A model for spatially embedded social networks

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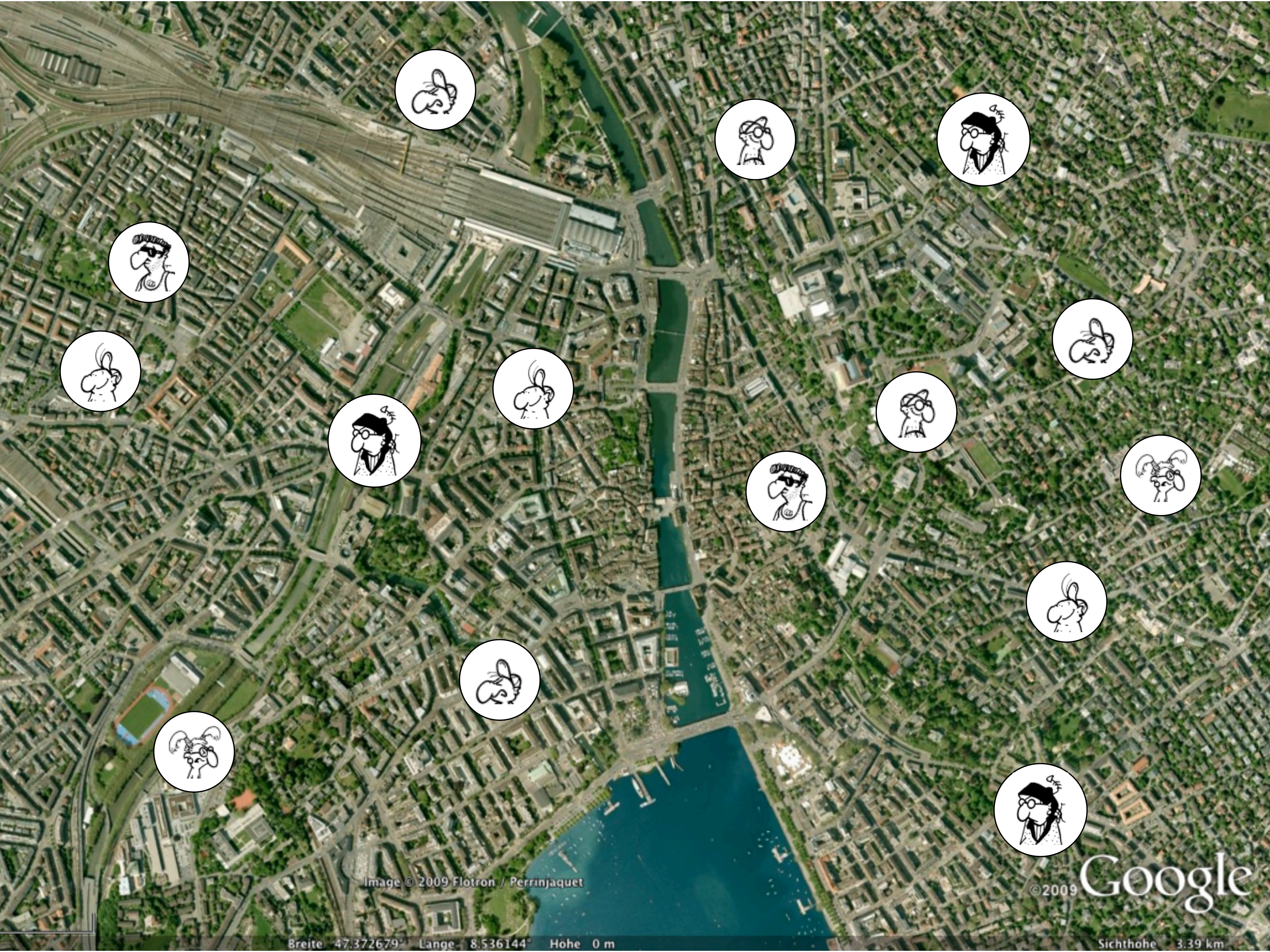
Breite 47.372679° Lange 8.536144° Hohe 0 m
Sichthohe 3.39 km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Breite 47.372679° Länge 8.536144° Höhe 0 m
Sichthöhe 3.39 km


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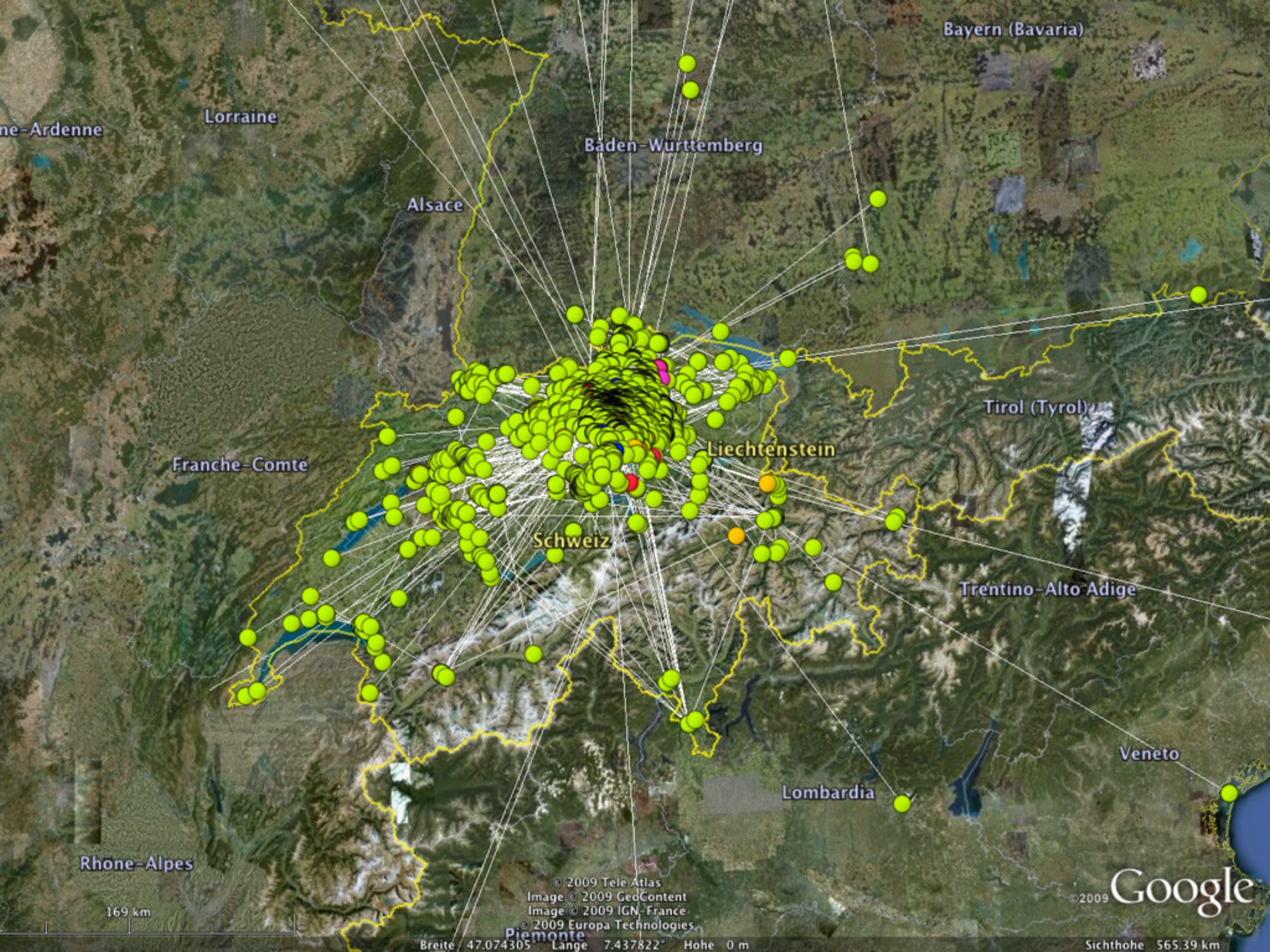
Breite 47.372679° Länge 8.536144° Höhe 0 m
Sichthöhe 3.39 km

Motivation

- Can we reproduce a social network with a model that uses easy to measure indicators
  - e.g. the spatial distribution of individuals?

