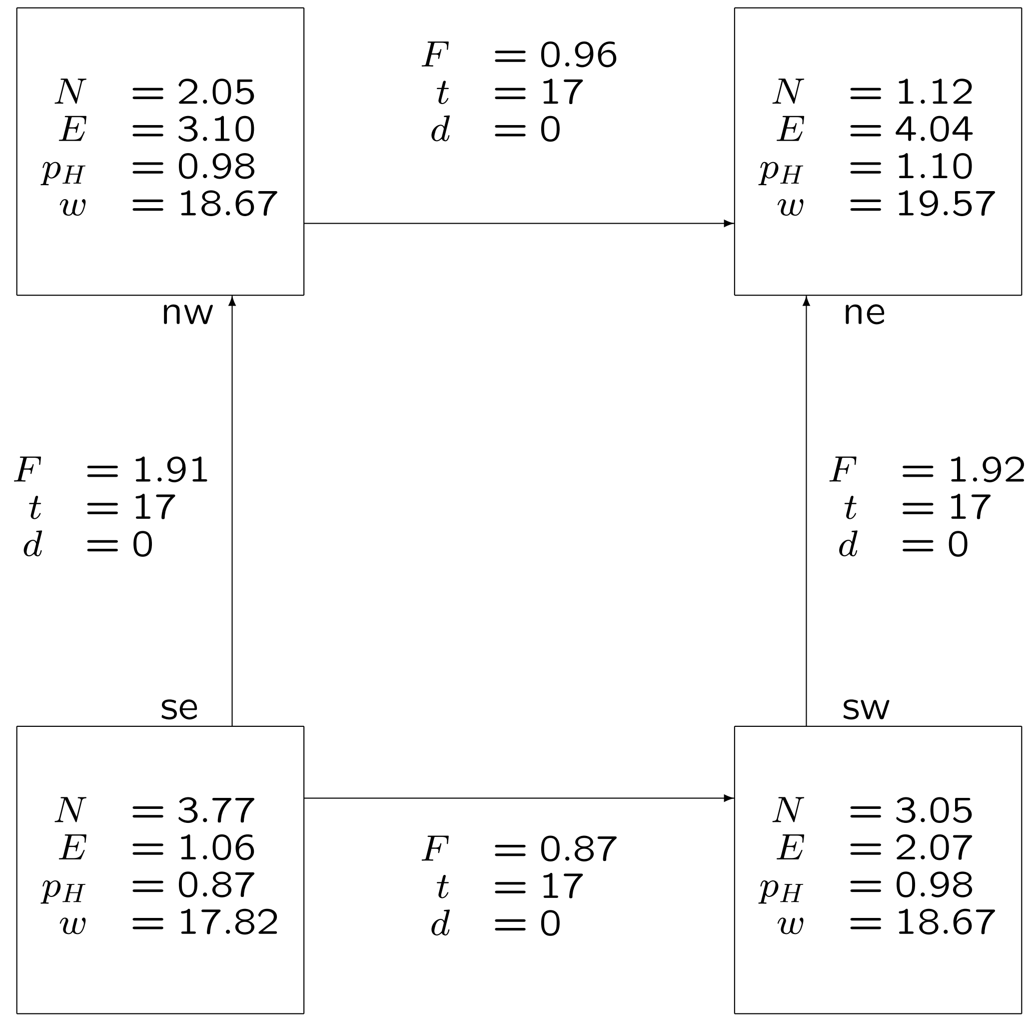
where $\bar{H}_i$ is the housing stock at node i.