Empirical Data


Bayern (Bavaria)
Lorraine
ne-Ardenne
Baden-Württemberg
Alsace
Tirol (Tyrol)
Liechtenstein
Franche-Comté
Schweiz
Trentino-Alto Adige
Veneto
Lombardia
Rhône-Alpes
Piemonte
169 km
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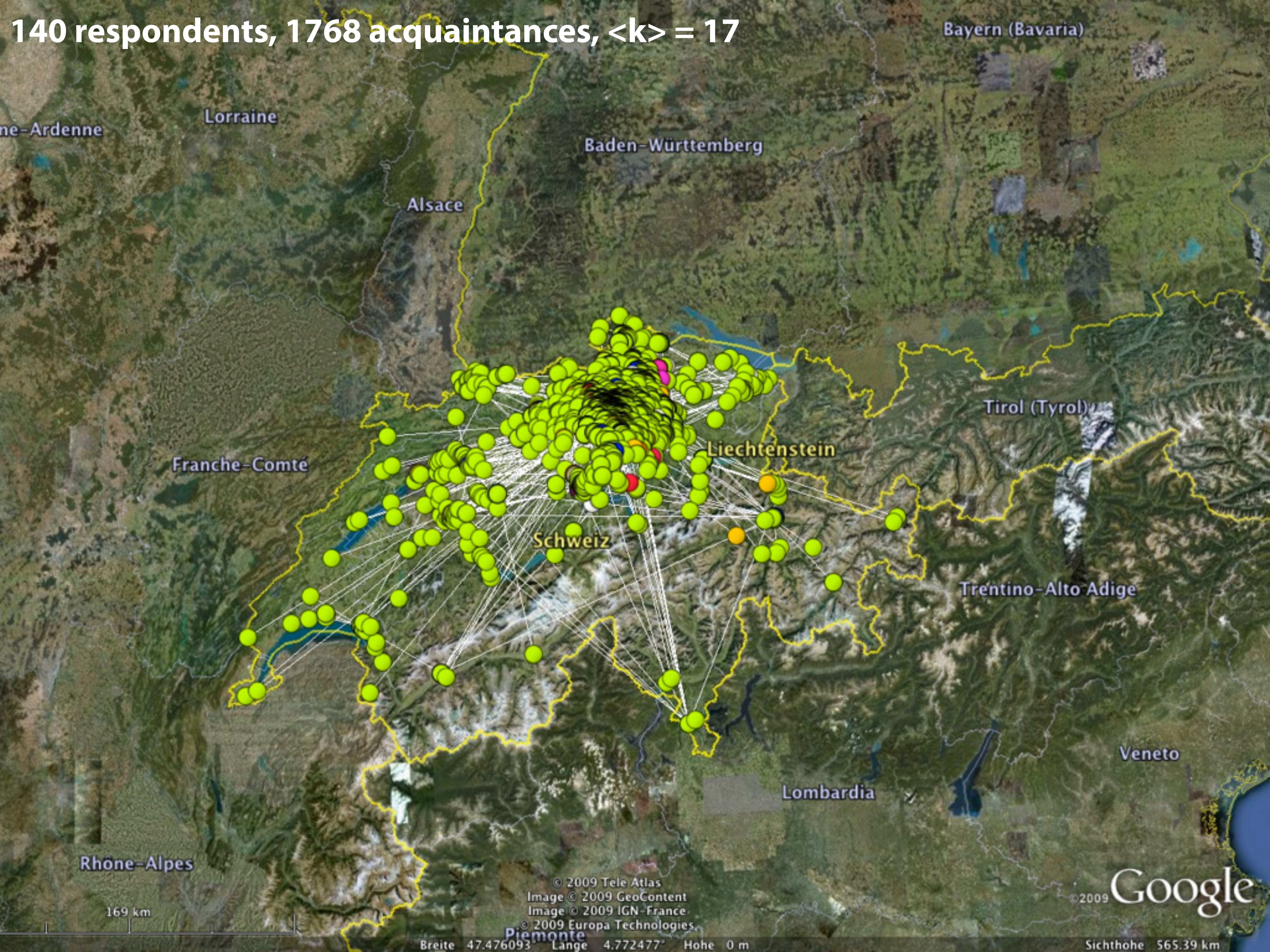
Breite 47.074305° Länge 7.437822° Höhe 0 m
Sichthöhe 565.39 km



140 respondents, 1768 acquaintances, <k> = 17
Bayern (Bavaria)
Lorraine
Baden-Württemberg
Alsace
Franche-Comté
Liechtenstein
Tirol (Tyrol)
Schweiz
Trentino-Alto Adige
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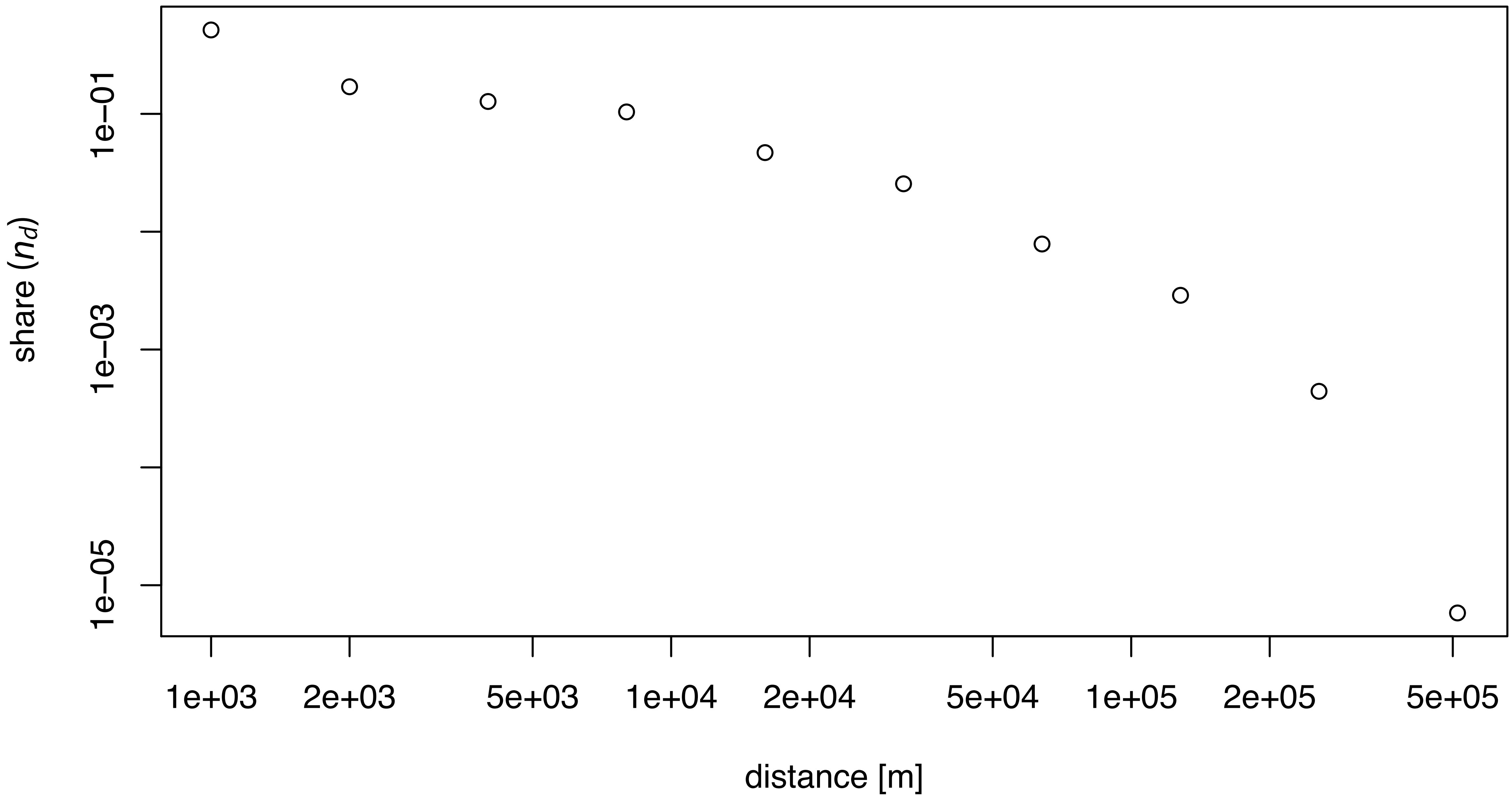
Breite 47.476093° Länge 4.772477° Höhe 0 m
Sichthöhe 565.39 km

Empirical Data

- Edge length distribution (edge length = beeline distance)



Empirical Data

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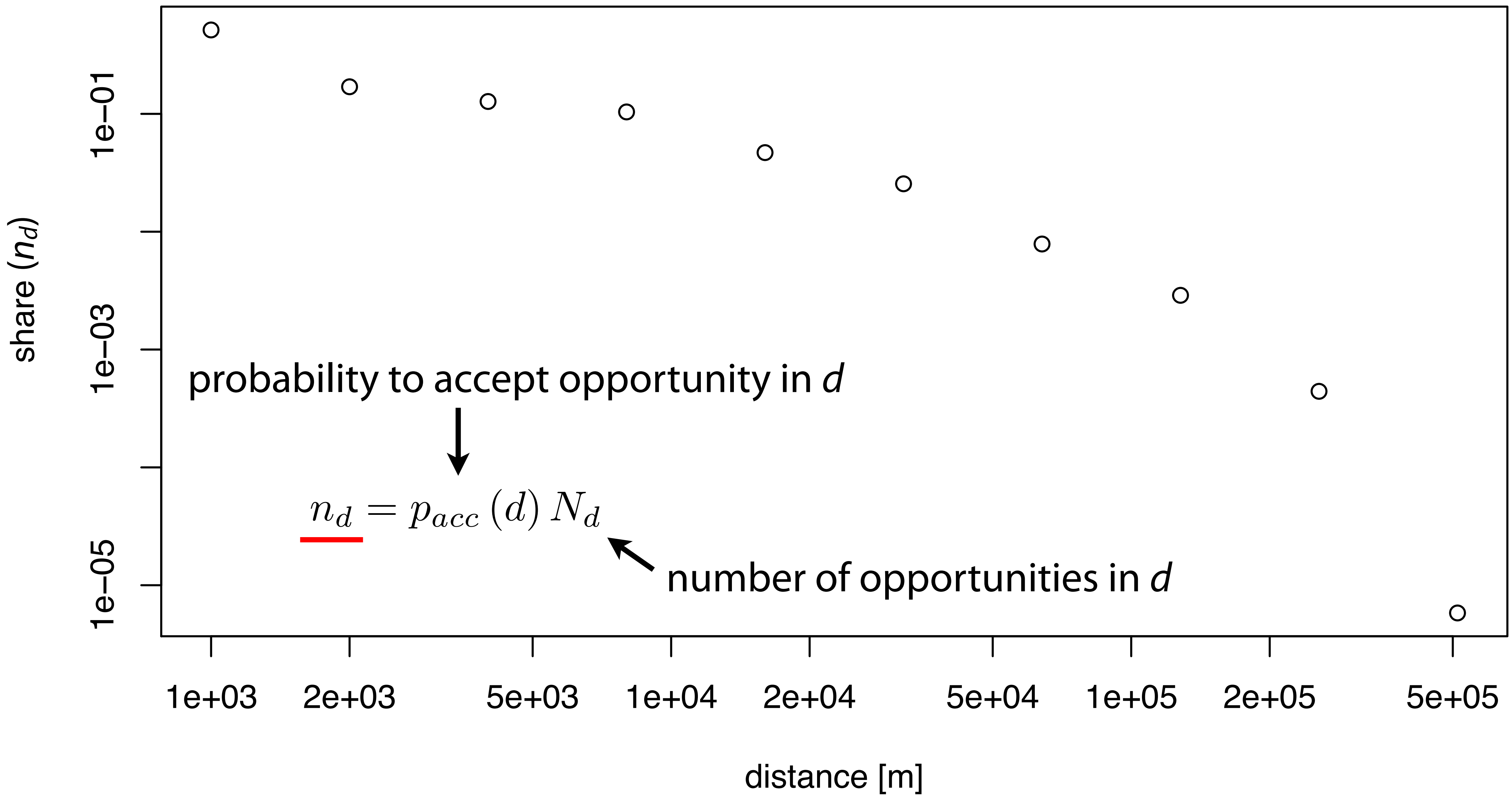
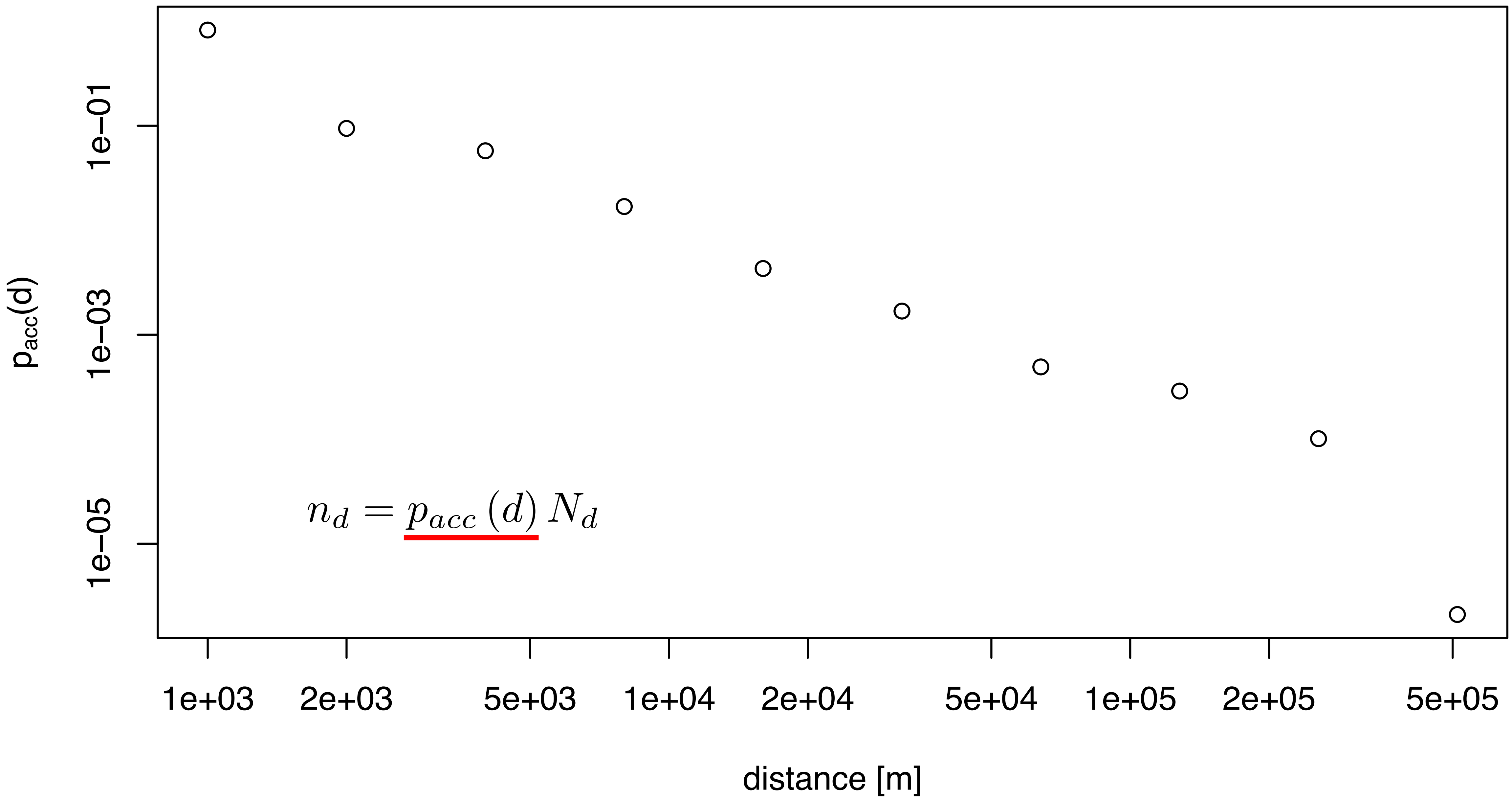