Closure

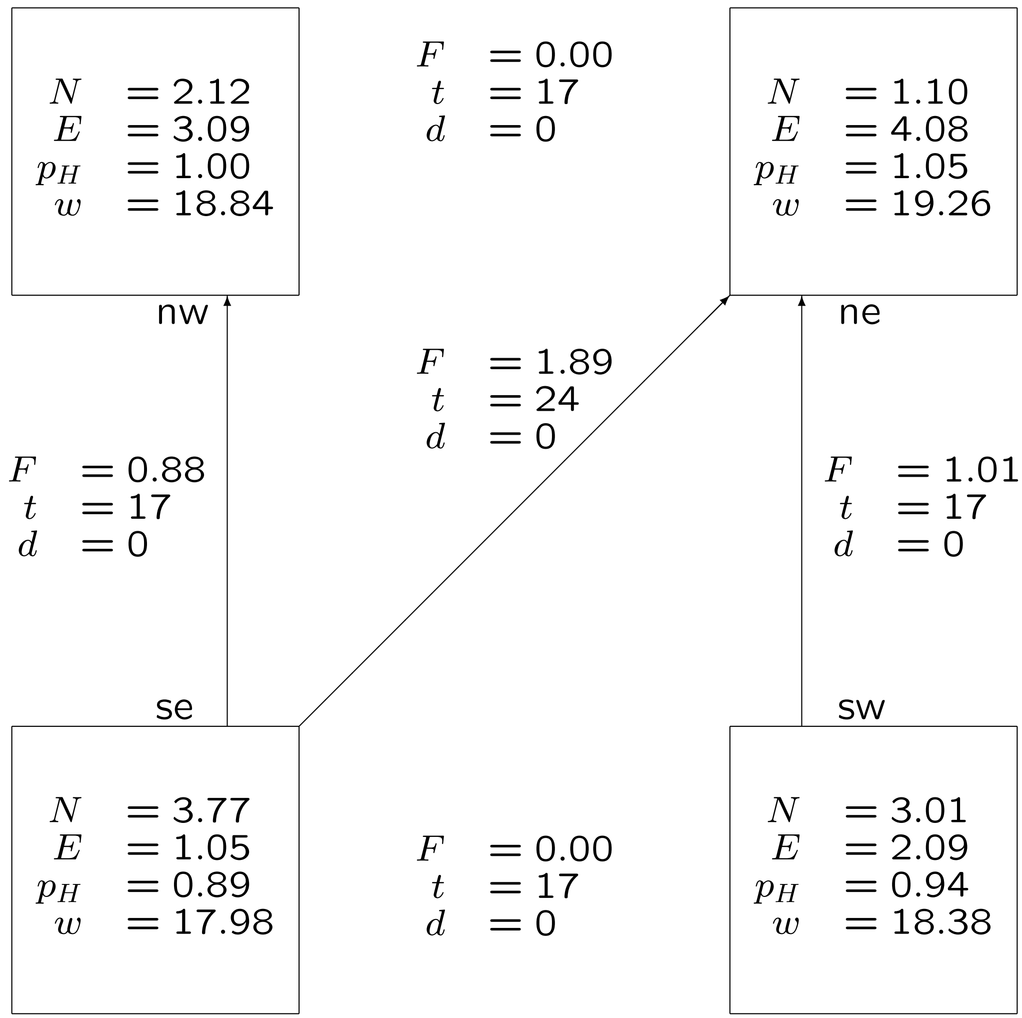
$$\phi \widehat{U}^\eta = \sum_{i,j} N_{ij} \quad \perp \quad \widehat{U}.$$

In the *closed city model*, $\eta = 0$, and in the *open city model*, $\widehat{U} = 0$ (i.e., $\eta = \infty$)