Empirical Data

- Acceptance probability (N_d from land use data)



Empirical Data

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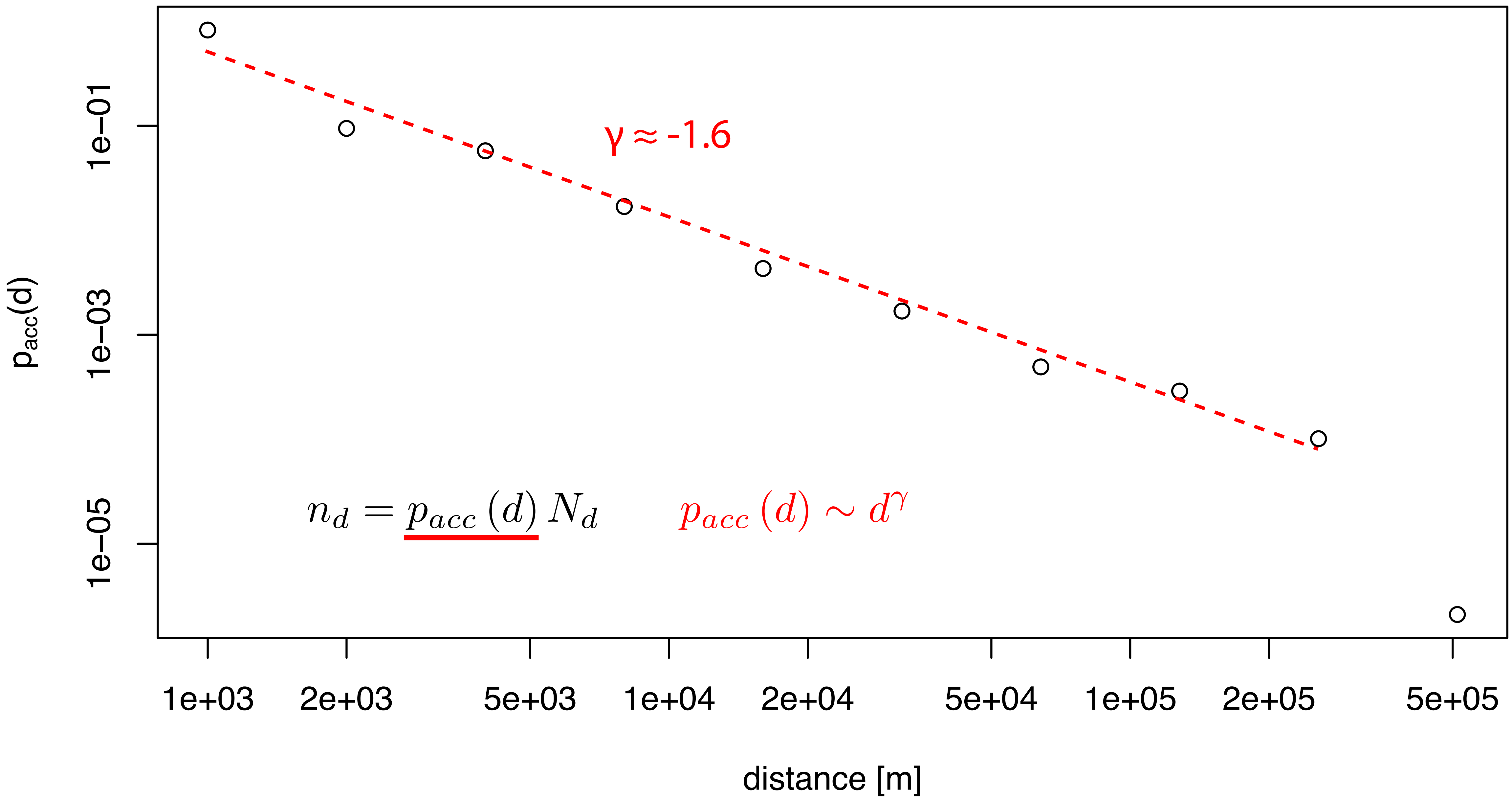


Methodology

Methodology

- Gravity model $$T_{ij} = K \frac{W_i W_j}{e^{c_{ij}}}$$
- Translated to a graph $$p_{ij} \sim e^{-c_{ij}}$$
- Empirical data $$p_{ij} \sim d_{ij}^{-\gamma}$$
- Costs of an edge $$c_{ij} = \gamma \ln d_{ij} + const$$

Methodology

- Assign each vertex a fixed budget C^*
- Create a graph with the constraint that
  - for each vertex i with k neighbours

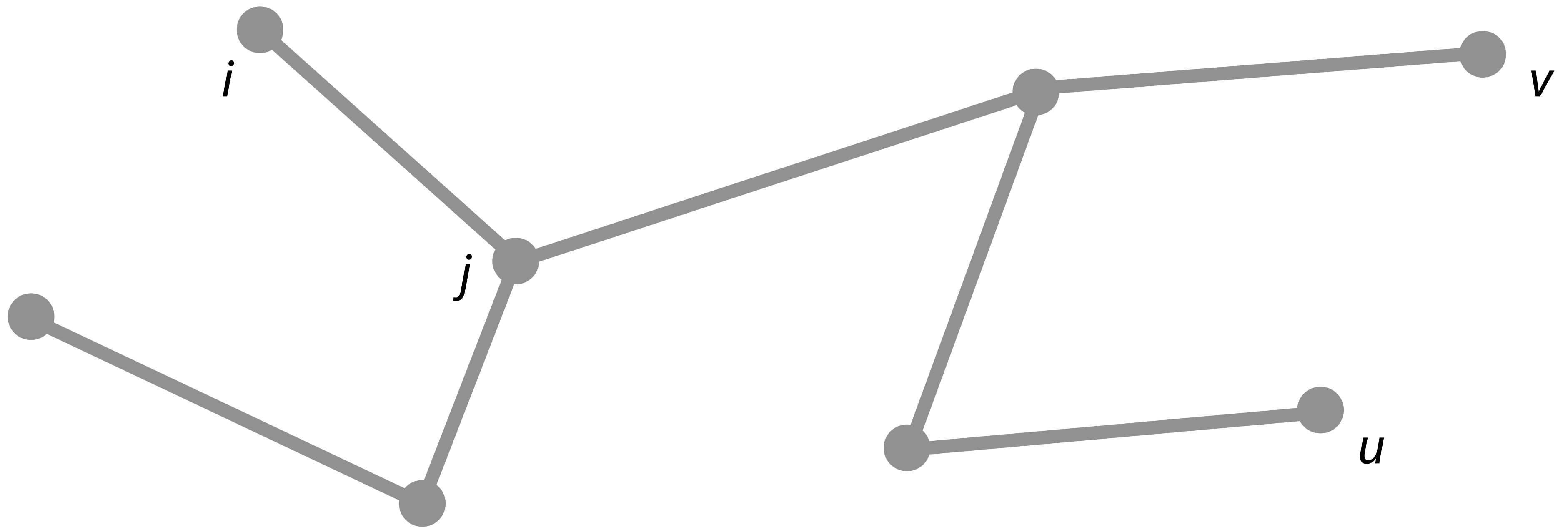
$$\sum_{j}^{k} c_{ij} = C_i \overset{!}{=} C^*$$

- Consider $P(Y)$ the probability distribution of all realisable graphs
- Choose $P(Y)$ so that the values of C_i meet as good as possible the desired value C^*
  - Exponential Random Graph Model

Methodology

- Generate a random graph with the desired number of edges
- Re-order edges until steady state distribution (Gibbs sampling)

$$P_{accept} = \frac{P\left(Y|y_{ij}=0, y_{uv}=1\right)}{P\left(Y|y_{ij}=0, y_{uv}=1\right) + P\left(Y|y_{ij}=1, y_{uv}=0\right)}$$



Methodology

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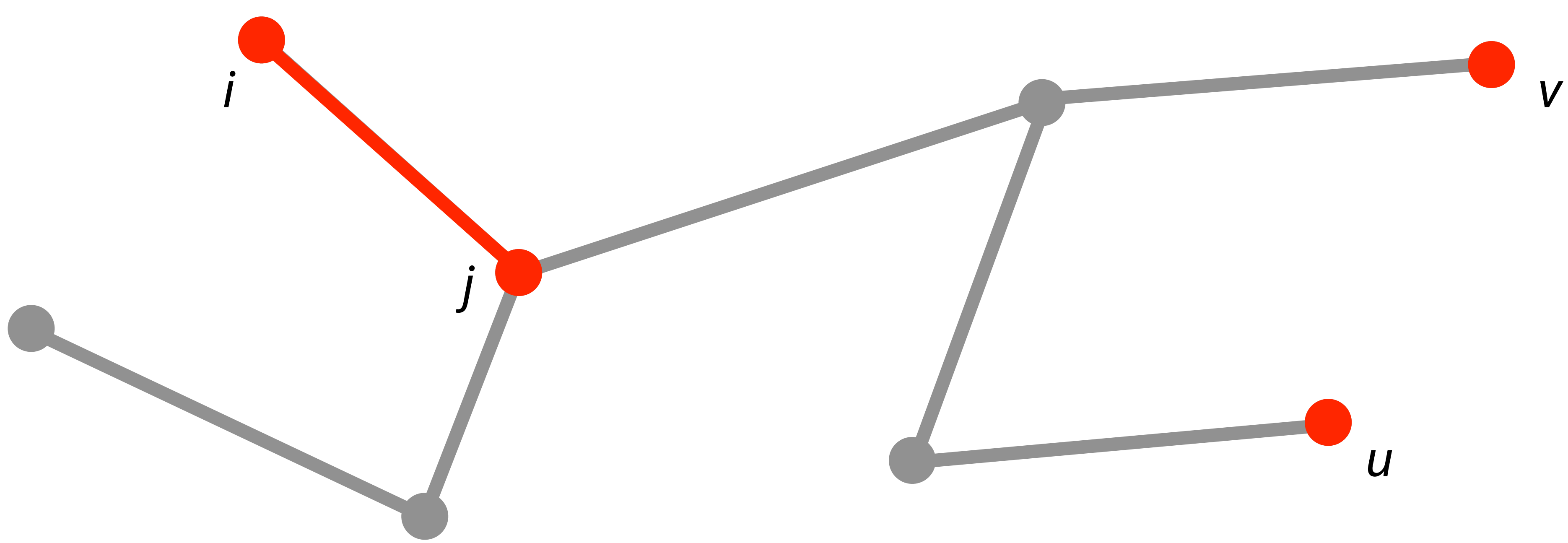
$$P_{accept} = \frac{P\left(Y|y_{ij}=0, y_{uv}=1\right)}{P\left(Y|y_{ij}=0, y_{uv}=1\right) + \underline{P\left(Y|y_{ij}=1, y_{uv}=0\right)}}$$



Methodology

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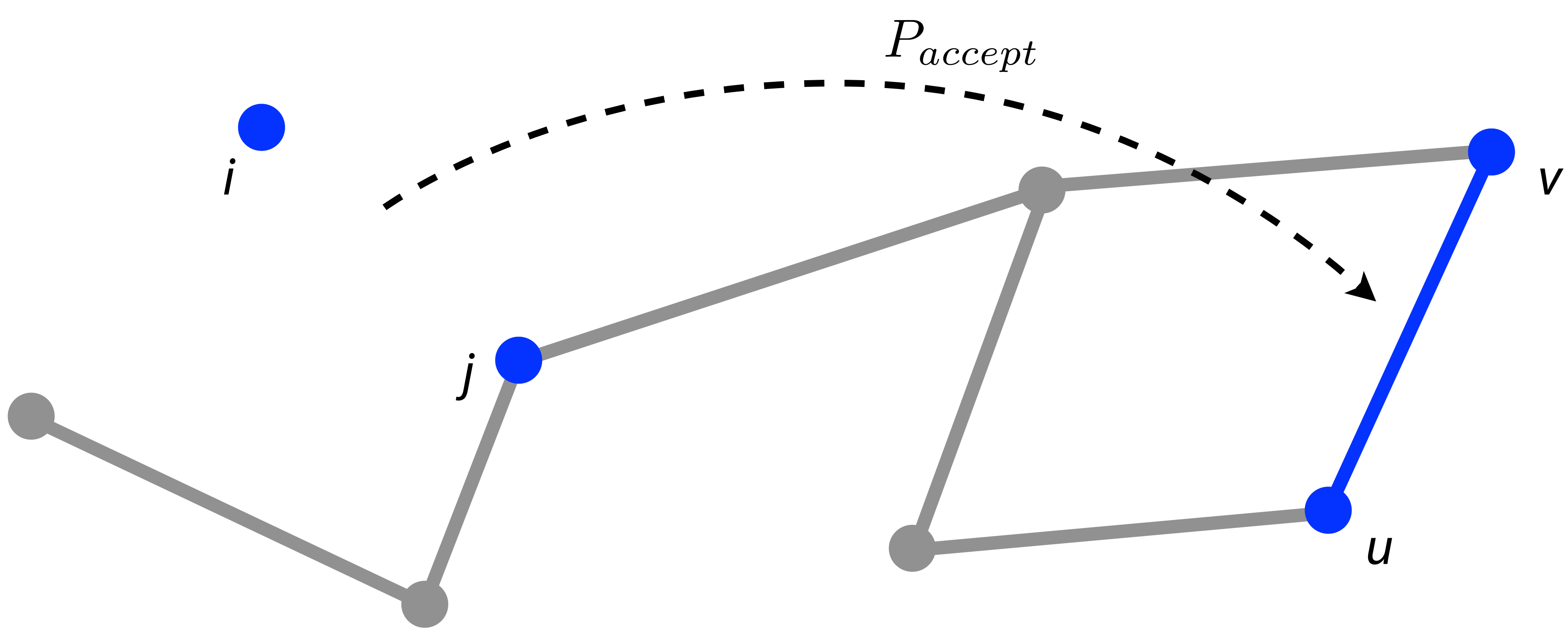