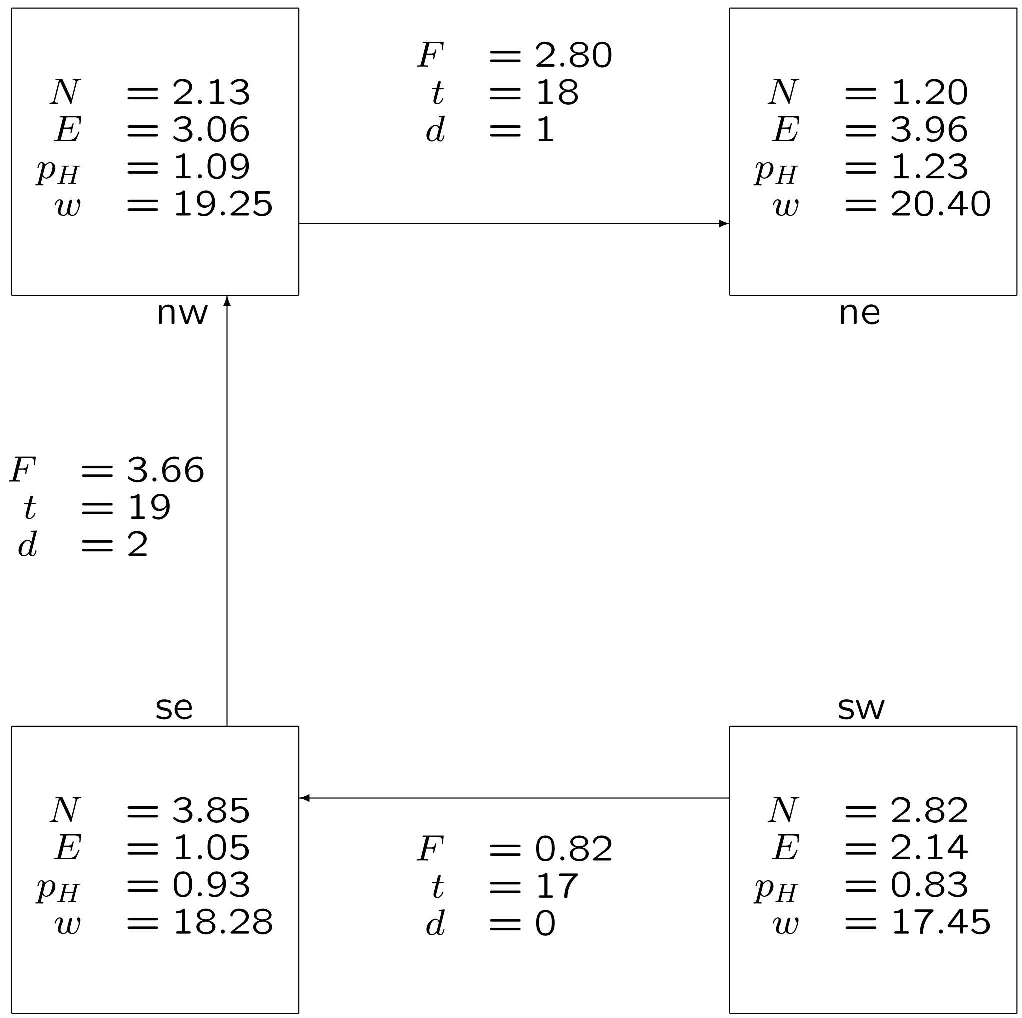
Reference Equilibrium



New Diagonal Highway (Hicksian EV=0.06)



Loss of SE-NE Highway (Hicksian EV =-0.12)



Income Accounting with Housing and Capital Earnings

Income for workers at node j equals wage income, shares of capital income and dividend payments on shares held outside the city (d_j):

$$M_j = \bar{w} W_j + \theta_j \Pi + d_j$$

Employment supply and demand:

$$E_j = \phi_j^L Y_j W_j^{-\epsilon}$$

Housing supply and demand:

$$\bar{H}_i = \frac{(1-\theta)\sum_j N_{ij} M_j}{P H_i}$$

Aggregate profit on capital and housing in the city:

$$\Pi = \sum_j R_j \bar{K}_j + \sum_i PH_i \bar{H}_i$$

Capital supply and demand:

$$\bar{K}_j = \phi_j^K Y_j RK_j^{-\epsilon}$$

Zero profit for production at location j. Wage payments plus capital earnings equal the value of output:

$$W_j E_j + RK_j K_j \geq \mu_j$$

Figure 1: Residence and Workplace Choices: Reference Equilibrium

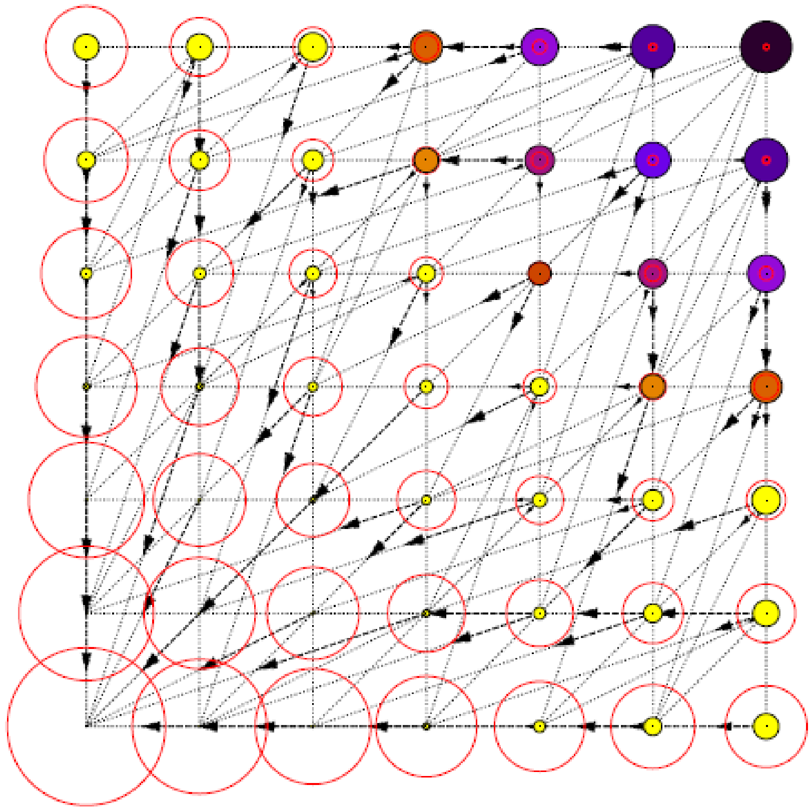


Figure 2: Traffic Congestion Rates: Reference Equilibrium

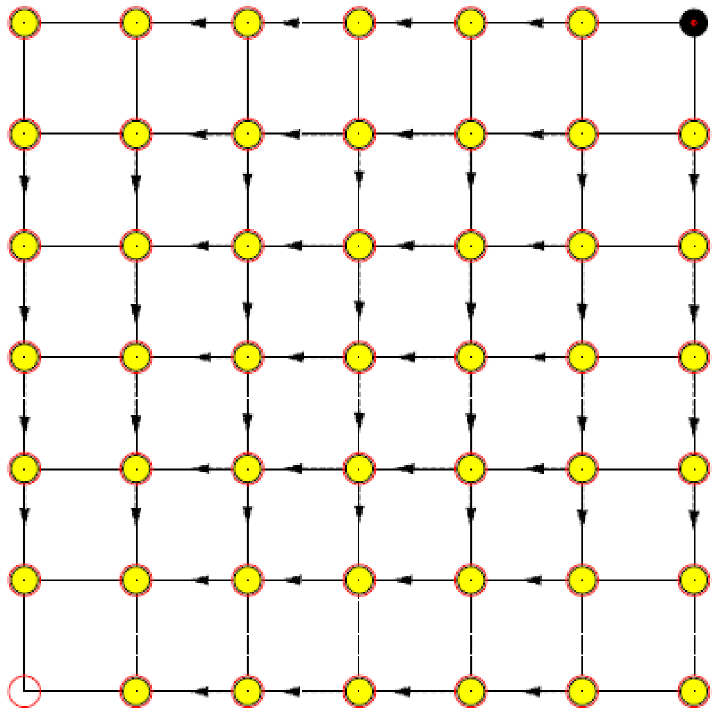