- probability of move is larger with larger *P(Y)* of new *Y*

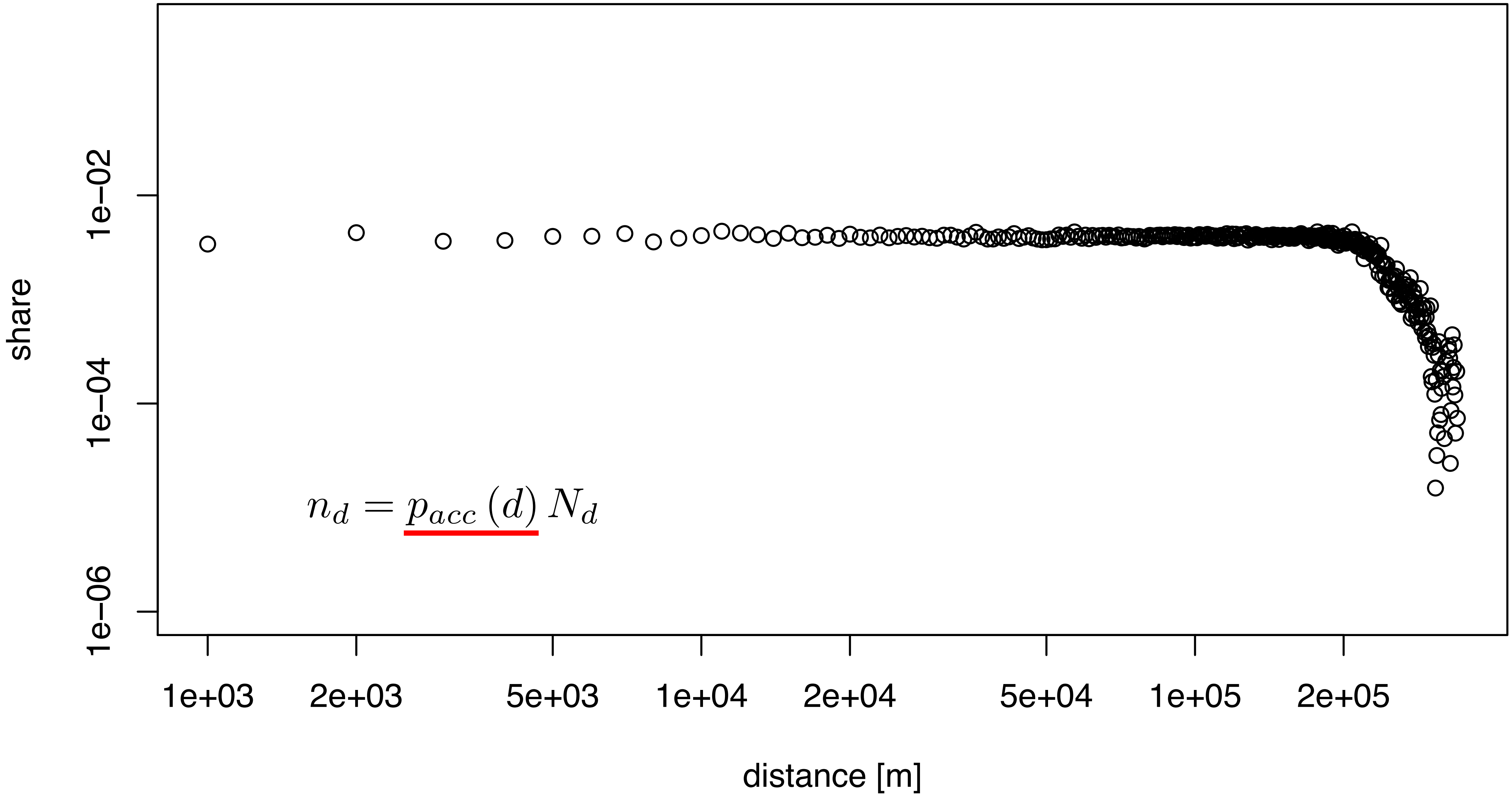
Simulation Scenario

Simulation Scenario

- Starting point: synthetic population of Switzerland (37k persons $\hat{\approx}$ 1% sample)
  - synthetic persons with residential locations (gender, age, income...)
  - i.e., fixed set of vertices with geographic coordinates
- Create a random graph with $\langle k \rangle = 17$
- Assign each vertex a budget C^*
- Reorder edges for $\sim 10^9$ iterations