Figure 3: Residence and Workplace Choices: Ring Road

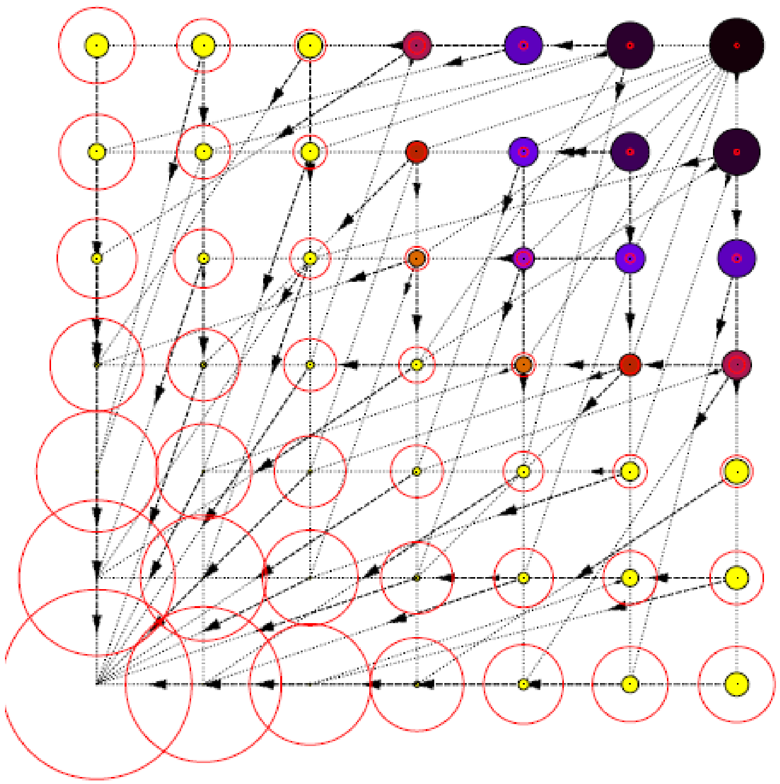


Figure 4: Ring Road without Congestion Effects

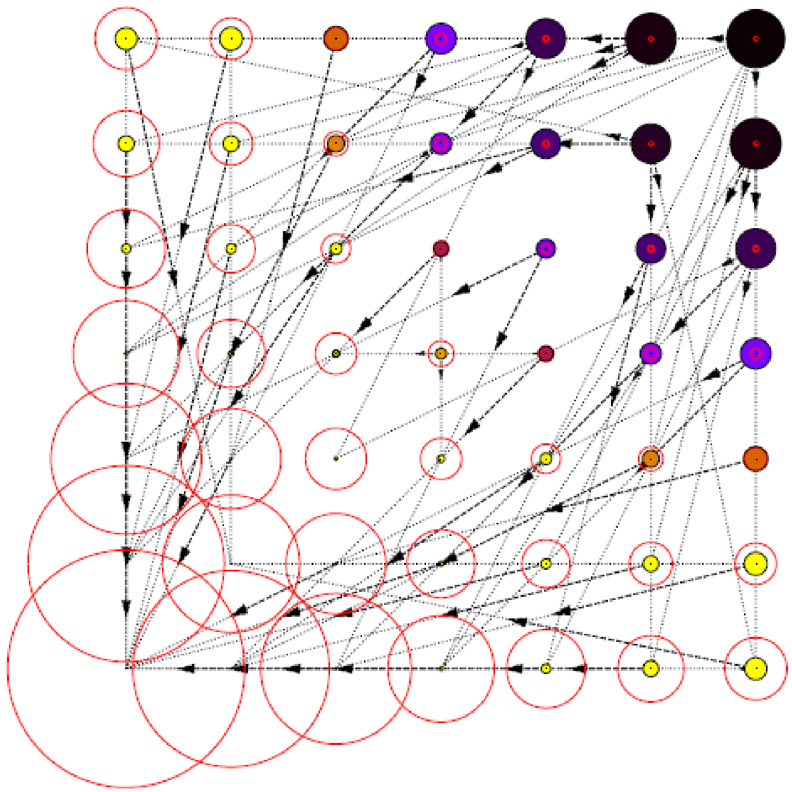
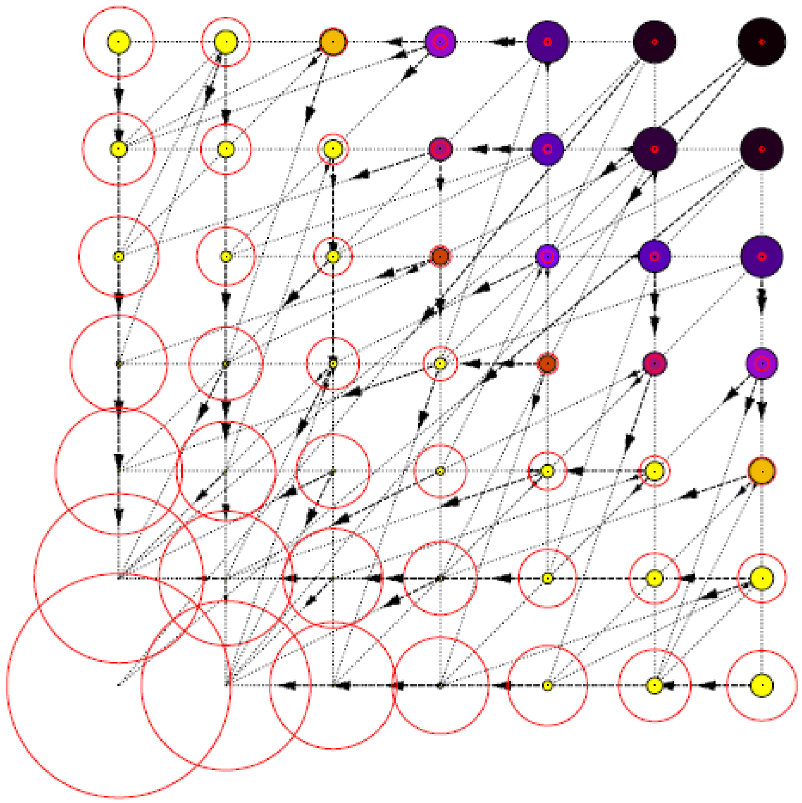
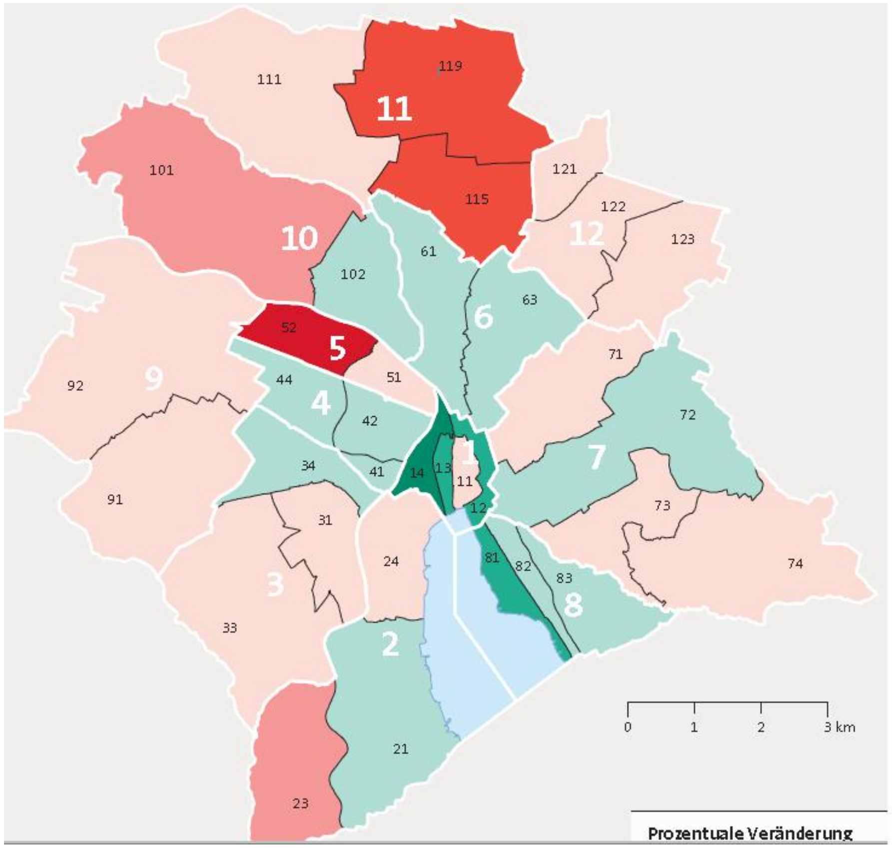