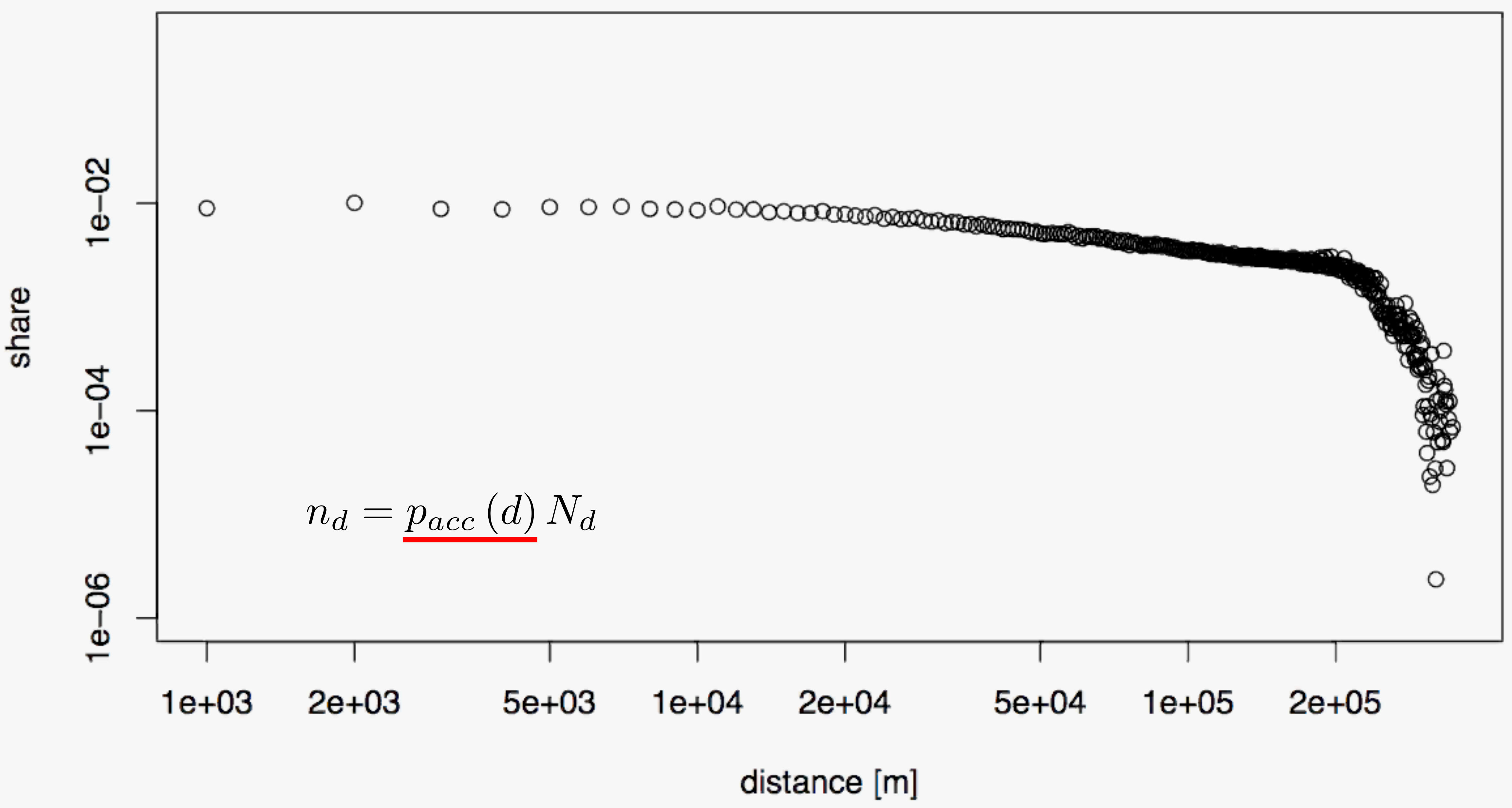
Simulation Results

- Acceptance probability



Simulation Results

- Acceptance probability



$$n_d = p_{acc}(d)\, N_d$$

Simulation Results

- How to control γ?

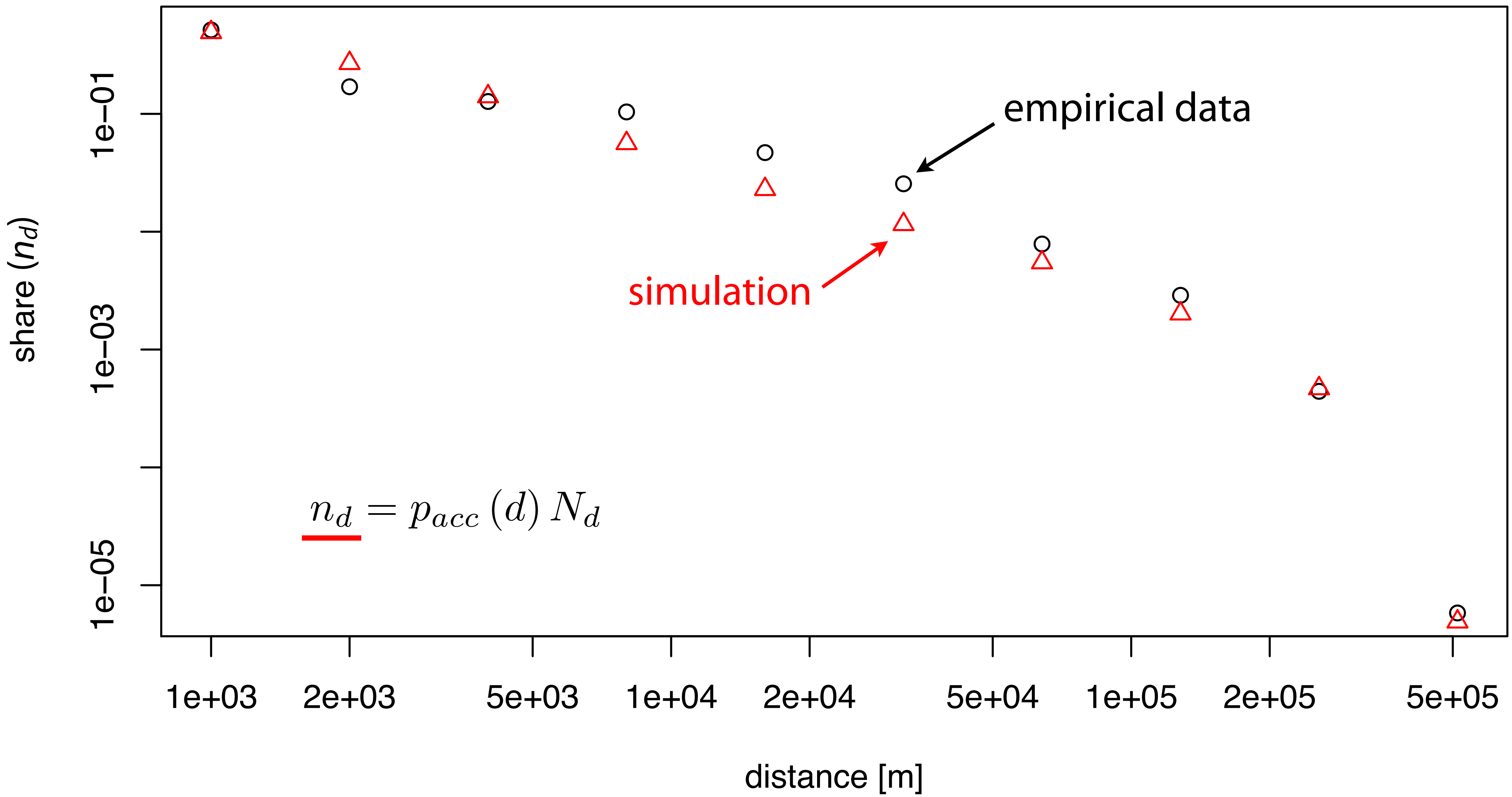
$$\sum_{j}^{k} c_{ij} = C_i \stackrel{!}{=} C^* \quad \text{where} \quad c_{ij} = \gamma \ln d_{ij} + const$$

C^* ↑	longer edges	γ ↓	flatter slope
C^* ↓	shorter edges	γ ↑	steeper slope

- by trial and error...
  - for <*k*> = 17 one obtains with $C^* = 25 \rightarrow \gamma = 1.6$
    (i.e. with <k> given from empirical data, adjust C* until *γ=1.6 from empirical data is achieved)*