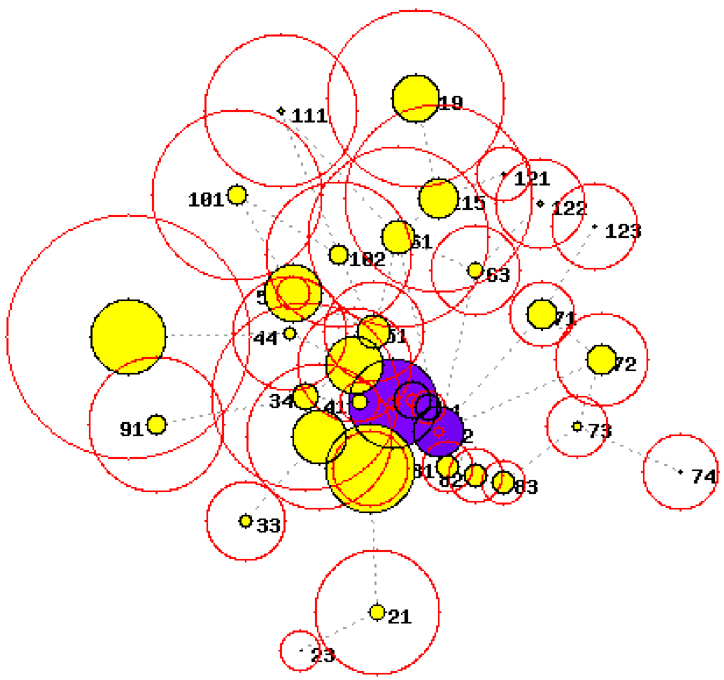
Figure 5: Residence and Workplace Choices: Rapid Transit



Zürich Quartiers



Zürich Employment and Residence



Conclusions – Future Work

- Positive Analysis

  1. Home ownership and demographics.

  2. General equilibrium closure (housing stocks and endogeneity of capital).

  3. Heterogeneity (employment categories and households).

  4. Public transit and modal choice.

  5. Calibration and estimation.

  6. Environmental goods and bads.

7. Schools.

8. Agglomeration effects.

- Normative Analysis

    1. Tolls and traffic controls

    2. Regulation of housing markets

    3. Property taxation.

- Computational issues

  1. Decomposition of traffic flow and sorting.

  2. Decomposition through column generation methods.

  3. Intertemporal responses.

  4. Visualization (GIS methods).