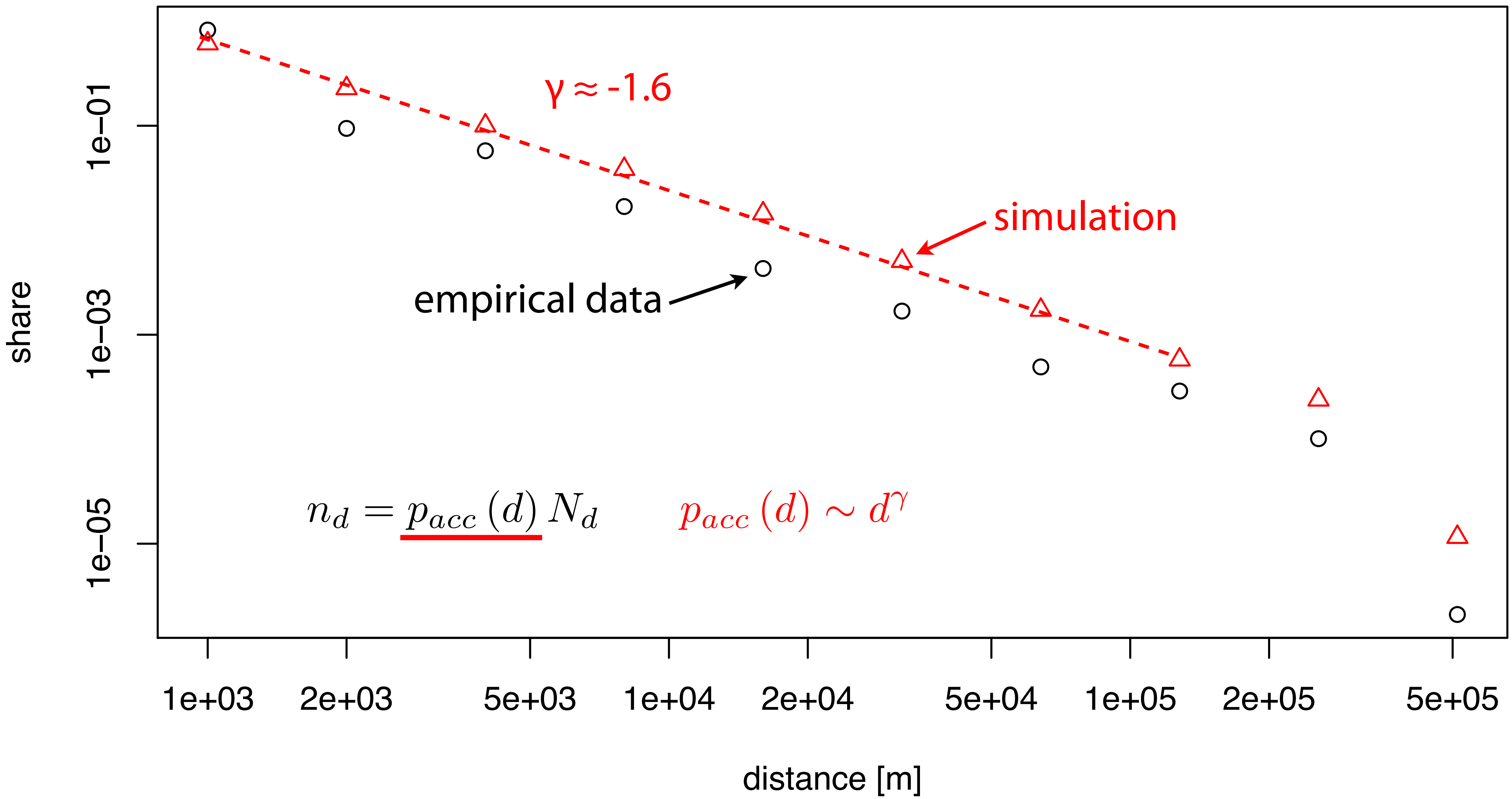
Simulation Results

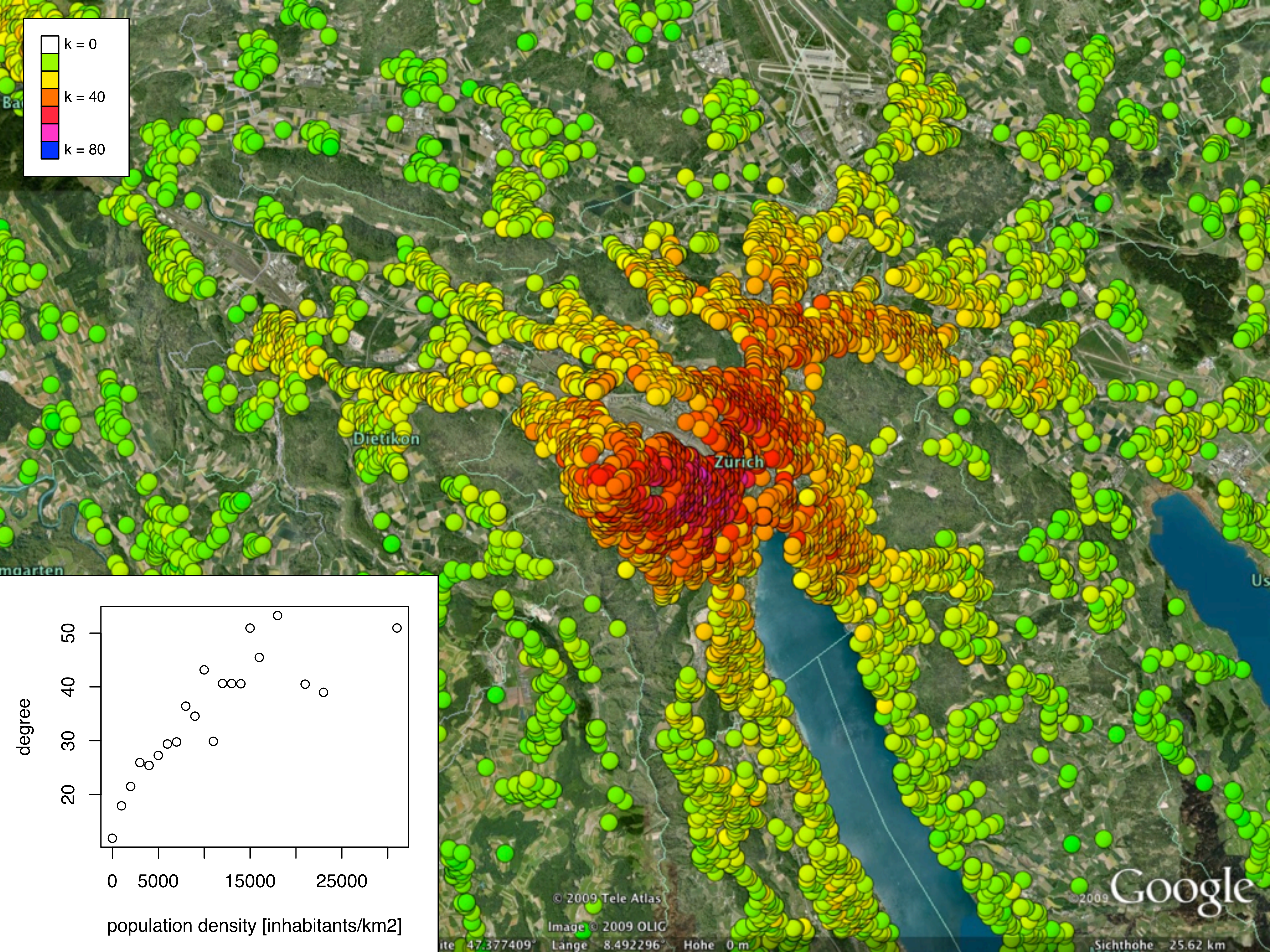
- Edge length distribution



Simulation Results

- Acceptance probability





k = 0
k = 40
k = 80
Dietikon
Zürich
degree
population density [inhabitants/km2]
0
5000
15000
25000
20
30
40
50
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Conclusion

- A model that uses only the spatial location of vertices as an explanatory variable
- Reproduces the edge length distribution of empirical data
- Spatial distribution of costs controls the spatial distribution of degrees
- No clustering. Small-World effects?

- Outlook
  - Calibrate cost function
  - Include socio-demographic attributes

Thanks for your attention!

Questions? Comments?

The following people contributed to the work presented in this talk:

- Gunnar Flötteröd
- Matthias Kowald
- Kai Nagel
- Kay W. Axhausen

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